

# Topology (*Topoloji*)

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**Problem 1.** Find subsets  $A$  and  $B$  of  $\mathbb{R}$  such that

$$\overline{A} \cap \overline{B} \neq \emptyset, \quad \overline{A \cap B} = \emptyset,$$

where  $\overline{X}$  is the closure of  $X$  in the usual topology.

**Problem 2.** Is the function  $d$  given by

$$d(\mathbf{x}, \mathbf{y}) = |x_0y_0| + |x_1y_1|$$

a metric on  $\mathbb{R}^2$ ?

**Problem 3.** Show that the metrics  $d_0$  and  $d_1$  given by

$$d_0(\mathbf{x}, \mathbf{y}) = |x_0 - y_0| + |x_1 - y_1|,$$
$$d_1(\mathbf{x}, \mathbf{y}) = \sqrt{(x_0 - y_0)^2 + (x_1 - y_1)^2}$$

define the same topology on  $\mathbb{R}^2$ . (You may assume that  $d_0$  and  $d_1$  are indeed metrics.)

**Problem 4.** In the Tychonoff topology on  $\mathcal{P}(\omega)$  are the following sets open, closed, both, or neither? Explain briefly.

(a)  $\{X \subseteq \omega : 16 \in X \wedge 17 \notin X\}$

(b)  $\{X \subseteq \omega : \forall y (y \in \omega \Rightarrow 2y \in X)\}$

(c)  $\{X \subseteq \omega : \forall y (y \in X \Rightarrow y + 1 \in X)\}$