Ekmatris ve ters

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Problem. $A = \begin{pmatrix} 1 & 3 & -3 \\ 4 & -1 & -1 \\ -2 & 1 & -2 \end{pmatrix}$ ise $\operatorname{Ek} A$ ve A^{-1} bulun.

Çözüm. İki yöntemden biri kullanılabilir.

1. Önce tanımından ekmatris hesaplanabilir:

$$\begin{aligned}\operatorname{Ek} A &= \operatorname{Ek}\begin{pmatrix} 1 & 3 & -3 \\ 4 & -1 & -1 \\ -2 & 1 & -2 \end{pmatrix}\\ &= \begin{pmatrix} \det\begin{pmatrix}-1 & -1\\ 1 & -2\end{pmatrix} & -\det\begin{pmatrix}3 & -3\\ 1 & -2\end{pmatrix} & \det\begin{pmatrix}3 & -3\\ -1 & -1\end{pmatrix} \\ -\det\begin{pmatrix}4 & -1\\ -2 & -2\end{pmatrix} & \det\begin{pmatrix}1 & -3\\ -2 & -2\end{pmatrix} & -\det\begin{pmatrix}1 & -3\\ 4 & -1\end{pmatrix} \\ \det\begin{pmatrix}4 & -1\\ -2 & 1\end{pmatrix} & -\det\begin{pmatrix}1 & 3\\ -2 & 1\end{pmatrix} & \det\begin{pmatrix}1 & 3\\ 4 & -1\end{pmatrix}\end{pmatrix} = \begin{pmatrix} 3 & 3 & -6 \\ 10 & -8 & -11 \\ 2 & -7 & -13 \end{pmatrix}.\end{aligned}$$

Kontrol ederiz:

$$\begin{aligned}\begin{pmatrix} 3 & 3 & -6 \\ 10 & -8 & -11 \\ 2 & -7 & -13 \end{pmatrix} \cdot A &= \begin{pmatrix} 3 & 3 & -6 \\ 10 & -8 & -11 \\ 2 & -7 & -13 \end{pmatrix}\begin{pmatrix} 1 & 3 & -3 \\ 4 & -1 & -1 \\ -2 & 1 & -2 \end{pmatrix}\\ &= \begin{pmatrix} 3+12+12 & 9-3-6 & -9-3+12 \\ 10-32+22 & 30+8-11 & -30+8+22 \\ 2-28+26 & 6+7-13 & -6+7+26 \end{pmatrix} = \begin{pmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{pmatrix}.\end{aligned}$$

Sonuç olarak $\det A = 27$, dolayısıyla

$$A^{-1} = \frac{1}{27}\operatorname{Ek} A = \begin{pmatrix} 3/27 & 3/27 & -6/27 \\ 10/27 & -8/27 & -11/27 \\ 2/27 & -7/27 & -13/27 \end{pmatrix}.$$

2. Gauss–Jordan indirgemesiyle

$$\left(\ A\ \middle|\ I\ \right) = \begin{pmatrix} 1 & 3 & -3 & 1 & 0 & 0 \\ 4 & -1 & -1 & 0 & 1 & 0 \\ -2 & 1 & -2 & 0 & 0 & 1 \end{pmatrix} \xrightarrow[2R_1+R_3]{-4R_1+R_2} \begin{pmatrix} 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & -13 & 11 & -4 & 1 & 0 \\ 0 & 7 & -8 & 2 & 0 & 1 \end{pmatrix} \xrightarrow{-\frac{1}{13}R_2}$$

$$\begin{pmatrix} 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & 1 & -11/13 & 4/13 & -1/13 & 0 \\ 0 & 7 & -8 & 2 & 0 & 1 \end{pmatrix} \xrightarrow{-7R_2+R_3} \begin{pmatrix} 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & 1 & -11/13 & 4/13 & -1/13 & 0 \\ 0 & 0 & -27/13 & -2/13 & 7/13 & 1 \end{pmatrix} \xrightarrow{-\frac{13}{27}R_3}$$

$$\begin{pmatrix} 1 & 3 & -3 & 1 & 0 & 0 \\ 0 & 1 & -11/13 & 4/13 & -1/13 & 0 \\ 0 & 0 & 1 & 2/27 & -7/27 & -13/27 \end{pmatrix} \xrightarrow[3R_3+R_1]{\frac{11}{13}R_3+R_2} \begin{pmatrix} 1 & 3 & 0 & 33/27 & -21/27 & -39/27 \\ 0 & 1 & 0 & 10/27 & -8/27 & -11/27 \\ 0 & 0 & 1 & 2/27 & -7/27 & -13/27 \end{pmatrix} \xrightarrow{-3R_2+R_1}$$

$$\begin{pmatrix} 1 & 0 & 0 & 3/27 & 3/27 & -6/27 \\ 0 & 1 & 0 & 10/27 & -8/27 & -11/27 \\ 0 & 0 & 1 & 2/27 & -7/27 & -13/27 \end{pmatrix}.$$

Tekrar kontrol ederiz:

$$\begin{pmatrix} 3/27 & 3/27 & -6/27 \\ 10/27 & -8/27 & -11/27 \\ 2/27 & -7/27 & -13/27 \end{pmatrix} \begin{pmatrix} 1 & 3 & -3 \\ 4 & -1 & -1 \\ -2 & 1 & -2 \end{pmatrix} = I,$$

dolayısıyla

$$A^{-1} = \begin{pmatrix} 3/27 & 3/27 & -6/27 \\ 10/27 & -8/27 & -11/27 \\ 2/27 & -7/27 & -13/27 \end{pmatrix}.$$

Kullandığımız satır işlemlerden $\det A = \left(-\frac{27}{13}\right) \cdot (-13) = 27$, dolayısıyla

$$\operatorname{Ek} A = \det A \cdot A^{-1} = \begin{pmatrix} 3 & 3 & -6 \\ 10 & -8 & -11 \\ 2 & -7 & -13 \end{pmatrix}.$$

Yukarıdaki problemin kaynağı: Koç & Esin, *Doğrusal Cebir* (Ankara: 2014), sayfa 127.

Problem. Herhangi kare matris yazıp ekmatrisini ve (varsa) tersini bulun!