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MSGSÜ, MAT 221, Sınav

10 Kasım 2014, Saat 13:00

Öğretmen: David Pierce

Yönergeler: Sınavda 4 sayfada 5 soru var. *Sadece 4 soruyu cevaplayın.* Lütfen dikkat ederek yazın. İngilizceyi veya Türkçeyi kullanabilirsiniz.

As in class, $\mathbb{N}$ is the set $\{1, 2, 3, \dots\}$ of **natural numbers,** and x' is the **successor** of x, so $1' = 2$, $2' = 3$, and so on. We also let $\omega = \{0\} \cup \mathbb{N}$ and $0' = 1$.

Problem 1. For a given value of n in $\mathbb{N}$, let $\bar{x}$ denote the congruence-class of x *modulo* n, and let $\mathbb{Z}_n = \{\bar{x} \colon x \in \mathbb{N}\} = \{\bar{1}, \dots, \bar{n}\}$. If $\overline{x'} = \overline{y'}$, then $\bar{x} = \bar{y}$. Therefore we can define $\bar{x}\,' = \overline{x'}$. The structure $(\mathbb{Z}_n, \bar{1}, ')$ allows proofs by induction. We have shown that addition and multiplication on $\mathbb{Z}_n$ can be defined recursively by

$$\bar{x} + \bar{1} = \bar{x}\,', \qquad \bar{x} + \bar{y}\,' = (\bar{x} + \bar{y})\,',$$
$$\bar{x} \cdot \bar{1} = \bar{x}, \qquad \bar{x} \cdot \bar{y}\,' = \bar{x} \cdot \bar{y} + \bar{x}.$$

(a) If $n = 6$, show that there is an operation on $\mathbb{Z}_n$ given by

$$\bar{x}^{\bar{1}} = \bar{x}, \qquad \bar{x}^{\bar{y}'} = \bar{x}^{\bar{y}} \cdot \bar{x}. \qquad (*)$$

It is enough to fill out the table

$\bar{x}^{\bar{y}}$		y: 1	2	3	4	5	6
x	1						
	2						
	3						
	4						
	5						
	6						

.

(b) If $n = 3$, show that there is *no* operation on $\mathbb{Z}_n$ as in $(*)$.

Problem 2. Using only the recursive definition of addition on $\mathbb{N}$ and induction, prove that addition is associative.

Problem 3. We know $2 \cdot \sum_{k=1}^{n} k = n \cdot (n+1)$. For all n in $\mathbb{N}$, prove $\left(\sum_{k=1}^{n} k\right)^2 = \sum_{k=1}^{n} k^3$.

Problem 4. We can define the so-called binomial coefficients recursively by

$$\binom{0}{0} = 1, \qquad \binom{0}{k+1} = 0, \qquad \binom{n+1}{0} = 1, \qquad \binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}.$$

Using only this definition, and induction, show that, for all n in ω, $\displaystyle\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

Problem 5. Let d be the greatest common divisor of 385 and 168.

(a) Find d.

(b) Find a solution from $\mathbb{N}$ of one of the equations

$$385x = 168y + d, \qquad\qquad 168x = 365y + d.$$