Pappus of Alexandria

Book 7 of the *Collection*

Part 1. Introduction, Text, and Translation

Edited
With Translation and Commentary by
Alexander Jones

In Two Parts
With 308 Figures

Springer Science+Business Media, LLC

Alexander Jones
6425 Adera Street
Vancouver, British Columbia V6M 3J7
Canada

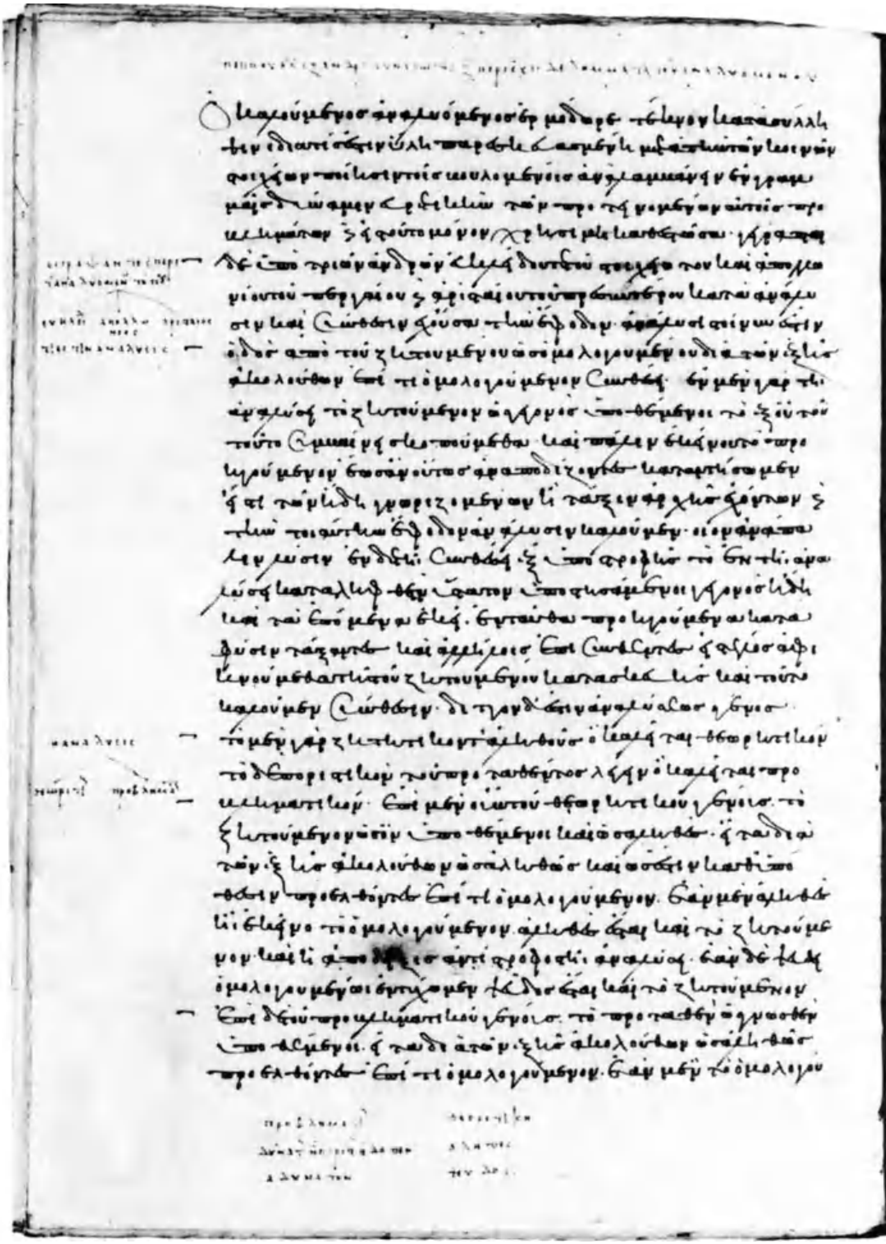
AMS Classification: 01A20

Library of Congress Cataloging in Publication Data
Pappus, of Alexandria.
Book 7 of the Collection.
(Sources in the history of mathematics and physical sciences ; 8)
English and Greek.
Revision of thesis (Ph. D.)—Brown University, 1985.
Bibliography: p.
Includes indexes.
Contents: pt. 1. Introduction, text, and translation—
pt. 2. Commentary, index, and figures.
1. Mathematics, Greek. I. Jones, Alexander.
II. Title. III. Title: Book seven of the Collection.
IV. Series.
QA22.P3713 1986 516.2 85-27788

Originally published by Springer-Verlag New York Inc. in 1986
Softcover reprint of the hardcover 1st edition 1986

9 8 7 6 5 4 3 2 1

ISBN 978-1-4612-9355-2 ISBN 978-1-4612-4908-5 (eBook)
DOI 10.1007/978-1-4612-4908-5



Vat. gr. 218, f. 118^{v}
Photo: Biblioteca Vaticana

Contents

Part 1

INTRODUCTION

Pappus and the *Collection*

§1. Biographical Data 1
§2. Works 3
§3. Integrity and Composition of the *Collection* 15
§4. Interpolations 18
§5. The Marginalia 20
§6. Early references 21
§7. Foul Papers 24
§8. The Proarchetypes 26
§9. Description of the Vaticanus 30
§10. Disturbances in the Vaticanus 33
§11. Byzantine Notices 36
§12. Witelo 42
§13. The Papal Inventories 45
§14. False Leads 48
§15. The Vaticanus in Florence and Rome 52
§16. The Recentiores 56
§17. Printed Editions 62

Introduction to Book 7

§18. The Domain of Analysis 66
§19. The Purpose and Plan of Book 7 70
§20. Mathematics in Book 7 71

Editorial Principles 75

Abbreviations Used in the Apparatus 79

TEXT AND TRANSLATION 82

Part 2

NOTES 377

ESSAYS ON THE LOST WORKS

A. The Minor Works of Apollonius

§1. Introduction 510
§2. The Cutting off of a Ratio 510

§3. The Cutting off of an Area 513
§4. The Determinate Section 514
§5. General Remarks on the 'Triple Section' 522
§6. The Neuses 527
§7. The Tangencies 534
§8. The Plane Loci 539
B. Euclid's Porisms
§1. Documents 547
§2. The Definitions 549
§3. The First Porism 554
§4. The 'Hyptios' Porisms 556
§5. Rectilinear Configurations 560
§6. Areas and Ranges 563
§7. Porisms on Circles 564
§8. Applications and Purpose 567
§9. Historical Note 569
C. The Loci of Aristaeus, Euclid, and Eratosthenes
§1. Introduction. 573
§2. Documents on Aristaeus 577
§3. Fragments from Pappus 582
§4. Book 3 of Apollonius's Conics 585
§5. The Four Line Locus 587
§6. Euclid's Loci on Surfaces 591
§7. The Derivation of Curvilinear Loci 595
§8. Eratosthenes's Loci with Respect to Means 599

APPENDICES
1. The Scholia to Book 7 600
2. The Book of Lemmas by "Aqātun" 603
3. Selections from the *Cutting off of a Ratio* 606

APPARATUS TO THE FIGURES 620

BIBLIOGRAPHIC ABBREVIATIONS 628

INDEX 647

GREEK INDEX 657

FIGURES 667

(193) Porisms, (Books) 1, 2, 3.

From Book 1:

1. *(Prop. 127 a– e)* For the first porism.

Let there be figure ABΓΔEZH, and, as is AZ to ZH, so let AΔ be to ΔΓ, and let ΘK be joined. That ΘK is parallel to AΓ.

Let ZΛ be drawn through Z parallel to BΔ.[1] Then since, as is AZ to ZH, so is AΔ to ΔΓ,[2] by inversion and *componendo* and *alternando* as is ΔA to AZ, that is, in parallels, as is BA to AΛ,[4] so is ΓA to AH.[3] Hence ΛH is parallel to BΓ.[5] Therefore as is EB to BΛ, so is <EΘ to ΘH.[6] But also as is EB to BΛ, so>, in parallels, is EK to KZ.[7] Thus as is EK to KZ, so is EΘ to ΘH.[8] ΘK is therefore parallel to AΓ.[9]

(194) *(Prop. 127 a– e)* By compound ratios, as follows:

Since, as is AZ to ZH, so is AΔ to ΔΓ,[1] by inversion, as is HZ to ZA, so is ΓΔ to ΔA.[2] *Componendo* and *alternando* and *convertendo,* as is AΔ to ΔZ, so is AΓ to ΓH.[3] But the (ratio) of AΔ to ΔZ is compounded out of that of <AB to BE and that of EK to KZ[4] *(see commentary),* while that of AΓ to ΓH (is compounded) out of that of> AB to BE and that of EΘ to ΘH[5] *(see commentary).* Therefore the ratio compounded out of that which AB has to BE and EK has to KZ is the same as the (ratio) compounded out of that which AB has to BE and EΘ has to ΘH.[6] And let the ratio of AB to BE be removed in common. Then there remains the ratio of EK to KZ equal to the ratio of EΘ to ΘH.[7] Thus ΘK is parallel to AΓ.[8]

(195) *(Prop. 128)* For the second porism.

Figure ABΓΔEZH. Let AZ be parallel to ΔB, and as is AE to EZ, so let ΓH be to HZ. That the (line) through Θ, K, Z is straight.

Let HΛ be drawn through H parallel to ΔE,[1] and let ΘK be joined and produced to Λ. Then since, as is AE to EZ, so is ΓH to HZ,[2] *alternando* as

(193) ΠΟΡΙΣΜΑΤΩΝ Α Β Γ

τοῦ πρώτου.

αʹ. εἰς τὸ πρῶτον πόρισμα.

ἔστω καταγραφὴ ἡ ΑΒΓΔΕΖΗ, καὶ ἔστω ὡς ΑΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, καὶ ἐπεζεύχθω ἡ ΘΚ. ὅτι παράλληλός ἐστιν ἡ ΘΚ τῆι ΑΓ. ἤχθω διὰ τοῦ Ζ τῆι ΒΔ παράλληλος ἡ ΖΛ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΑΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, ἀνάπαλιν καὶ συνθέντι καὶ ἐναλλάξ ἐστιν ὡς ἡ ΔΑ πρὸς τὴν ΑΖ, τουτέστιν ἐν παραλλήλωι, ὡς ἡ ΒΑ πρὸς τὴν ΑΛ, οὕτως ἡ ΓΑ πρὸς τὴν ΑΗ. παράλληλος ἄρα ἐστὶν ἡ ΛΗ τῆι ΒΓ. ἔστιν ἄρα ὡς ἡ ΕΒ πρὸς τὴν ΒΛ, οὕτως ἡ <ΕΘ πρὸς τὴν ΘΗ. ἔστιν δὲ καὶ ὡς ἡ ΕΒ πρὸς τὴν ΒΛ, οὕτως> ἐν παραλλήλωι ἡ ΕΚ πρὸς τὴν ΚΖ. καὶ ὡς ἄρα ἡ ΕΚ πρὸς τὴν ΚΖ, οὕτως ἐστὶν ἡ ΕΘ πρὸς τὴν ΘΗ. παράλληλος ἄρα ἐστὶν ἡ ΘΚ τῆι ΑΓ.

(194) διὰ δὲ τοῦ συνημμένου οὕτως. ἐπεί ἐστιν ὡς ἡ ΑΖ 868 πρὸς τὴν ΖΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, ἀνάπαλίν ἐστιν ὡς ἡ ΗΖ πρὸς τὴν ΖΑ, οὕτως ἡ ΓΔ πρὸς τὴν ΔΑ. συνθέντι καὶ ἐναλλὰξ καὶ ἀναστρέψαντί ἐστιν ὡς ΑΔ πρὸς τὴν ΔΖ, |οὕτως ἡ ΑΓ πρὸς |158v τὴν ΓΗ. ἀλλ' ὁ μὲν τῆς ΑΔ πρὸς τὴν ΔΖ συνῆπται ἔκ τε τοῦ τῆς <ΑΒ πρὸς τὴν ΒΕ καὶ τοῦ τῆς ΕΚ πρὸς τὴν ΚΖ, ὁ δὲ τῆς ΑΓ πρὸς τὴν ΓΗ ἔκ τε τοῦ τῆς> ΑΒ πρὸς τὴν ΒΕ καὶ τοῦ τῆς ΕΘ πρὸς τὴν ΘΗ. ὁ ἄρα συνημμένος λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΑΒ πρὸς τὴν ΒΕ καὶ ἡ ΕΚ πρὸς τὴν ΚΖ ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ ὃν ἔχει ἡ ΑΒ πρὸς τὴν ΒΕ καὶ ἡ ΕΘ πρὸς τὴν ΘΗ. καὶ κοινὸς ἐκκεκρούσθω ὁ τῆς ΑΒ <πρὸς> τὴν ΒΕ λόγος. λοιπὸν ἄρα ὁ τῆς ΕΚ πρὸς τὴν ΚΖ λόγος <ὁ αὐτός> ἐστιν τῶι τῆς ΕΘ πρὸς τὴν ΘΗ λόγωι. <παράλληλος> ἄρα ἐστὶν ἡ ΘΚ τῆι ΑΓ.

(195) εἰς τὸ δεύτερον πόρισμα.

καταγραφὴ ἡ ΑΒΓΔΕΖΗΘ. ἔστω δὲ παράλληλος ἡ ΑΖ τῆι ΔΒ, ὡς δὲ ἡ ΑΕ πρὸς τὴν ΕΖ, οὕτως ἡ ΓΗ πρὸς τὴν ΗΖ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Θ, Κ, Ζ. ἤχθω διὰ τοῦ Η παράλληλος τῆι ΔΕ ἡ ΗΛ, καὶ ἐπιζευχθεῖσα ἡ ΘΚ ἐκβεβλήσθω ἐπὶ τὸ Λ. ἐπεὶ οὖν

‖ 4 αʹ mg A ‖ 5 post ὡς add ἡ Ge (BS) ‖ 11 ΛΗ] ΑΗ A^1 Λ supra A^2 ‖ 12 ἡ (ΕΘ) del Hu ἐν παραλλήλωι ἡ Heiberg$_3$ | ΕΘ – οὕτως add Heiberg$_3$ ‖ 13 ΕΚ πρὸς τὴν ΚΖ A, post quae add καὶ ἡ ΕΘ πρὸς τὴν ΘΗ Co ‖ 17 ΗΖ Co ΝΖ A ‖ 19 post ὡς add ἡ Ge (BS) | ΔΖ Co ΑΖ A ‖ 21 ΑΒ – τῆς (ΑΒ) add Heiberg$_3$ ‖ 26 κοινὸς] signum quasi κ° A | πρὸς add Ge (BS) ‖ 27 ὁ αὐτός add Co ‖ 28 λόγωι. παράλληλος] λόγος A παράλληλος Co ‖ 32 παράλληλος τῆι] παρὰ τὴν A ‖ 33 ἐπιζευχθεῖσα Hu ἐπεζεύχθω A | post τὸ Λ spatium litterarum fere septem relictum A is AE to ΓH, so is EZ to ZH.[3] But as is AE to ΓH, so is EΘ to HΛ,[4] and *alternando*, because there are two by two (parallel lines). Therefore as is EZ to ZH, so is EΘ to HΛ.[5] And EΘ is parallel to HΛ.[6] Thus (VI, 32) the (line) through Θ, Λ, Z is straight.[7] Q.E.D.

(196) *(Prop. 129 a– h)* Let two straight lines ΘE, ΘΔ be drawn onto three straight lines AB, ΓA, ΔA. That, as is the rectangle contained by ΘE, HZ to the rectangle contained by ΘH, ZE, so is the rectangle contained by ΘB, ΔΓ to the rectangle contained by ΘΔ, BΓ.

Let KΛ be drawn through Θ parallel to ZΓA,[1] and let ΔA and AB intersect it at points K and Λ; and (let there be drawn) ΛM through Λ parallel to ΔA,[2] and let it intersect EΘ at M. Then since, as is EZ to ZA, so is EΘ to ΘΛ,[3] while as is AZ to ZH, so is ΘΛ to ΘM,[5] because ΘK is to ΘH also (as is ΘΛ to ΘM) in parallels,[4] therefore *ex aequali* as is EZ to ZH, so is EΘ to ΘM.[6] Therefore the rectangle contained by ΘE, HZ equals the rectangle contained by EZ, ΘM.[7] But (let) the rectangle contained by EZ, ΘH (be) another arbitrary quantity. Then as is the rectangle contained by EΘ, HZ to the rectangle contained by EZ, HΘ, so is the rectangle contained by EZ, ΘM to the rectangle contained by EZ, HΘ,[8] that is ΘM to ΘH,[9] that is ΛΘ to ΘK.[10] By the same argument also as is KΘ to ΘΛ, so is the rectangle contained by ΘΔ, BΓ to the rectangle contained by ΘB, ΓΔ.[11] By inversion, therefore, as is ΛΘ to ΘK, so is the rectangle contained by ΘB, ΓΔ to the rectangle contained by ΘΔ, BΓ.[12] But as is ΛΘ to ΘK, so the rectangle contained by EΘ, HZ was shown to be to the rectangle contained by EZ, HΘ. And thus as is the rectangle contained by EΘ, HZ to the rectangle contained by EZ, HΘ, so is the rectangle contained by ΘB, ΓΔ to the rectangle contained by ΘΔ, BΓ.[13]

(197) *(Prop. 129 a– h)* By means of compounded ratios, as follows:

Since the ratio of the rectangle contained by ΘE, HZ to the rectangle contained by ΘH, ZE is compounded out of that which ΘE has to EZ and that which ZH has to HΘ,[1] and as is ΘE to EZ, so is ΘΛ to ZA,[2] while as is ZH to HΘ, so is ZA to ΘK,[3] therefore the (ratio of the) rectangle contained by ΘE, HZ to the rectangle contained by ΘH, EZ is compounded out of that which ΘΛ has to ZA and that which ZA has to ΘK.[4] But the (ratio) compounded out of that which ΘΛ has to ZA and that which ZA has to ΘK is the same as that of ΘΛ to ΘK.[5] Hence as is the rectangle contained by ΘE, HZ to the rectangle contained by ΘH, ZE, so is ΘΛ to ΘK.[6] For the same reasons also as is the rectangle contained by ΘΔ, BΓ to ἔστιν ὡς ἡ ΑΕ πρὸς τὴν ΕΖ, οὕτως ἡ ΓΗ πρὸς τὴν ΗΖ, ἐναλλὰξ ἔστιν ὡς ἡ ΑΕ πρὸς τὴν ΓΗ, οὕτως ἡ ΕΖ πρὸς τὴν ΖΗ. ὡς δὲ ἡ ΑΕ πρὸς τὴν ΓΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΗΛ, καὶ ἐναλλάξ, διὰ τὸ εἶναι δύο παρὰ δύο· καὶ ὡς ἄρα ἡ ΕΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΗΛ. καὶ ἔστιν παράλληλος ἡ ΕΘ τῆι ΗΛ· εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Θ, Λ, Ζ. ὅ(περ):— 870

(196) εἰς τρεῖς εὐθείας τὰς ΑΒ, ΓΑ, ΔΑ διήχθωσαν δύο εὐθεῖαι αἱ ΘΕ, ΘΔ. ὅτι ἐστὶν ὡς τὸ ὑπὸ ΘΕ, ΗΖ πρὸς τὸ ὑπὸ ΘΗ, ΖΕ, οὕτως τὸ ὑπὸ ΘΒ, ΔΓ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ. ἤχθω διὰ μὲν τοῦ Θ τῆι ΖΓΑ παράλληλος ἡ ΚΛ, καὶ αἱ ΔΑ, ΑΒ συμπιπτέτωσαν αὐτῆι κατὰ τὰ Κ, Λ σημεῖα· διὰ δὲ τοῦ Λ τῆι ΔΑ παράλληλος ἡ ΛΜ, καὶ συμπιπτέτω τῆι ΕΘ ἐπὶ τὸ Μ. ἐπεὶ οὖν ἐστιν ὡς μὲν ἡ ΕΖ πρὸς τὴν ΖΑ, οὕτως ἡ ΕΘ πρὸς τὴν ΘΛ, ὡς δὲ ἡ ΑΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΘΛ πρὸς τὴν ΘΜ (καὶ γὰρ ἡ ΘΚ πρὸς τὴν ΘΗ ἐν παραλλήλωι) δι' ἴσου ἄρα ἐστὶν ὡς ἡ ΕΖ πρὸς ΖΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΘΜ· τὸ ἄρα ὑπὸ |τῶν ΘΕ, ΗΖ ἴσον ἐστὶν τῶι ὑπὸ τῶν |159 ΕΖ, ΘΜ. ἄλλο δέ τι τυχὸν τὸ ὑπὸ τῶν ΕΖ, ΘΗ. ἔστιν ἄρα ὡς τὸ ὑπὸ τῶν ΕΘ, ΗΖ πρὸς τὸ ὑπὸ τῶν ΕΖ, ΗΘ, οὕτως τὸ ὑπὸ ΕΖ, ΘΜ πρὸς τὸ ὑπὸ ΕΖ, ΗΘ, τουτέστιν ἡ ΘΜ πρὸς ΘΗ, τουτέστιν ἡ ΛΘ πρὸς τὴν ΘΚ· κατὰ τὰ αὐτὰ καὶ ὡς ἡ ΚΘ πρὸς τὴν ΘΛ, οὕτως τὸ ὑπὸ ΘΔ, ΒΓ πρὸς τὸ ὑπὸ ΘΒ, ΓΔ. ἀνάπαλιν ἄρα γίνεται ὡς ἡ ΛΘ πρὸς τὴν ΘΚ, οὕτως τὸ ὑπὸ ΘΒ, ΓΔ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ. ὡς δὲ ἡ 872 ΛΘ πρὸς τὴν ΘΚ, οὕτως ἐδείχθη τὸ ὑπὸ ΕΘ, ΗΖ πρὸς τὸ ὑπὸ ΕΖ, ΗΘ. καὶ ὡς ἄρα τὸ ὑπὸ ΕΘ, ΗΖ πρὸς τὸ <ὑπὸ> ΕΖ, ΗΘ, οὕτως τὸ ὑπὸ ΘΒ, ΓΔ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ.

(197) διὰ δὲ τοῦ συνημμένου οὕτως. ἐπεὶ <ὁ> τοῦ ὑπὸ ΘΕ, ΗΖ πρὸς τὸ ὑπὸ ΘΗ, ΖΕ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΘΕ πρὸς τὴν ΕΖ καὶ τοῦ ὃν ἔχει ἡ ΖΗ πρὸς τὴν ΗΘ, καὶ ἔστιν ὡς μὲν ἡ ΘΕ πρὸς τὴν ΕΖ, οὕτως ἡ ΘΛ πρὸς τὴν ΖΑ, ὡς δὲ ἡ ΖΗ πρὸς τὴν ΗΘ, οὕτως ἡ ΖΑ πρὸς τὴν ΘΚ, τὸ ἄρα ὑπὸ ΘΕ, ΗΖ πρὸς τὸ ὑπὸ ΘΗ, ΕΖ συνῆπται ἔκ τε τοῦ ὃν ἔχει ἡ ΘΛ πρὸς τὴν ΖΑ καὶ τοῦ ὃν ἔχει ἡ ΖΑ πρὸς τὴν ΘΚ· ὁ δὲ συνημμένος ἔκ τε τοῦ τῆς ΘΛ πρὸς τὴν ΖΑ καὶ τοῦ τῆς ΖΑ πρὸς τὴν ΘΚ ὁ αὐτός ἐστιν τῶι τῆς ΘΛ πρὸς τὴν ΘΚ· ἔστιν ἄρα ὡς τὸ ὑπὸ ΘΕ, ΗΖ πρὸς τὸ ὑπὸ

‖ 3 καὶ ἐναλλὰξ in ras. A, post διὰ τὸ εἶναι δύο παρὰ δύο transp. Hu, quae omnia del Heiberg$_3$ ‖ 6 post Θ add Κ Ge (S) | post Ζ add τουτέστιν ἡ διὰ τῶν Θ, Κ, Ζ Hu ‖ 11 Δ(Α) in ras. A | pro ἡ ΛΜ καὶ coni. διαχθεῖσα ἡ ΛΜ Hu app ‖ 17 pro τυχὸν coni. ἔχομεν Hu app ‖ 21 ἀνάπαλιν Co ἀνάλογον A ‖ 24 ὑπὸ (ΕΖ) add Ge (S) ‖ 26 ὁ add Heiberg$_3$ ‖ 28 πρὸς τὴν ΕΖ — ΖΗ bis A corr Co the rectangle contained by ΘB, ΓΔ, so is ΘK to ΘΛ.[7] And by inversion, as is the rectangle contained by ΘB, ΓΔ to the rectangle contained by ΘΔ, BΓ, so is ΛΘ to ΘK.[8] But as is the rectangle contained by ΘE, ZH to the rectangle contained by ΘH, ZE, <so was ΘΛ to ΘK. Thus, as is the rectangle contained by ΘE, ZH to the rectangle contained by ΘH, ZE,> so is the rectangle contained by ΘB, ΓΔ to the rectangle contained by ΘΔ, BΓ.[9]

(198) *(Prop. 130 a – h)* Figure ABΓΔEZHΘKΛ. As is the rectangle contained by AZ, BΓ to the rectangle contained by AB, ΓZ, so let the rectangle contained by AZ, ΔE be to the rectangle contained by AΔ, EZ. That the (line) through points Θ, H, Z is straight.

Since, as is the rectangle contained by AZ, BΓ to the rectangle contained by AB, ΓZ, so is the rectangle contained by AZ, ΔE to the rectangle contained by AΔ, EZ,[1] *alternando* as is the rectangle contained by AZ, BΓ to the rectangle contained by AZ, ΔE, that is as is BΓ to ΔE,[3] so is the rectangle contained by AB, ΓZ to the rectangle contained by AΔ, EZ.[2] But the ratio of BΓ to ΔE is compounded, if KM is drawn through K parallel to AZ,[4] out of that which BΓ has to KN and that which KN has to KM, and as well that which KM has to ΔE.[5] But the (ratio) of the rectangle contained by AB, ΓZ to the rectangle contained by AΔ, EZ is compounded out of that of BA to AΔ and that of ΓZ to ZE.[6] Let the (ratio) of BA to AΔ be removed in common, this being the same as that of NK to KM.[7] Then the remaining (ratio) of ΓZ to ZE is compounded out of that of BΓ to KN, that is that of ΘΓ to KΘ,[9] and that of KM to ΔE, that is that of KH to HE.[10] [8] Thus the (line) through Θ, H, Z is straight.

For if I draw EΞ through E parallel to ΘΓ,[11] and ΘH is joined and produced to Ξ, the ratio of KH to HE is the same as that of KΘ to EΞ,[12] while the (ratio) compounded out of that of ΓΘ to ΘK and that of ΘK to EΞ is converted into the ratio of ΘΓ to EΞ,[13] and the ratio of ΓZ to ZE is the same as that of ΓΘ to EΞ.[14] Because ΓΘ is (therefore) parallel to EΞ,[15] the (line) through Θ, Ξ, Z is straight;[16] for that is obvious. Therefore the (line) through Θ, H, Z is also straight.[17]

(199) *(Prop. 131)* If there is figure ABΓΔEZHΘ, then as AΔ is to ΔΓ, so is AB to BΓ. So let AB be to BΓ as is AΔ to ΔΓ. That the (line) through A, H, Θ is straight.

ΘΗ, ΖΕ, οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ. διὰ ταὐτὰ καὶ ὡς τὸ ὑπὸ ΘΔ, ΒΓ πρὸς τὸ ὑπὸ ΘΒ, ΓΔ, οὕτως ἐστὶν ἡ ΘΚ πρὸς τὴν ΘΛ. καὶ ἀνάπαλίν ἐστιν ὡς τὸ ὑπὸ ΘΒ, ΓΔ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ, οὕτως ἡ ΛΘ πρὸς τὴν ΘΚ. ἦν δὲ καὶ ὡς τὸ ὑπὸ τῶν ΘΕ, ΖΗ πρὸς τὸ ὑπὸ ΘΗ, ΖΕ, <οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ. καὶ ὡς ἄρα τὸ ὑπὸ τῶν ΘΕ, ΖΗ πρὸς τὸ ὑπὸ ΘΗ, ΖΕ,> οὕτως τὸ ὑπὸ ΘΒ, ΓΔ πρὸς τὸ ὑπὸ ΘΔ, ΒΓ.

(198) |καταγραφὴ ἡ ΑΒΓΔΕΖΗΘΚΛ. ἔστω δὲ ὡς τὸ ὑπὸ ΑΖ, ΒΓ |159v πρὸς τὸ ὑπὸ ΑΒ, ΓΖ, οὕτως τὸ ὑπὸ ΑΖ, ΔΕ πρὸς τὸ ὑπὸ ΑΔ, ΕΖ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Θ, Η, Ζ σημείων. ἐπεί ἐστιν ὡς τὸ ὑπὸ ΑΖ, ΒΓ πρὸς τὸ ὑπὸ ΑΒ, ΓΖ, οὕτως τὸ ὑπὸ ΑΖ, ΔΕ πρὸς τὸ ὑπὸ ΑΔ, ΕΖ, ἐναλλάξ ἐστιν ὡς τὸ ὑπὸ ΑΖ, ΒΓ πρὸς τὸ ὑπὸ ΑΖ, ΔΕ, 874 τουτέστιν ὡς ἡ ΒΓ πρὸς τὴν ΔΕ, οὕτως τὸ ὑπὸ ΑΒ, ΓΖ πρὸς τὸ ὑπὸ ΑΔ, ΕΖ. ἀλλ' ὁ μὲν τῆς ΒΓ πρὸς τὴν ΔΕ συνῆπται λόγος, ἐὰν διὰ τοῦ Κ τῆι ΑΖ παράλληλος ἀχθῆι ἡ ΚΜ, ἔκ τε τοῦ τῆς ΒΓ πρὸς ΚΝ καὶ <τοῦ> τῆς ΚΝ πρὸς ΚΜ, καὶ ἔτι τοῦ τῆς ΚΜ πρὸς ΔΕ. ὁ δὲ τοῦ ὑπὸ ΑΒ, ΓΖ πρὸς τὸ ὑπὸ ΑΔ, ΕΖ συνῆπται ἔκ τε τοῦ τῆς ΒΑ πρὸς ΑΔ καὶ τοῦ τῆς ΓΖ πρὸς τὴν ΖΕ. κοινὸς ἐκκεκρούσθω ὁ τῆς ΒΑ πρὸς ΑΔ, ὁ αὐτὸς ὢν τῶι τῆς ΝΚ πρὸς ΚΜ. λοιπὸς ἄρα ὁ τῆς ΓΖ πρὸς τὴν ΖΕ συνῆπται ἔκ τε τοῦ τῆς ΒΓ πρὸς τὴν ΚΝ, τουτέστιν τοῦ τῆς ΘΓ πρὸς τὴν ΚΘ, καὶ τοῦ τῆς ΚΜ πρὸς τὴν ΔΕ, τουτέστιν <τοῦ> τῆς ΚΗ πρὸς τὴν ΗΕ. εὐθεῖα ἄρα ἡ διὰ τῶν Θ, Η, Ζ. ἐὰν γὰρ διὰ τοῦ Ε τῆι ΘΓ παράλληλον ἀγάγω τὴν ΕΞ, καὶ ἐπιζευχθεῖσα ἡ ΘΗ ἐκβληθῆι ἐπὶ τὸ Ξ, ὁ μὲν τῆς ΚΗ πρὸς τὴν ΗΕ λόγος ὁ αὐτός ἐστιν τῶι τῆς ΚΘ πρὸς τὴν ΕΞ, ὁ δὲ συνημμένος ἔκ τε τοῦ τῆς ΓΘ πρὸς τὴν ΘΚ καὶ τοῦ τῆς ΘΚ πρὸς τὴν ΕΞ μεταβάλλεται εἰς τὸν τῆς ΘΓ πρὸς ΕΞ λόγον, καὶ ὁ τῆς ΓΖ πρὸς ΖΕ λόγος ὁ αὐτὸς τῶι τῆς ΓΘ πρὸς τὴν ΕΞ. παραλλήλου οὔσης τῆς ΓΘ τῆι ΕΞ, εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Θ, Ξ, Ζ· τοῦτο γὰρ φανερόν. ὥστε καὶ ἡ διὰ τῶν Θ, Η, Ζ εὐθεῖά ἐστιν.

(199) |ἐὰν ἦι καταγραφὴ ἡ ΑΒΓΔΕΖΗΘ, γίνεται ὡς ἡ ΑΔ πρὸς |160 τὴν ΔΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΓ. ἔστω οὖν ὡς ἡ ΑΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΓ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Η, Θ. ἤχθω διὰ τοῦ Η τῆι ΑΔ παράλληλος ἡ ΚΛ. ἐπεὶ οὖν ἐστιν ὡς ἡ

‖ 4 ἦν Ge (S) τὴν A ‖ 5 οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ add Ge | καὶ ὡς ἄρα — (τῶν) ΘΗ, ΖΕ add Co (τῶν) del Ge ‖ 7 ΑΒΓΔΕΖΗΘΚΛ Co ΑΒΓΔΕΖΗΘΙΚΛ A ‖ 15 ΚΝ (καὶ) Co ΚΗ A | (ἔτι) τοῦ Co τὸ A ‖ 17 ΖΕ Co ΔΕ A | κοινὸς] κ° A ‖ 19 λοιπὸς Ge λοιπὸν A ‖ 21 τοῦ add Hu ‖ 22 Ζ Co Κ A | ΘΓ Co ΒΓ A ‖ 23 ΕΞ Co ΕΖ A | ἐπιζευχθεῖσα ἡ Hu ἐπιζευχθείσης τῆς A ‖ 26 μεταβάλλεται Hu μεταβαλλόμενος A | τὸν Ge (S) τὸ A ‖ 27 ΕΞ Co ΘΖ A

Let ΚΛ be drawn through Η parallel to ΑΔ.[1] Then since as is ΑΔ to ΔΓ, so is ΑΒ to ΒΓ,[2] while as is ΑΔ to ΔΓ, so is ΚΛ to ΛΗ,[3] and as is ΑΒ to ΒΓ, so is ΚΗ to ΗΜ,[4] therefore as is ΚΛ to ΛΗ, so is ΚΗ to ΗΜ.[5] And remainder ΗΛ is to remainder ΛΜ as is ΚΛ to ΛΗ,[6] that is as ΑΔ is to ΔΓ.[7] *Alternando* as is ΑΔ to ΗΛ, so is ΓΔ to ΛΜ,[8] that is ΔΘ to ΘΛ.[9] And ΗΛ is parallel to ΑΒ.[10] Hence the (line) through points Α, Η, Θ is straight;[11] for this is obvious.

(200) *(Prop. 132)* Again if there is a figure (ΑΒΓΔΕΖΗ), and ΔΖ is parallel to ΒΓ, then ΑΒ equals ΒΓ. So let it be equal. That (ΔΖ) is parallel (to ΒΓ).

But it is. For if, with ΕΒ drawn through, I make ΒΘ equal to ΗΒ,[1] and I join ΑΘ and ΘΓ, then there results a parallelogram ΑΘΓΗ,[2] and because of this, as is ΑΔ to ΔΕ, so is ΓΖ to ΖΕ.[4] For each of the foregoing (ratios) is the same as the ratio of ΘΗ to ΗΕ.[3] Thus (VI, 2) ΔΖ is parallel to ΑΓ.[5]

(201) *(Prop. 133)* Let there be a figure (ΑΒΓΔΕΖΗΘ), and let ΒΑ be a mean proportional between ΔΒ and ΒΓ. That ΖΗ is parallel to ΑΓ.

Let ΕΒ be produced, and let ΑΚ be drawn through Α parallel to straight line ΔΖ,[1] and let ΓΚ be joined. Then since as is ΓΒ to ΒΑ, so is ΑΒ to ΒΔ,[2] while as is ΑΒ to ΒΔ, so is ΚΒ to ΒΘ,[3] therefore as is ΓΒ to ΒΑ, so is ΚΒ to ΒΘ.[4] Hence ΑΘ is parallel to ΚΓ.[5] Therefore again, as is ΑΖ to ΖΕ, so is ΓΗ to ΗΕ;[7] for either of the foregoing ratios is the same as that of ΚΘ to ΕΘ.[6] Thus ΖΗ is parallel to ΑΔ.[8]

(202) *(Prop. 134)* Let there be an "altar" ΑΒΓΔΕΖΗ, and let ΔΕ be parallel to ΒΓ, and ΕΗ to ΒΖ. That ΔΖ too is parallel to ΓΗ.

Let ΒΕ, ΔΓ, and ΖΗ be joined. Then triangle ΔΒΕ equals triangle ΔΓΕ.[1] Let triangle ΔΑΕ be added in common. Then all triangle ΑΒΕ equals all triangle ΓΔΑ.[2] Again, since ΒΖ is parallel to ΕΗ,[3] triangle ΒΖΕ equals triangle ΒΖΗ.[4] Let triangle ΑΒΖ be subtracted in common. Then the remaining triangle ΑΒΕ equals the remaining triangle ΑΗΖ.[5] But

ΑΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΓ, ἀλλ' ὡς μὲν ἡ ΑΔ πρὸς 876
τὴν ΔΓ, οὕτως ἡ ΚΛ πρὸς τὴν ΛΗ, ὡς δὲ ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΚΗ πρὸς τὴν ΗΜ, καὶ ὡς ἄρα ἡ ΚΛ πρὸς τὴν ΛΗ, οὕτως ἡ ΚΗ πρὸς τὴν ΗΜ. καὶ λοιπὴ ἡ ΗΛ πρὸς λοιπὴν τὴν ΛΜ ἐστὶν ὡς ἡ ΚΛ πρὸς τὴν ΛΗ, τουτέστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΓ. ἐναλλάξ ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΗΛ, οὕτως ἡ ΓΔ πρὸς τὴν ΛΜ, τουτέστιν ἡ ΔΘ πρὸς ΘΛ. καὶ ἔστι παράλληλος ἡ ΗΛ τῆι ΑΒ. εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Α, Η, Θ σημείων· τοῦτο γὰρ φανερόν.

(200) πάλιν ἐὰν ἦι καταγραφή, καὶ παράλληλος ἡ ΔΖ τῆι ΒΓ, γίνεται ἴση ἡ ΑΒ τῆι ΒΓ. ἔστω οὖν ἴση. ὅτι παράλληλος. ἔστιν δέ. ἐὰν γὰρ διαχθείσης τῆς ΕΒ θῶ τῆι ΗΒ ἴσην τὴν ΒΘ, καὶ ἐπιζεύξω τὰς ΑΘ, ΘΓ, γίνεται παραλληλόγραμμον τὸ ΑΘΓΗ, καὶ διὰ τοῦτό ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΕ, οὕτως ἡ ΓΖ πρὸς τὴν ΖΕ. ἑκάτερος γὰρ τῶν εἰρημένων ὁ αὐτός ἐστιν τῶι τῆς ΘΗ πρὸς τὴν ΗΕ λόγωι. ὥστε παράλληλός ἐστιν ἡ ΔΖ τῆι ΑΓ.

(201) ἔστω καταγραφή, καὶ τῶν ΔΒ, ΒΓ μέση ἀνάλογον ἔστω ἡ ΒΑ. ὅτι παράλληλός ἐστιν ἡ ΖΗ τῆι ΑΓ. ἐκβεβλήσθω ἡ ΕΒ, καὶ διὰ τοῦ Α τῆι ΔΖ εὐθείαι παράλληλος ἤχθω ἡ ΑΚ, καὶ ἐπεζεύχθω ἡ ΓΚ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΓΒ πρὸς τὴν ΒΑ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΔ, ὡς δὲ ἡ ΑΒ πρὸς τὴν ΒΔ, οὕτως ἡ ΚΒ πρὸς τὴν ΒΘ, καὶ ὡς ἄρα ἡ ΓΒ πρὸς τὴν ΒΑ, οὕτως ἡ ΚΒ πρὸς τὴν ΒΘ. 878
παράλληλος ἄρα ἐστὶν ἡ ΑΘ τῆι ΚΓ. ἔστιν οὖν πάλιν ὡς ἡ ΑΖ πρὸς τὴν ΖΕ, οὕτως ἡ ΓΗ πρὸς τὴν ΗΕ. ἑκάτερος γὰρ τῶν εἰρημένων λόγος ὁ αὐτός ἐστιν τῶι τῆς ΚΘ πρὸς τὴν ΕΘ. ὥστε παράλληλός ἐστιν ἡ ΖΗ τῆι ΑΔ.

(202) |ἔστω βωμίσκος ὁ ΑΒΓΔΕΖΗ, καὶ ἔστω παράλληλος ἡ μὲν |160v
ΔΕ τῆι ΒΓ, ἡ δὲ ΕΗ τῆι ΒΖ. ὅτι καὶ ἡ ΔΖ τῆι ΓΗ παράλληλός ἐστιν. ἐπεζεύχθωσαν αἱ ΒΕ, ΔΓ, ΖΗ. ἴσον ἄρα ἐστὶν τὸ ΔΒΕ τρίγωνον τῶι ΔΓΕ τριγώνωι. κοινὸν προσκείσθω τὸ ΔΑΕ τρίγωνον. ὅλον ἄρα τὸ ΑΒΕ τρίγωνον ὅλωι τῶι ΓΔΑ τριγώνωι ἴσον ἐστίν. πάλιν ἐπεὶ παράλληλός ἐστιν ἡ ΒΖ τῆι ΕΗ, ἴσον ἐστὶν τὸ ΒΖΕ τρίγωνον τῶι ΒΖΗ τριγώνωι. κοινὸν ἀφῃρήσθω τὸ ΑΒΖ τρίγωνον. λοιπὸν ἄρα τὸ ΑΒΕ τρίγωνον λοιπῶι τῶι ΑΗΖ

‖ 2 ΚΛ Co ΚΑ A | ΛΗ Co ΛΜ A | ΑΒ Co ΑΕ A ‖ 3 ΚΛ ... ΛΗ Ge ΗΛ ... ΛΜ A ‖ 4 καὶ λοιπὴ — τὴν ΛΗ del Co ‖ 5 ΚΛ ... ΛΗ Ge ΚΜ ... ΛΜ A | ΔΓ Co ΑΓ A | ἐναλλάξ Co ἀνάλογον A ‖ 6 ΗΛ Co ΗΔ A ‖ 7 ΑΒ] ΔΘ A ΑΔ Co ‖ 11 διαχθείσης τῆς ΕΒ θῶ] διὰ τὴν ΕΒ θῶ A ἐπὶ τῆς ΕΒ θῶ Hu τῆι ΕΒ προσθῶ Heiberg$_3$, del Co ‖ 14 ἑκάτερος Heiberg$_3$ ἑκάτερα A ἑκατέρων Hu ‖ 15 λόγωι Heiberg$_3$ λόγον A λόγος Ge (BS) ‖ 16 καὶ Co κατὰ A | ΔΒ, ΒΓ μέση Hu ΑΒ, ΒΓ μέση A ΑΒ, ΒΓ τρίτη Co ΓΒ, ΑΒ τρίτη Breton ‖ 17 ΒΑ Hu ΒΔ A | ἐκβεβλήσθω Co ἐκβληθεῖσα A | ΕΒ Co ΑΒ A ‖ 21 ΒΑ Co ΒΛ A ‖ 22 ΑΘ Co ΛΘ A ‖ 23 ΖΕ Co ΖΓ A | ἑκάτερος Heiberg$_3$ ἑκάτερα A ἑκατέρων Hu ‖ 24 ΕΘ] ΒΘ A ΘΕ Co ‖ 25 ΑΔ] ΑΓ Breton ‖ 26 ὁ] ἡ Ge ‖ 31 ἡ ... τῆι] τῆι ... ἡ coni Hu app ‖ 32 ἀφῃρήσθω Ge (BS) ἀφαιρησθω A triangle ABE equals triangle AΓΔ. Therefore triangle AΓΔ too equals triangle AZH.[6] Let triangle AΓH be added in common. Then all triangle ΓΔH equals all triangle ΓZH.[7] And they are on the same base, ΓH. Hence (I, 39) ΓH is parallel to ΔZ.[8]

(203) *(Prop. 135)* Let there be triangle ABΓ, and let AΔ and AE be drawn through it, and let ZH be drawn parallel to BΓ, and let ZΘH be inflected. Let ΔΘ be to ΘE as is BΘ to ΘΓ. That KΛ is parallel to BΓ.

For since ΔΘ is to ΘE as is BΘ to ΘΓ,[1] therefore remainder BΔ is to remainder ΓE as is ΔΘ to ΘE.[2] But as is BΔ to EΓ, so is ZM to NH.[3] <Hence as is ZM to NH,> so is ΔΘ to ΘE.[4] *Alternando* as is ZM to ΔΘ, so is NH to ΘE.[5] But as is ZM to ΔΘ, so is ZK to KΘ in parallels;[6] while as HN is to ΘE, so is HΛ to ΛΘ.[7] Therefore as is ZK to KΘ, so is HΛ to ΛΘ.[8] Thus KΛ is parallel to HZ,[9] and therefore also to ΓB.[10]

(204) *(Prop. 136)* Let two straight lines ΔΘ, ΘE be drawn onto two straight lines BAE, ΔAH from point Θ. Let the rectangle contained by ΘH, ZE be to the rectangle contained by ΘE, ZH as is the rectangle contained by ΔΘ, BΓ to the rectangle contained by ΔΓ, BΘ. That the (line) through Γ, A, Z is straight.

Let KΛ be drawn through Θ parallel to ΓA,[1] and let it intersect AB and AΔ at points K and Λ. And let ΛM be drawn through Λ parallel to AΔ,[2] and let EΘ be produced to M. And let KN be drawn through K parallel to AB,[3] and let ΔΘ be produced to N.

Then since because of the parallels ΔΓ is to ΓB as is ΔΘ to ΘN,[4] therefore the rectangle contained by ΔΘ, ΓB equals the rectangle contained by ΔΓ, ΘN.[5] (Let) the rectangle contained by ΔΓ, BΘ (be) some other arbitrary quantity. Then as is the rectangle contained by ΔΘ, BΓ to the rectangle contained by ΔΓ, BΘ, so is the rectangle contained by ΓΔ, ΘN to the rectangle contained by ΔΓ, BΘ,[6] that is ΘN to ΘB.[7] But as is the rectangle contained by ΘΔ, BΓ to the rectangle contained by ΔΓ, BΘ, so was the rectangle contained by ΘH, ZE assumed to be to the rectangle contained by ΘE, ZH,[8] while as is ΘN to ΘB, so is KΘ to ΘΛ,[9] that is in parallels HΘ to ΘM,[10] that is the rectangle contained by ΘH, ZE to the rectangle contained by ΘM, ZE.[11] Hence as is the rectangle contained by ΘH, ZE to the rectangle contained by ΘE, ZH, so is the rectangle contained by ΘH, ZE to the rectangle contained by ΘM, ZE.[12] Therefore <the rectangle contained by ΘE, ZH> equals <the rectangle contained by ΘM, ZE.[13] In ratio, therefore,> as is MΘ to ΘE, so is HZ to ZE.[14] *Componendo*[15] and *alternando* as is ME to EH, so is ΘE to EZ.[16] But ΛE is to EA as is ME to EH.[17] Therefore as is ΛE to EA, so is ΘE to EZ.[18] Hence AZ is parallel to KΛ.[19] But ΓA is also (parallel) to (KΛ).[20] Thus ΓAZ is straight.[21] Q.E.D.

τριγώνωι ἴσον ἐστίν. ἀλλὰ τὸ ΑΒΕ τρίγωνον τῶι ΑΓΔ τριγώνωι ἐστὶν ἴσον. καὶ τὸ ΑΓΔ ἄρα τρίγωνον τῶι ΑΖΗ τριγώνωι ἴσον ἐστίν. κοινὸν προσκείσθω τὸ ΑΓΗ τρίγωνον. ὅλον ἄρα τὸ ΓΔΗ τρίγωνον ὅλωι τῶι ΓΖΗ τριγώνωι ἴσον ἐστίν. καὶ ἔστιν ἐπὶ τῆς αὐτῆς βάσεως τῆς ΓΗ. παράλληλος ἄρα ἐστὶν ἡ ΓΗ τῆι ΔΖ.

(203) ἔστω τρίγωνον τὸ ΑΒΓ, καὶ ἐν αὐτῶι διήχθωσαν αἱ ΑΔ, ΑΕ, καὶ τῆι ΒΓ παράλληλος ἤχθω ἡ ΖΗ, καὶ κεκλάσθω ἡ ΖΘΗ. ἔστω δὲ ὡς ἡ ΒΘ πρὸς τὴν ΘΓ, οὕτως ἡ ΔΘ πρὸς τὴν ΘΕ. ὅτι παράλληλός ἐστιν ἡ ΚΛ τῆι ΒΓ. ἐπεὶ γάρ ἐστιν ὡς ἡ ΒΘ πρὸς τὴν ΘΓ, οὕτως ἡ ΔΘ πρὸς τὴν ΘΕ, λοιπὴ ἄρα ἡ ΒΔ πρὸς λοιπὴν τὴν ΓΕ ἐστὶν ὡς ἡ ΔΘ πρὸς τὴν ΘΕ. ὡς δὲ ἡ ΒΔ πρὸς τὴν ΕΓ, οὕτως ἐστὶν ἡ ΖΜ πρὸς τὴν ΝΗ. <καὶ ὡς ἄρα ἡ ΖΜ πρὸς ΝΗ,> 880 οὕτως ἐστὶν ἡ ΔΘ πρὸς τὴν ΘΕ. ἐναλλάξ ἐστιν ὡς ἡ ΖΜ πρὸς τὴν ΔΘ, οὕτως ἡ ΝΗ πρὸς τὴν ΘΕ. ἀλλ' ὡς μὲν ἡ ΖΜ πρὸς τὴν ΔΘ, οὕτως ἐστὶν ἐν παραλλήλωι ἡ ΖΚ πρὸς τὴν ΚΘ· ὡς δὲ ἡ ΗΝ πρὸς τὴν ΘΕ, οὕτως ἐστὶν ἡ ΗΛ πρὸς τὴν ΛΘ. καὶ ὡς ἄρα ἡ ΖΚ πρὸς τὴν ΚΘ, οὕτως ἐστὶν ἡ ΗΛ πρὸς τὴν ΛΘ. παράλληλος ἄρα ἐστὶν ἡ ΚΛ τῆι ΗΖ. ὥστε καὶ τῆι ΓΒ.

(204) |εἰς δύο εὐθείας τὰς ΒΑΕ, ΔΑΗ ἀπὸ τοῦ Θ σημείου δύο διήχθωσαν εὐθεῖαι αἱ ΔΘ, ΘΕ. |161 ἔστω δὲ ὡς τὸ ὑπὸ τῶν ΔΘ, ΒΓ πρὸς τὸ ὑπὸ ΔΓ, ΒΘ, οὕτως τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΕ, ΖΗ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Γ, Α, Ζ. ἤχθω διὰ τοῦ Θ τῆι ΓΑ παράλληλος ἡ ΚΛ, καὶ συμπιπτέτω ταῖς ΑΒ, ΑΔ κατὰ τὰ Κ, Λ σημεῖα. καὶ διὰ τοῦ Λ τῆι ΑΔ παράλληλος ἤχθω ἡ ΛΜ, καὶ ἐκβεβλήσθω ἡ ΕΘ ἐπὶ τὸ Μ. διὰ δὲ τοῦ Κ τῆι ΑΒ παράλληλος ἤχθω ἡ ΚΝ, καὶ ἐκβεβλήσθω ἡ ΔΘ ἐπὶ τὸ Ν. ἐπεὶ οὖν διὰ τὰς παραλλήλους γίνεται ὡς ἡ ΔΘ πρὸς τὴν ΘΝ, οὕτως ἡ ΔΓ πρὸς τὴν ΓΒ, τὸ ἄρα ὑπὸ τῶν ΔΘ, ΓΒ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΔΓ, ΘΝ. ἄλλο δέ τι τυχὸν τὸ ὑπὸ ΔΓ, ΒΘ. ἔστιν ἄρα ὡς τὸ ὑπὸ ΔΘ, ΒΓ πρὸς τὸ ὑπὸ ΔΓ, ΒΘ, οὕτως τὸ ὑπὸ ΓΔ, ΘΝ πρὸς τὸ ὑπὸ ΔΓ, ΒΘ, τουτέστιν ἡ ΘΝ πρὸς ΘΒ. ἀλλ' ὡς μὲν τὸ ὑπὸ ΘΔ, ΒΓ πρὸς τὸ 882 ὑπὸ ΔΓ, ΒΘ ὑπόκειται τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΕ, ΖΗ. ὡς δὲ ἡ ΘΝ πρὸς ΘΒ, οὕτως ἡ ΚΘ πρὸς ΘΛ, τουτέστιν ἐν παραλλήλωι ἡ

‖ 2 ἐστὶν –ΑΖΗ τριγώνωι om A¹ add mg A² ‖ 6 ἡ... τῆι] τῆι ... ἡ coni. Hu app ‖ 8 ΖΘΗ Co ΖΗ A ‖ 11 λοιπὴ Ge (BS) λοιπὸν A ‖ 13 τὴν (ΝΗ) om Hu | καὶ – ΝΗ add Co ‖ 17 καὶ ὡς ἄρα – ΛΘ tris A corr Co ‖ 21 διήχθωσαν Ge (BS) διήχθω A ‖ 27 ἐκβεβλήσθω Hu ἐκβληθῆι A ‖ 28 παραλλήλους Ge (S) παράλληλα A ‖ 29 ΘΝ Co ΘΗ A

The characteristics of the cases of this (proposition are) as the foregoing ones, of which it is the converse.

(205) *(Prop. 137)* Triangle ΑΒΓ, and ΑΔ parallel to ΒΓ, and let ΔΕ be drawn through and intersect ΒΓ at point Ε. That ΓΒ is to ΒΕ as is the rectangle contained by ΔΕ, ΖΗ to the rectangle contained by ΕΖ, ΗΔ.

Let ΓΘ be drawn through Γ parallel to ΔΕ,[1] and let ΑΒ be produced to Θ. Then since ΓΘ is to ΖΗ as is ΓΑ to ΑΗ,[2] while ΕΔ is to ΔΗ as is ΓΑ to ΑΗ,[3] therefore ΘΓ is to ΖΗ as is ΕΔ to ΔΗ.[4] Hence the rectangle contained by ΓΘ, ΔΗ equals the rectangle contained by ΕΔ, ΖΗ.[5] (Let) the rectangle contained by ΕΖ, ΗΔ (be) some other arbitrary quantity. Then as is the rectangle contained by ΔΕ, ΖΗ to the rectangle contained by ΔΗ, ΕΖ, so is the rectangle contained by ΓΘ, ΔΗ to the rectangle contained by ΔΗ, ΕΖ,[6] that is ΓΘ to ΕΖ,[7] that is ΓΒ to ΒΕ.[8] Thus as is the rectangle contained by ΔΕ, ΖΗ to the rectangle contained by ΕΖ, ΗΔ, so is ΓΒ to ΒΕ. The same if parallel ΑΔ is drawn on the other side, and the straight line (ΔΕ) is drawn through from Δ outside (the triangle) in the direction of Γ.

(206) *(Prop. 138)* Now that these things have been proved, let it be required to prove that, if ΑΒ and ΓΔ are parallel, and some straight lines ΑΔ, ΑΖ, ΒΓ, ΒΖ intersect them, and ΕΔ and ΕΓ are joined, it results that the (line) through Η, Μ, and Κ is straight.

For since ΔΑΖ is a triangle, and ΑΕ is parallel to ΔΖ,[1] and ΕΓ has been drawn through intersecting ΔΖ at Γ, by the foregoing (lemma) it turns out that as ΔΖ is to ΖΓ, so is the rectangle contained by ΓΕ, ΗΘ to the rectangle contained by ΓΗ, ΘΕ.[2] Again, since ΓΒΖ is a triangle, and ΒΕ

ΗΘ πρὸς τὴν ΘΜ, τουτέστιν τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΜ, ΖΕ. καὶ ὡς ἄρα τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΕ, ΖΗ, οὕτως ἐστὶν τὸ ὑπὸ ΘΗ, ΖΕ πρὸς τὸ ὑπὸ ΘΜ, ΖΕ. ἴσον ἄρα ἐστὶν <τὸ ὑπὸ ΘΕ, ΖΗ τῶι ὑπὸ ΘΜ, ΖΕ. ἀνάλογον ἄρα ἐστὶν> ὡς ἡ ΜΘ πρὸς τὴν ΘΕ, οὕτως ἡ ΗΖ πρὸς τὴν ΖΕ. συνθέντι καὶ ἐναλλάξ ἐστιν ὡς ἡ ΜΕ πρὸς τὴν ΕΗ, οὕτως ἡ ΘΕ πρὸς τὴν ΕΖ. ἀλλ' ὡς ἡ ΜΕ πρὸς τὴν ΕΗ, οὕτως ἐστὶν ἡ ΛΕ πρὸς τὴν ΕΑ. καὶ ὡς ἄρα ἡ ΛΕ πρὸς τὴν ΕΑ, οὕτως ἡ ΘΕ πρὸς τὴν ΕΖ. παράλληλος ἄρα ἐστὶν ἡ ΑΖ τῆι ΚΛ. ἀλλὰ καὶ ἡ ΓΑ. εὐθεῖα ἄρα ἐστὶν ἡ ΓΑΖ. ὅ(περ): —

τὰ δὲ πτωτικὰ αὐτοῦ ὁμοίως τοῖς προγεγραμμένοις, ὧν ἐστιν ἀναστρόφιον.

(205) τρίγωνον τὸ ΑΒΓ, καὶ τῆι ΒΓ παράλληλος ἡ ΑΔ, καὶ διαχθεῖσα ἡ ΔΕ τῆι ΒΓ συμπιπτέτω κατὰ τὸ Ε σημεῖον. ὅτι ἐστὶν ὡς τὸ ὑπὸ ΔΕ, ΖΗ πρὸς τὸ ὑπὸ ΕΖ, ΗΔ, οὕτως ἡ ΓΒ πρὸς τὴν ΒΕ. ἤχθω διὰ τοῦ Γ τῆι ΔΕ παράλληλος ἡ ΓΘ, καὶ ἐκβεβλήσθω ἡ ΑΒ ἐπὶ τὸ Θ. ἐπεὶ οὖν ἐστιν |ὡς ἡ ΓΑ πρὸς τὴν |161v
ΑΗ, οὕτως ἡ ΓΘ πρὸς τὴν ΖΗ, ὡς δὲ ἡ ΓΑ πρὸς τὴν ΑΗ, οὕτως ἐστὶν ἡ ΕΔ πρὸς τὴν ΔΗ, καὶ ὡς ἄρα ἡ ΕΔ πρὸς τὴν ΔΗ, οὕτως ἐστὶν ἡ ΘΓ πρὸς τὴν ΖΗ. τὸ ἄρα ὑπὸ τῶν ΓΘ, ΔΗ ἴσον ἐστὶν τῶι ὑπὸ τῶν ΕΔ, ΖΗ. ἄλλο δέ τι τυχὸν τὸ ὑπὸ ΕΖ, ΗΔ. ἔστιν ἄρα ὡς τὸ ὑπὸ ΔΕ, ΖΗ πρὸς τὸ ὑπὸ ΔΗ, ΕΖ, οὕτως τὸ ὑπὸ ΓΘ, ΔΗ πρὸς τὸ 884
ὑπὸ ΔΗ, ΕΖ, τουτέστιν ἡ ΓΘ πρὸς ΕΖ, τουτέστιν ἡ ΓΒ πρὸς ΒΕ. ἔστιν οὖν ὡς τὸ ὑπὸ ΔΕ, ΖΗ πρὸς τὸ ὑπὸ ΕΖ, ΗΔ, οὕτως ἡ ΓΒ πρὸς ΒΕ. τὰ δ' αὐτὰ κἂν ἐπὶ τὰ ἕτερα μέρη ἀχθῆι ἡ ΑΔ παράλληλος, καὶ ἀπὸ τοῦ Δ ἐκτὸς ὡς ἐπὶ τὸ Γ διαχθῆι ἡ εὐθεῖα.

(206) ἀποδεδειγμένων νῦν τούτων, ἔστω δεῖξαι ὅτι ἐὰν παράλληλοι ὦσιν αἱ ΑΒ, ΓΔ, καὶ εἰς αὐτὰς ἐμπίπτωσιν εὐθεῖαί τινες αἱ ΑΔ, ΑΖ, ΒΓ, ΒΖ, καὶ ἐπιζευχθῶσιν αἱ ΕΔ, ΕΓ, ὅτι γίνεται εὐθεῖα ἡ διὰ τῶν Η, Μ, Κ. ἐπεὶ γὰρ τρίγωνον τὸ ΔΑΖ, καὶ τῆι ΔΖ παράλληλος ἡ ΑΕ, καὶ διῆκται ἡ ΕΓ συμπίπτουσα τῆι ΔΖ κατὰ τὸ Γ, διὰ τὸ προγεγραμμένον γίνεται ὡς ἡ ΔΖ πρὸς τὴν ΖΓ, οὕτως τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ. πάλιν

‖ 3 τὸ ὑπὸ ΘΕ, ΖΗ — ἄρα ἐστὶν ὡς] τὸ ὑπὸ ΘΕ, ΖΗ τῶι ὑπὸ ΘΜ, ΘΕ. καὶ ὡς ἄρα add Co ‖ 9 ΓΑ Co ΓΔ A | ΓΑΖ ὅ(περ) Ge (V) ΓΑΖΟ ο: A ‖ 25 ἐκτὸς — εὐθεῖα Heiberg$_3$ ἐκτὸς ὡς ἐπὶ τὸ Γ διὰ τὴν εὐθεῖαν A ἐκτὸς τοῦ Γ ὡς ἐπὶ τὸ Ε ἀχθῆι ἡ ΔΕ Co, quorum ὡς ἐπὶ τὸ Ε del Hu ‖ 27 νῦν] οὖν coni. Hu app | ἔστω] ἔσται A | ὅτι del Ge ‖ 29 ὅτι secl Hu has been drawn parallel to ΓΔ,[3] and ΔE has been drawn through intersecting ΓZΔ at Δ, it turns out that as ΓZ is to ZΔ, so is the rectangle contained by ΔE, ΛK to the rectangle contained by ΔK, ΛE.[4] By inversion, therefore, as ΔZ is to ZΓ, so is the rectangle contained by ΔK, ΛE to the rectangle contained by ΔE, ΛK.[5] But also as ΔZ is to ZΓ, so was the rectangle contained by ΓE, HΘ to the rectangle contained by ΓH, ΘE. Therefore as the rectangle contained by ΓE, HΘ to the rectangle contained by ΓH, ΘE, so is the rectangle contained by ΔK, ΛE to the rectangle contained by ΔE, KΛ.[6] This has been reduced to the (lemma) before last. Then since two straight lines EΓ, EΔ have been drawn onto two straight lines ΓMΛ, ΔMΘ, and as the rectangle contained by ΓE, HΘ is to the rectangle contained by ΓH, ΘE, so is the rectangle contained by ΔK, EΛ to the rectangle contained by ΔE, ΛK, therefore the (line) through H, M, K is straight;[7] for this was proved before (lemma 7.204).

(207) *(Prop. 139)* But now let AB and ΓΔ not be parallel, but let them intersect at N. That again the (line) through H, M, and K is straight.

Since two (straight lines) ΓE and ΓΔ have been drawn through from the same point Γ onto three straight lines AN, AZ, AΔ, it turns out that as is the rectangle contained by ΓE, HΘ to the rectangle contained by ΓH, ΘE, so is the rectangle contained by ΓN, ZΔ to the rectangle contained by NΔ, ΓZ (lemma 7.196).[1] Again, since two (straight lines) ΔE, ΔN have been drawn through from the same point Δ onto three straight lines BN, BΓ, ΓZ, as is the rectangle contained by NΓ, ZΔ to the rectangle contained by NΔ, ZΓ, so is the rectangle contained by ΔK, EΛ to the rectangle contained by ΔE, KΛ.[2] But as is the rectangle contained by NΓ, ZΔ to the rectangle contained by NΔ, ΓZ, so the rectangle contained by ΓE, HΘ was proved to be to the rectangle contained by ΓH, ΘE. Therefore as is the rectangle contained by ΓE, ΘH to the rectangle contained by ΓH, ΘE, so is the rectangle contained by ΔK, EΛ to the rectangle contained by ΔE, KΛ.[3] It has been reduced to the (lemma) which (it was reduced to) also in the case of the parallels. Because of the foregoing (lemma 7.204) the (line) through H, M, K is straight.[4]

(208) *(Prop. 140)* Let AB be parallel to ΓΔ, and let AE and ΓB be drawn through, and (let) Z (be) a point on BH, so that as is ΔE to EΓ, so will the rectangle contained by ΓB, HZ be to the rectangle contained by ZB, ΓH. That the (line) through A, Z, Δ is straight.

Let ΔΘ be drawn through Δ parallel to BΓ,[1] and let AE be produced to Θ; and let ΘK be drawn through Θ parallel to ΓΔ,[2] and let BΓ be produced to K. Then since as is ΔE to EΓ, so is the rectangle contained by ΓB, ZH to the rectangle contained by BZ, ΓH (lemma 7.205),[4] while as is ΔE to EΓ, so are ΔΘ to ΓH and (consequently) the rectangle contained by ΔΘ, BZ to the rectangle contained by ΓH, BZ,[3] therefore the rectangle contained by BΓ, ZH equals the rectangle contained by ΔΘ, BZ.[5] Hence in ἐπεὶ τρίγωνόν ἐστιν τὸ ΓΒΖ, καὶ τῆι ΓΔ παράλληλος ἦκται ἡ ΒΕ, καὶ διῆκται ἡ ΔΕ συμπίπτουσα τῆι ΓΖΔ κατὰ τὸ Δ, γίνεται ὡς ἡ ΓΖ πρὸς τὴν ΖΔ, οὕτως τὸ ὑπὸ ΔΕ, ΛΚ πρὸς τὸ ὑπὸ ΔΚ, ΛΕ. ἀνάπαλιν ἄρα γίνεται ὡς ἡ ΔΖ πρὸς τὴν ΖΓ, οὕτως τὸ ὑπὸ ΔΚ, ΛΕ πρὸς τὸ ὑπὸ ΔΕ, ΛΚ. ἦν δὲ καὶ ὡς ἡ ΔΖ πρὸς τὴν ΖΓ, οὕτως τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ. καὶ ὡς ἄρα τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ, οὕτως ἐστὶν τὸ ὑπὸ ΔΚ, ΛΕ πρὸς τὸ ὑπὸ ΔΕ, ΚΛ. ἀπῆκται εἰς τὸ πρὸ ἑνός. ἐπεὶ οὖν εἰς δύο εὐθείας τὰς ΓΜΛ, ΔΜΘ, δύο εὐθεῖαι διηγμέναι εἰσὶν αἱ ΕΓ, ΕΔ, καὶ ἔστιν ὡς τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ, οὕτως τὸ ὑπὸ ΔΚ, ΕΛ πρὸς τὸ ὑπὸ ΔΕ, ΛΚ, εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Η, Μ, Κ. τοῦτο γὰρ προδέδεικται.

(207) ἀλλὰ δὴ μὴ ἔστωσαν αἱ ΑΒ, ΓΔ παράλληλοι, ἀλλὰ συμπιπτέτωσαν κατὰ τὸ Ν. ὅτι πάλιν εὐθεῖά ἐστιν ἡ διὰ τῶν Η, Μ, Κ. ἐπεὶ εἰς τρεῖς εὐθείας τὰς ΑΝ, ΑΖ, ΑΔ ἀπὸ τοῦ αὐτοῦ |σημείου (|162) τοῦ Γ, δύο διηγμέναι εἰσὶν αἱ ΓΕ, ΓΔ, γίνεται ὡς τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ, οὕτως τὸ ὑπὸ τῶν ΓΝ, ΖΔ πρὸς τὸ ὑπὸ τῶν ΝΔ, ΓΖ. πάλιν ἐπεὶ ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Δ εἰς τρεῖς εὐθείας τὰς ΒΝ, ΒΓ, ΓΖ δύο εἰσὶν διηγμέναι αἱ ΔΕ, ΔΝ, ἔστιν ὡς τὸ ὑπὸ ΝΓ, ΖΔ πρὸς τὸ ὑπὸ ΝΔ, ΖΓ, οὕτως τὸ ὑπὸ ΔΚ, ΕΛ πρὸς τὸ ὑπὸ ΔΕ, ΚΛ. ἀλλ' ὡς τὸ ὑπὸ ΝΓ, ΖΔ πρὸς τὸ ὑπὸ ΝΔ, ΓΖ, οὕτως ἐδείχθη τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ. καὶ ὡς ἄρα τὸ ὑπὸ ΓΕ, ΘΗ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ, οὕτως ἐστὶν τὸ ὑπὸ ΔΚ, ΕΛ πρὸς τὸ ὑπὸ ΔΕ, ΚΛ. ἀπῆκται εἰς ὃ καὶ ἐπὶ τῶν παραλλήλων. διὰ δὴ τὸ προγεγραμμένον εὐθεῖά ἐστιν ἡ διὰ τῶν Η, Μ, Κ.

(208) ἔστω παράλληλος ἡ ΑΒ τῆι ΓΔ, καὶ διήχθωσαν αἱ ΑΕ, ΓΒ, καὶ σημεῖον ἐπὶ τῆς ΒΗ τὸ Ζ, ὥστε εἶναι ὡς τὴν ΔΕ πρὸς τὴν ΕΓ, οὕτως τὸ ὑπὸ ΓΒ, ΗΖ πρὸς τὸ ὑπὸ ΖΒ, ΓΗ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Ζ, Δ. ἤχθω διὰ μὲν τοῦ Δ τῆι ΒΓ παράλληλος ἡ ΔΘ, καὶ ἐκβεβλήσθω ἡ ΑΕ ἐπὶ τὸ Θ, διὰ δὲ τοῦ Θ τῆι ΓΔ παράλληλος ἡ ΘΚ, καὶ ἐκβεβλήσθω ἡ ΒΓ ἐπὶ τὸ Κ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΔΕ πρὸς τὴν ΕΓ, οὕτως τὸ ὑπὸ ΓΒ, ΖΗ πρὸς τὸ ὑπὸ ΒΖ, ΓΗ, ὡς δὲ ἡ ΔΕ πρὸς τὴν ΕΓ, οὕτως ἐστὶν ἥ τε ΔΘ πρὸς τὴν ΓΗ καὶ τὸ ὑπὸ ΔΘ, ΒΖ πρὸς τὸ ὑπὸ τῶν ΓΗ, ΒΖ, ἴσον ἄρα

‖ 1 ΓΔ] ΓΖ coni. Hu app ‖ 8 ἀπῆκται Hu p. 1263 ἀνῆκται A | ἀπῆκται — ἑνός secl Hu ‖ 9 ΓΜΛ Co ΓΜΔ A ‖ 10 ΘΗ Co ΓΕ A | ΕΛ A^2 ex ΔΛ ‖ 14 Ν Co Η A ‖ 16 Γ Co Κ A | ΓΔ Co ΝΔ A ‖ 17 ΓΝ Co ΓΗ A ‖ 24 ἀπῆκται] ἀνῆκται Ge | ἀπῆκται — παραλλήλων secl Hu | ὃ καὶ] τὸ δέκατον coni. Hu app ‖ 27 ἐπὶ Ge (BS) ἐπεὶ A | ΒΗ Co ΖΗ A ‖ 30 ἐκβεβλήσθω Ge ἐκβληθῆι A ‖ 33 ΒΖ, ΓΗ $Heiberg_3$ ΒΓ, ΖΗ A ΖΒ, ΓΗ Co | ἐστὶν del coni. Hu app ratio as ΓB is to BZ, so is ΔΘ, that is ΓK,[7] to HZ.[6] Hence the sum KB is to the sum BH as KΓ is to ZH,[8] that is as ΔΘ is to ZH.[9] But as is KB to BH, so in parallels are ΘA to AH, and ΔΘ to ZH.[10] And ΔΘ and ZH are parallel.[11] Thus the (line) through points A, Z, Δ is straight.[12]

(209) *(Prop. 141)* Now that this has been proved, let AB be parallel to ΓΔ, and let straight lines AZ, ZB, ΓE, EΔ intersect them, and let BΓ and HK be joined. That the (line) through A, M, Δ is straight.

Let ΔM be joined and produced to Θ. Then since, having a triangle BΓZ, BE has been drawn parallel to ΓΔ from the apex point B (and falling) outside (the triangle), and ΔE has been drawn through, it turns out (lemma 7.205) that as ΓZ is to ZΔ, so is the rectangle contained by ΔE, KΛ to the rectangle contained by EΛ, KΔ.[1] Thus as the rectangle contained by ΔE, KΛ is to the rectangle contained by ΔK, ΛE, so is the rectangle contained by ΓH, ΘE to the rectangle contained by ΓE, HΘ (lemma 7.196);[2] for two (straight lines) EΓ, EΔ have been drawn through from the same point E onto three straight lines ΓΛ, ΔΘ, HK. And so as is ΔZ to ZΓ, so is the rectangle contained by ΓE, HΘ to the rectangle contained by ΓH, ΘE.[3] And the (line) through H, M, K is straight.[4] Hence by the foregoing (lemma 7.208) the (line) through A, M, Δ is also straight.[5]

(210) *(Prop. 142 a – b)* Let two (straight lines) ΔB, ΔE be drawn across two straight lines AB, AΓ from the same point Δ, and let points H, Θ be chosen on them. And as is the rectangle contained by EH, ZΔ to the rectangle contained by ΔE, HZ, so let the rectangle contained by BΘ, ΓΔ be to the rectangle contained by BΔ, ΓΘ. That the (line) through A, H, Θ is straight.

Let KΛ be drawn through H parallel to BΔ.[1] Then since as the rectangle contained by EH, ZΔ is to the rectangle contained by ΔE, ZH, so is the rectangle contained by BΘ, ΓΔ to the rectangle contained by BΔ, ΓΘ,[2] while the ratio of the rectangle contained by EH, ZΔ to the rectangle contained by ΔE, HZ is compounded out of that which HE has to EΔ, that is KH to BΔ,[4] and that which ΔZ has to ZH, that is ΔΓ to HΛ;[5] [3] and the ratio of the rectangle contained by BΘ, ΓΔ to the rectangle contained by BΔ, ΓΘ is compounded out of that which ΘB has to BΔ and that which ΔΓ has to ΓΘ,[6] therefore the (ratio compounded) out of that of KH to BΔ and that of ΔΓ to HΛ is the same as that compounded out of that of BΘ to BΔ and that of ΔΓ to ΓΘ.[7] But the (ratio) of KH to BΔ is compounded out of that of KH to BΘ and that of BΘ to BΔ.[8] Therefore the (ratio) compounded ἐστὶν τὸ ὑπὸ τῶν ΒΓ, ΖΗ τῶι ὑπὸ ΔΘ, ΒΖ. ἀνάλογον ἄρα ἐστὶν 888 ὡς ἡ ΓΒ πρὸς τὴν ΒΖ, οὕτως ἡ ΔΘ, τουτέστιν ὡς ἡ ΓΚ, πρὸς τὴν ΗΖ. καὶ ὅλη ἄρα ἡ ΚΒ πρὸς ὅλην τὴν ΒΗ ἐστὶν ὡς ἡ ΚΓ πρὸς ΖΗ, τουτέστιν ὡς ἡ ΔΘ πρὸς ΖΗ. ἀλλ' ὡς ἡ ΚΒ πρὸς ΒΗ, ἐν παραλλήλωι οὕτως ἐστὶν ἡ ΘΑ πρὸς ΑΗ, καὶ ἡ ΔΘ πρὸς ΖΗ· καὶ εἰσὶν παράλληλοι αἱ ΔΘ, ΖΗ· εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Α, Ζ, Δ σημείων.

(209) τούτου προτεθεωρημένου ἔστω παράλληλος ἡ ΑΒ τῆι ΓΔ, καὶ εἰς αὐτὰς ἐμπιπτέτωσαν εὐθεῖαι ΑΖ, ΖΒ, ΓΕ, ΕΔ, καὶ ἐπεζεύχθωσαν αἱ ΒΓ, ΗΚ· ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Μ, Δ. ἐπιζευχθεῖσα ἡ ΔΜ ἐκβεβλήσθω ἐπὶ τὸ Θ. ἐπεὶ οὖν τριγώνου τοῦ ΒΓΖ ἐκτὸς ἀπὸ τῆς κορυφῆς τοῦ Β |σημείου τῆι ΓΔ |162v παράλληλος ἦκται ἡ ΒΕ, καὶ διῆκται ἡ ΔΕ, γίνεται ὡς ἡ ΓΖ πρὸς ΖΔ, οὕτως τὸ ὑπὸ ΔΕ, ΚΛ πρὸς τὸ ὑπὸ ΕΛ, ΚΔ. ὡς ἄρα τὸ ὑπὸ ΔΕ, ΚΛ πρὸς τὸ ὑπὸ ΔΚ, ΛΕ, οὕτως ἐστὶν τὸ ὑπὸ ΓΗ, ΘΕ πρὸς τὸ ὑπὸ ΓΕ, ΗΘ· εἰς τρεῖς <γὰρ> εὐθείας τὰς ΓΛ, ΔΘ, ΗΚ δύο εἰσὶν διηγμέναι ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Ε αἱ ΕΓ, ΕΔ. καὶ ὡς ἄρα ἡ ΔΖ πρὸς ΖΓ, οὕτως ἐστὶν τὸ ὑπὸ ΓΕ, ΗΘ πρὸς τὸ ὑπὸ ΓΗ, ΘΕ. καὶ ἔστιν εὐθεῖα ἡ διὰ τῶν Η, Μ, Κ. διὰ τὸ προγεγραμμένον ἄρα καὶ ἡ διὰ τῶν Α, Μ, Δ ἐστὶν εὐθεῖα.

890

(210) εἰς δύο εὐθείας τὰς ΑΒ, ΑΓ ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Δ δύο διήχθωσαν αἱ ΔΒ, ΔΕ, καὶ ἐπ' αὐτῶν εἰλήφθω σημεῖα τὰ Η, Θ. ἔστω δὲ ὡς τὸ ὑπὸ ΕΗ, ΖΔ πρὸς τὸ ὑπὸ ΔΕ, ΗΖ, οὕτως τὸ ὑπὸ ΒΘ, ΓΔ πρὸς τὸ ὑπὸ ΒΔ, ΓΘ. ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Η, Θ. ἤχθω διὰ τοῦ Η τῆι ΒΔ παράλληλος ἡ ΚΛ. ἐπεὶ οὖν ἐστιν ὡς τὸ ὑπὸ ΕΗ, ΖΔ πρὸς τὸ ὑπὸ ΔΕ, ΖΗ, οὕτως τὸ ὑπὸ ΒΘ, ΓΔ πρὸς τὸ ὑπὸ ΒΔ, ΓΘ, ἀλλ' <ὁ τοῦ> ὑπὸ ΕΗ, ΖΔ πρὸς τὸ ὑπὸ ΔΕ, ΗΖ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΗΕ πρὸς ΕΔ, τουτέστιν ἡ ΚΗ πρὸς ΒΔ, καὶ ἐξ οὗ ὃν ἔχει ἡ ΔΖ πρὸς ΖΗ, τουτέστιν ἡ ΔΓ πρὸς τὴν ΗΛ, ὁ δὲ τοῦ ὑπὸ ΒΘ, ΓΔ πρὸς τὸ ὑπὸ ΒΔ, ΓΘ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΘΒ πρὸς ΒΔ καὶ ἐξ οὗ ὃν ἔχει ἡ ΔΓ πρὸς ΓΘ, καὶ ὁ <ἔκ τε τοῦ> τῆς ΚΗ ἄρα πρὸς ΒΔ καὶ τοῦ τῆς ΔΓ πρὸς ΗΛ ὁ αὐτός ἐστιν τῶι συνημμένωι ἔκ τε τοῦ τῆς ΒΘ πρὸς ΒΔ καὶ τοῦ τῆς ΔΓ πρὸς ΓΘ. ὁ δὲ τῆς ΚΗ πρὸς ΒΔ

‖ 2 post ΔΘ add πρὸς τὴν ΗΖ Hu ‖ 3 ὅλη Ge (BS) ὅληι A ‖ 6 εὐθεῖα Ge (S) εὐθειαι A ‖ 10 ἐπεζεύχθωσαν Ge (BS) ἐπεζεύχθω A | Α, Μ, Δ Co ΗΜΚ A ‖ 11 ἐπιζευχθεῖσα ἡ ΔΜ] ἐπεζεύχθω ἡ ΔΜ A post quae add καὶ Co | Θ Co Κ A ‖ 12 ἐκτὸς secl Hu (Simson$_1$) ‖ 13 ΔΕ Co ΔΒ A ‖ 14 ΖΔ Co ΖΓ A | ἄρα] δὲ A ‖ 15 ΛΕ Co ΛΒ A ‖ 16 γὰρ add Hu ‖ 19 καὶ – Η, Μ, Κ del Heiberg$_3$ | Η, Μ, Κ] Δ, Μ, Θ Co Θ, Μ, Δ Hu ‖ 20 καὶ del Heiberg$_3$ ‖ 22 διήχθωσαν Ge (BS) διήχθη A ‖ 23 δὲ Hu δὴ A ‖ 27 ἀλλ'] ἀλλὰ A ὁ τοῦ add Ge (BS) ‖ 32 ΓΘ Co ΓΕ A | ἔκ τε τοῦ add Hu ‖ 34 ΒΔ Co ΘΔ A | ΔΓ Co ΑΓ A out of that of KH to BΘ and that of BΘ to BΔ and furthermore of that of ΔΓ to HΛ is the same as the (ratio) compounded out of that of BΘ to BΔ and that of ΔΓ to ΓΘ.[9] Let the ratio of ΘB to BΔ be removed in common. Then the remaining (ratio) compounded out of that of KH to BΘ and that of ΔΓ to HΛ is the same as that of ΔΓ to ΓΘ,[10] that is the (ratio) compounded out of that of ΔΓ to HΛ and that of HΛ to ΘΓ.[11] And again, let the ratio of ΔΓ to HΛ be removed in common. Then the remaining ratio of KH to BΘ is the same as that of HΛ to ΘΓ.[12] And *alternando*, as is KH to HΛ, so is BΘ to ΘΓ.[13] And KΛ and BΓ are parallel.[14] Therefore the (line) through points A, H, Θ is straight.[15]

(211) 18. *(Prop. 143)* But now let AB not be parallel to ΓΔ, but let it intersect it at N.

Then since two straight lines ΔE, ΔN have been drawn from the same point Δ across three straight lines BN, BΓ, BZ, as the rectangle contained by NΔ, ΓZ is to the rectangle contained by NΓ, ΔZ, so is the rectangle contained by ΔE, KΛ to the rectangle contained by EΛ, KΔ (lemma 7.196).[1] But as is the rectangle contained by EΔ, KΛ to the rectangle contained by EΛ, KΔ, so is the rectangle contained by EΘ, ΓH to the rectangle contained by EΓ, ΘH;[2] for again two (straight lines) EΓ, EΔ have been drawn from the same point E across three (straight lines) ΓΛ, ΔΘ, HK. Therefore as is the rectangle contained by EΘ, ΓH to the rectangle contained by EΓ, ΘH, so is the rectangle contained by NΔ, ΓZ to the rectangle contained by NΓ, ZΔ.[3] By the foregoing (lemma) the (line) through A, Θ, Δ is straight.[4] Thus the (line) through A, M, Δ too is straight.[5]

(212) *(Prop. 144)* (Let there be) triangle ABΓ, and let AΔ be drawn parallel to BΓ, and let ΔE, ZH be drawn across. And as the square of EB is to the rectangle contained by EΓ, ΓB, so let BH be to HΓ. That, if BΔ is joined, the (line) through Θ, K, Γ is straight.

Since, as is the square of EB to the rectangle contained by EΓ, ΓB, so is BH to HΓ,[1] let the ratio of ΓE to EB be applied in common, this being the same as that of the rectangle contained by EΓ, ΓB to the rectangle contained by EB, BΓ.[2] Then *ex aequali* the ratio of the square of EB to the rectangle contained by EB, BΓ, that is the (ratio) of EB to BΓ, is the same as the (ratio) compounded out of that of BH to HΓ and that of the rectangle contained by EΓ, ΓB to the rectangle contained by EB, BΓ,[3] which is the same as that of EΓ to EB.[4] Therefore the (ratio) of the square of EB to the

συνῆπται ἔκ τε τοῦ τῆς ΚΗ πρὸς ΒΘ καὶ τοῦ τῆς ΒΘ πρὸς ΒΔ. ὁ
ἄρα συνημμένος ἔκ τε τοῦ τῆς ΚΗ πρὸς ΒΘ καὶ τοῦ τῆς ΒΘ πρὸς
ΒΔ, καὶ ἔτι τοῦ τῆς ΔΓ πρὸς ΗΛ ὁ αὐτός ἐστιν τῶι συνημμένωι
ἔκ τε τοῦ τῆς ΒΘ πρὸς ΒΔ καὶ τοῦ τῆς ΔΓ πρὸς ΓΘ. κοινὸς
ἐκκεκρούσθω ὁ τῆς ΘΒ πρὸς ΒΔ λόγος. λοιπὸς ἄρα ὁ
συνημμένος ἔκ τε τοῦ τῆς ΚΗ πρὸς ΒΘ καὶ τοῦ τῆς ΔΓ πρὸς ΗΛ
ὁ αὐτός ἐστιν τῶι τῆς ΔΓ πρὸς τὴν ΓΘ, τουτέστιν τῶι
συνημμένωι ἔκ τε τοῦ τῆς ΔΓ πρὸς τὴν ΗΛ καὶ τοῦ τῆς ΗΛ πρὸς
τὴν ΘΓ. καὶ πάλιν κοινὸς ἐκκεκρούσθω ὁ τῆς ΔΓ πρὸς τὴν ΗΛ
λόγος. λοιπὸς ἄρα ὁ τῆς ΚΗ πρὸς τὴν ΒΘ λόγος ὁ αὐτός ἐστὶν
τῶι τῆς ΗΛ πρὸς τὴν ΘΓ. |καὶ ἐναλλάξ ἐστιν ὡς ἡ ΚΗ πρὸς τὴν 892
ΗΛ, οὕτως ἡ ΒΘ πρὸς τὴν ΘΓ. καὶ εἰσὶν αἱ ΚΛ, ΒΓ παράλληλοι. |163
εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Α, Η, Θ σημείων.

(211) ιη΄. ἀλλὰ δὴ μὴ ἔστω παράλληλος ἡ ΑΒ τῆι ΓΔ, ἀλλὰ
συμπιπτέτω κατὰ τὸ Ν. ἐπεὶ οὖν ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Δ
εἰς τρεῖς εὐθείας τὰς ΒΝ, ΒΓ, ΒΖ δύο εὐθεῖαι διηγμέναι
εἰσὶν αἱ ΔΕ, ΔΝ, ἔστιν ὡς τὸ ὑπὸ ΝΔ, ΓΖ πρὸς τὸ ὑπὸ ΝΓ, ΔΖ,
οὕτως τὸ ὑπὸ ΔΕ, ΚΛ πρὸς τὸ ὑπὸ ΕΛ, ΚΔ. ὡς δὲ τὸ ὑπὸ ΕΔ, ΚΛ
πρὸς τὸ ὑπὸ ΕΛ, ΚΔ, οὕτως ἐστὶν τὸ ὑπὸ ΕΘ, ΓΗ πρὸς τὸ ὑπὸ ΕΓ,
ΘΗ. πάλιν γὰρ εἰς τρεῖς τὰς ΓΛ, ΔΘ, ΗΚ ἀπὸ τοῦ αὐτοῦ σημείου
τοῦ Ε δύο ἠγμέναι εἰσὶν αἱ ΕΓ, ΕΔ. καὶ ὡς ἄρα τὸ ὑπὸ ΕΘ, ΓΗ
πρὸς τὸ ὑπὸ ΕΓ, ΘΗ, οὕτως τὸ ὑπὸ ΝΔ, ΓΖ πρὸς τὸ ὑπὸ ΝΓ, ΖΔ.
διὰ τὸ προγεγραμμένον εὐθεῖά ἐστιν ἡ διὰ τῶν Α, Θ, Δ. καὶ ἡ
διὰ τῶν Α, Μ, Δ ἄρα εὐθεῖά ἐστιν.

(212) τρίγωνον τὸ ΑΒΓ, καὶ τῆι ΒΓ παράλληλος ἤχθω ἡ ΑΔ,
καὶ διήχθωσαν αἱ ΔΕ, ΖΗ. ἔστω δὲ ὡς τὸ ἀπὸ ΕΒ πρὸς τὸ ὑπὸ
ΕΓΒ, οὕτως ἡ ΒΗ πρὸς τὴν ΗΓ. ὅτι ἐὰν ἐπιζευχθῆι ἡ ΒΔ,
γίνεται εὐθεῖα ἡ διὰ τῶν Θ, Κ, Γ. ἐπεί ἐστιν ὡς τὸ ἀπὸ τῆς
ΕΒ πρὸς τὸ ὑπὸ ΕΓΒ, οὕτως ἡ ΒΗ πρὸς ΗΓ, κοινὸς ἄρα
προσκείσθω ὁ τῆς ΓΕ πρὸς ΕΒ λόγος, ὁ αὐτὸς ὢν τῶι τοῦ ὑπὸ
ΕΓΒ πρὸς τὸ ὑπὸ ΕΒΓ. δι' ἴσου ἄρα ὁ τοῦ ἀπὸ ΕΒ πρὸς τὸ ὑπὸ 894
ΕΒΓ λόγος, τουτέστιν ὁ τῆς ΕΒ πρὸς τὴν ΒΓ, ὁ αὐτός ἐστιν τῶι
συνημμένωι ἔκ τε τοῦ τῆς ΒΗ πρὸς ΗΓ καὶ τοῦ τοῦ ὑπὸ ΕΓΒ
πρὸς τὸ ὑπὸ ΕΒΓ, ὅς ἐστιν ὁ αὐτὸς τῶι τῆς ΕΓ πρὸς ΕΒ. ὥστε ὁ

‖ 4 κοινὸς] κ° A ‖ 9 κοινὸς] κ° A ‖ 14 ιη΄ mg A ‖ 17 ΔΝ Co ΔΗ A | ΝΔ Co ΝΛ A ‖ 21 ΕΔ Co ΕΑ A ‖ 22 τὸ ὑπὸ ΝΔ, ΓΖ bis A corr Co ‖ 23 post διὰ add δὴ Ge ‖ 26 τὸ (ἀπὸ ΕΒ) Ge (S) τὰ A ‖ 29 κοινὸς Ge (S) κοινὸν A | ἄρα secl Hu ‖ 33 τοῦ Hu τῶι A del Ge rectangle contained by EB, BΓ is compounded out of that which BH has to HΓ and that which EΓ has to EB,[5] which is the same as that of the rectangle contained by EΓ, BH to the rectangle contained by EB, ΓH.[6] But as is EB to BΓ, so, by the foregoing lemma (7.205), is *the rectangle contained by ΔE, ZΘ to the rectangle contained by ΔZ, ΘE.[7] And therefore as is the rectangle contained by ΓE, BH to the rectangle contained by ΓH, EB, so is the rectangle contained by ΔE, ZΘ to the rectangle contained by ΔZ, ΘE.[8] * Therefore the (line) through Θ, K, Γ is straight;[9] for that is in the case-variants of the converses.

(213) *(Prop. 145)* Let two (straight lines) EZ, EB be drawn from some point E across three straight lines AB, AΓ, AΔ, and, as EZ is to ZH, so let ΘE be to ΘH. That also as BE is to BΓ, so is EΔ to ΔΓ.

Let ΛK be drawn through H parallel to BE.[1] Then since as is EZ to ZH, so is EΘ to ΘH,[2] but as is EZ to ZH, so is EB to HK,[3] while as is EΘ to ΘH, so is ΔE to HΛ,[4] therefore as is BE to HK, so is ΔE to HΛ.[5] *Alternando*, as is EB to EΔ, so is KH to HΛ.[6] But as is KH to HΛ, so is BΓ to ΓΔ.[7] Therefore as is BE to EΔ, so is BΓ to ΓΔ.[8] *Alternando*, as is EB to BΓ, so is EΔ to ΔΓ.[9] The case-variants likewise.

(214) *(Prop. 146)* Let there be two triangles ABΓ, ΔEZ that have angles A, Δ equal. That, as is the rectangle contained by BA, AΓ to the rectangle contained by EΔ, ΔZ, so is triangle ABΓ to triangle EΔZ.

Let perpendiculars BH, EΘ be drawn.[1] Then since angle A equals Δ, and H (equals) Θ,[2] therefore as is AB to BH, so is ΔE to EΘ.[3] But as AB is to BH, so is the rectangle contained by BA, AΓ to the rectangle contained by BH, AΓ,[4] while as is ΔE to EΘ, so is the rectangle contained by EΔ, ΔZ to the rectangle contained by EΘ, ΔZ.[5] Therefore as is the rectangle contained by BA, AΓ to the rectangle contained by BH, AΓ, so is the rectangle contained by EΔ, ΔZ to the rectangle contained by EΘ, ΔZ;[6] and *alternando*.[7] But as is the rectangle contained by BH, AΓ to the rectangle contained by EΘ, ΔZ, so is triangle ABΓ to triangle ΔEZ;[8] for each of BH and EΘ is a perpendicular of each of the triangles named. Therefore as is the rectangle contained by BA, AΓ to the rectangle contained by EΔ, ΔZ, so is triangle ABΓ to triangle ΔEZ.[9]

τοῦ ἀπὸ ΕΒ *πρὸς τὸ ὑπὸ* ΕΒΓ *συνῆπται ἔκ τε τοῦ ὃν ἔχει ἡ* ΒΗ *πρὸς* ΗΓ *καὶ τοῦ ὃν ἔχει ἡ* ΕΓ *πρὸς* ΕΒ, *ὅς ἐστιν ὁ αὐτὸς τῶι τοῦ ὑπὸ* ΕΓ, ΒΗ *πρὸς τὸ ὑπὸ* ΕΒ, ΓΗ. *ὡς δὲ ἡ* ΕΒ *πρὸς τὴν* ΒΓ, *οὕτως ἐστὶν διὰ τὸ προγεγραμμένον λῆμμα τὸ ὑπὸ* ΔΕ, ΖΘ *πρὸς τὸ ὑπὸ* ΔΖ, ΘΕ. *καὶ ὡς ἄρα τὸ ὑπὸ* ΓΕ, ΒΗ *πρὸς τὸ ὑπὸ* ΓΗ, ΕΒ, *οὕτως ἐστὶν τὸ ὑπὸ* ΔΕ, ΖΘ *πρὸς τὸ ὑπὸ* ΔΖ, ΘΕ. *εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν* Θ, Κ, Γ· *τοῦτο γὰρ ἐν τοῖς πτωτικοῖς τῶν ἀναστροφίων*.

(213) |*εἰς τρεῖς εὐθείας τὰς* ΑΒ, ΑΓ, ΑΔ *ἀπό τινος σημείου* |163v *τοῦ* Ε *δύο διήχθωσαν αἱ* ΕΖ, ΕΒ. *ἔστω δὲ ὡς ἡ* ΕΖ *πρὸς τὴν* ΖΗ, *οὕτως ἡ* ΘΕ *πρὸς τὴν* ΘΗ. *ὅτι γίνεται καὶ ὡς ἡ* ΒΕ *πρὸς τὴν* ΒΓ, *οὕτως ἡ* ΕΔ *πρὸς τὴν* ΔΓ. *ἤχθω διὰ τοῦ* Η *τῆι* ΒΕ *παράλληλος ἡ* ΛΚ. *ἐπεὶ οὖν ἐστιν ὡς ἡ* ΕΖ *πρὸς τὴν* ΖΗ, *οὕτως ἡ* ΕΘ *πρὸς τὴν* ΘΗ, *ἀλλ' ὡς μὲν ἡ* ΕΖ *πρὸς τὴν* ΖΗ, *οὕτως ἡ* ΕΒ *πρὸς τὴν* ΗΚ, *ὡς δὲ ἡ* ΕΘ *πρὸς τὴν* ΘΗ, *οὕτως ἐστὶν ἡ* ΔΕ *πρὸς τὴν* ΗΛ, *καὶ ὡς ἄρα ἡ* ΒΕ *πρὸς τὴν* ΗΚ, *οὕτως ἐστὶν ἡ* ΔΕ *πρὸς τὴν* ΗΛ. *ἐναλλὰξ ἐστιν ὡς ἡ* ΕΒ *πρὸς τὴν* ΕΔ, *οὕτως ἡ* ΚΗ *πρὸς τὴν* ΗΛ. *ὡς δὲ ἡ* ΚΗ *πρὸς τὴν* ΗΛ, *οὕτως ἐστὶν ἡ* ΒΓ *πρὸς τὴν* ΓΔ. *καὶ ὡς ἄρα ἡ* ΒΕ *πρὸς τὴν* ΕΔ, *οὕτως ἡ* ΒΓ *πρὸς τὴν* ΓΔ. *ἐναλλὰξ ἐστιν ὡς ἡ* ΕΒ *πρὸς τὴν* ΒΓ, *οὕτως ἡ* ΕΔ *πρὸς τὴν* ΔΓ. *τὰ δὲ πτωτικὰ ὁμοίως*.

(214) *ἔστω δύο τρίγωνα τὰ* ΑΒΓ, ΔΕΖ *ἴσας ἔχοντα τὰς* Α, Δ *γωνίας. ὅτι ἐστὶν ὡς τὸ ὑπὸ* ΒΑΓ *πρὸς τὸ ὑπὸ* ΕΔΖ, *οὕτως τὸ* ΑΒΓ *τρίγωνον πρὸς τὸ* ΕΔΖ *τρίγωνον. ἤχθωσαν κάθετοι αἱ* ΒΗ, 896 ΕΘ. *ἐπεὶ οὖν ἴση ἐστὶν ἡ μὲν* Α *γωνία τῆι* Δ, *ἡ δὲ* Η *τῆι* Θ, *ἔστιν ἄρα ὡς ἡ* ΑΒ *πρὸς τὴν* ΒΗ, *οὕτως ἡ* ΔΕ *πρὸς τὴν* ΕΘ. *ἀλλ' ὡς μὲν ἡ* ΑΒ *πρὸς τὴν* ΒΗ, *οὕτως ἐστὶν τὸ ὑπὸ* ΒΑΓ *πρὸς τὸ ὑπὸ* ΒΗ, ΑΓ, *ὡς δὲ ἡ* ΔΕ *πρὸς τὴν* ΕΘ, *οὕτως ἐστὶν τὸ ὑπὸ* ΕΔΖ *πρὸς τὸ ὑπὸ* ΕΘ, ΔΖ. *ἔστιν ἄρα ὡς τὸ ὑπὸ* ΒΑΓ *πρὸς τὸ ὑπὸ* ΒΗ, ΑΓ, *οὕτως τὸ ὑπὸ* ΕΔΖ *πρὸς τὸ ὑπὸ* ΕΘ, ΔΖ· *καὶ ἐναλλάξ. ἀλλ' ὡς τὸ ὑπὸ* ΒΗ, ΑΓ *πρὸς τὸ ὑπὸ* ΕΘ, ΔΖ, *οὕτως ἐστὶν τὸ* ΑΒΓ *τρίγωνον πρὸς τὸ* ΔΕΖ *τρίγωνον· ἑκατέρα γὰρ τῶν* ΒΗ, ΕΘ *κάθετός ἐστιν ἑκατέρου τῶν εἰρημένων τριγώνων. καὶ ὡς ἄρα τὸ ὑπὸ* ΒΑΓ *πρὸς τὸ ὑπὸ* ΕΔΖ, *οὕτως ἐστὶν τὸ* ΑΒΓ *τρίγωνον πρὸς τὸ* ΔΕΖ *τρίγωνον*.

‖ 1 *τοῦ ἀπὸ* Hu *ἀπὸ τοῦ* A | *συνῆπται* Ge (BS) *συνῆκται* A | ΒΗ Co ΒΝ A ‖ 4 ΔΕ, ΖΘ... ΔΖ, ΘΕ] ΔΖ, ΘΕ... ΔΕ, ΖΘ Simson$_1$ ‖ 5 ΕΒ Co ΘΒ A ‖ 6 ΔΕ, ΖΘ... ΔΖ, ΘΕ] ΔΖ, ΘΕ... ΔΕ, ΖΘ Simson$_1$ ‖ 12 *ἤχθω* Ge (S) *ἤχθη* A ‖ 15 *ἐστὶν* secl Hu ‖ 25 ΕΘ Co ΗΘ A ‖ 26 ΒΗ Co ΒΕ A