PAPPI ALEXANDRINI
COLLECTIONIS
QUAE SUPERSUNT

E LIBRIS MANU SCRIPTIS EDIDIT
LATINA INTERPRETATIONE ET COMMENTARIIS
INSTRUXIT

FRIDERICUS HULTSCH.

VOLUMEN II.

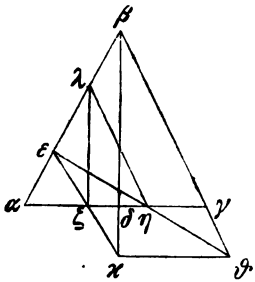
INSUNT LIBRORUM VI ET VII RELIQUIAE.

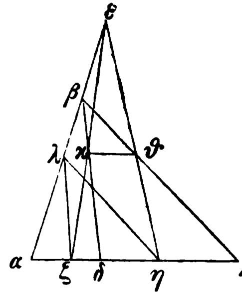
BEROLINI
APUD WEIDMANNOS
MDCCCLXXVII.

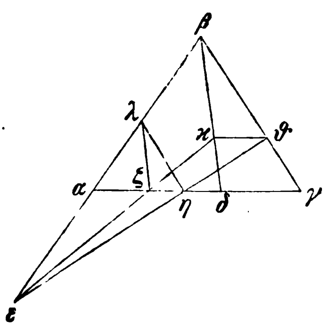
Πορισμάτων α' β' γ'.

Τοῦ πρώτου εἰς τὸ πρῶτον πόρισμα.

193 *α'. Ἔστω καταγραφὴ ἡ ΑΒΓΔΕΖΗ, καὶ ἔστω ὡς ἡ ΑΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, καὶ ἐπεζεύχθω ἡ ΘΚ· ὅτι παράλληλός ἐστιν ἡ ΘΚ τῇ ΑΓ.*







Ἤχθω διὰ τοῦ Ζ τῇ ΒΔ παράλληλος ἡ ΖΛ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΑΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, ἀνάπαλιν καὶ συνθέντι καὶ ἐναλλάξ ἐστιν ὡς ἡ ΔΑ πρὸς τὴν ΑΖ, τουτέστιν ἐν παραλλήλῳ ὡς ἡ ΒΑ πρὸς τὴν ΑΛ, οὕτως ἡ ΓΑ πρὸς τὴν ΑΗ· παράλληλος ἄρα ἐστὶν ἡ ΛΗ τῇ ΒΓ· ἔστιν ἄρα ὡς ἡ ΕΒ πρὸς τὴν ΒΛ, οὕτως ἐν παραλλήλῳ ἡ ΕΚ πρὸς τὴν ΚΖ, καὶ ἡ ΕΘ πρὸς τὴν ΘΗ· *καὶ ὡς ἄρα ἡ ΕΚ πρὸς τὴν ΚΖ, οὕτως ἐστὶν ἡ ΕΘ πρὸς τὴν ΘΗ· παράλληλος ἄρα ἐστὶν ἡ ΘΚ τῇ ΑΓ.*

194 *Διὰ δὲ τοῦ συνημμένου οὕτως. ἐπεί ἐστιν ὡς ἡ ΑΖ*

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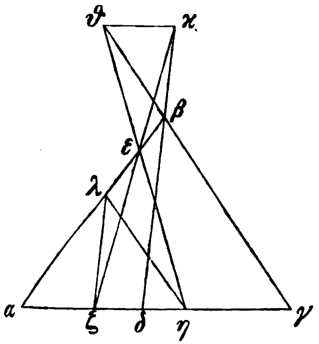
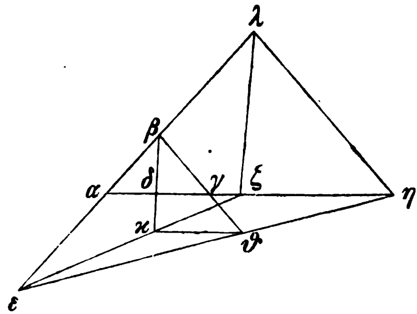
1. *Α' Β' Γ'* AB, *τρία* S 3. *ᾱ* in A vs. 2 ante *Τοῦ πρώτου* servatum est, *α'* ante *Ἔστω* in BS *ἡ* (post *ὡς*) add. BS 15. *ἄρα ἐστὶν ἡ* $\overline{\text{ΑΗ}}$ AB, corr. A[1] super vs. S 18. *καὶ ἡ ΕΘ πρὸς τὴν ΘΗ* add. *Co*

LEMMATA IN PORISMATUM LIBROS I II III.

In libri primi primum porisma.

I. Sit figura $\alpha\beta\gamma\delta\varepsilon\zeta\eta$, sitque $\alpha\zeta : \zeta\eta = \alpha\delta : \delta\gamma$, et du- Prop. 127
catur $\vartheta\varkappa$; dico parallelas esse rectas $\alpha\gamma$ $\vartheta\varkappa$.





Ducatur per ζ rectae $\beta\delta$ parallela $\zeta\lambda$. Quoniam igitur est $\alpha\zeta : \zeta\eta = \alpha\delta : \delta\gamma$, e contrario est $\zeta\eta : \alpha\zeta = \delta\gamma : \alpha\delta$, et componendo $\alpha\eta : \alpha\zeta = \alpha\gamma : \alpha\delta$, et vicissim $\alpha\eta : \alpha\gamma = \alpha\zeta : \alpha\delta$, *denique e contrario*

$\alpha\gamma : \alpha\eta = \alpha\delta : \alpha\zeta$, id est propter parallelas $\beta\delta$ $\lambda\zeta$
$= \alpha\beta : \alpha\lambda$.

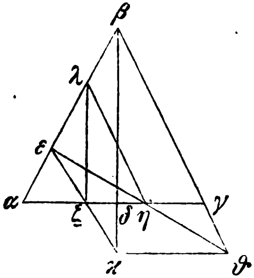
Ergo parallelae sunt $\beta\gamma$ $\lambda\eta$; est igitur propter parallelas $\beta\varkappa$ $\lambda\zeta$
$\varepsilon\beta : \beta\lambda = \varepsilon\varkappa : \varkappa\zeta$, et propter parallelas $\beta\vartheta$ $\lambda\eta$
$= \varepsilon\vartheta : \vartheta\eta$;

ergo, quia $\varepsilon\varkappa : \varkappa\zeta = \varepsilon\vartheta : \vartheta\eta$, parallelae sunt $\varkappa\vartheta$ $\alpha\gamma$.

Per formulam compositae proportionis sic. Quoniam est

PROPOS. 127: Simson p. 398 sq., Breton p. 219 sq., Chasles p. 74. 87. 108 sqq., Vincent p. 33 sqq. Propositionem et hanc et proximas accuratius enuntiat Simsonus; quas cum omnes repetere alienum sit ab hac editione, exempli gratia hanc unam afferamus: "Si in recta linea fuerint puncta α ζ δ η γ, ita ut $\alpha\zeta$ sit ad $\zeta\eta$, ut $\alpha\delta$ ad $\delta\gamma$, et ad rectam lineam $\alpha\beta$ inflectantur $\zeta\varepsilon$ $\eta\varepsilon$, et ad eandem inflectantur $\delta\beta$ $\gamma\beta$, et inflexae a punctis ζ δ sibi mutuo occurrant in $\varkappa$, inflexae vero a punctis η γ occurrant in ϑ, et $\vartheta\varkappa$ iungatur, erit $\varkappa\vartheta$ parallela ipsi $\alpha\gamma$". Figurae quinque, ut hic descriptae sunt, exstant in codicibus.

πρὸς τὴν ΖΗ, οὕτως ἡ ΑΔ πρὸς τὴν ΔΓ, ἀνάπαλίν ἐστιν ὡς ἡ ΗΖ πρὸς τὴν ΖΑ, οὕτως ἡ ΓΔ πρὸς τὴν ΔΑ. συνθέντι καὶ ἐναλλὰξ καὶ ἀναστρέψαντί ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΖ, οὕτως ἡ ΑΓ πρὸς τὴν ΓΗ. ἀλλ' ὁ μὲν τῆς ΑΔ πρὸς τὴν ΔΖ συνῆπται ἔκ τε τοῦ τῆς ΑΒ πρὸς τὴν ΒΕ καὶ τοῦ τῆς ΕΘ πρὸς τὴν ΘΗ· ὁ ἄρα συνημμένος λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΑΒ πρὸς τὴν ΒΕ καὶ ἡ ΕΚ πρὸς τὴν ΚΖ ὁ αὐτός ἐστιν τῷ συνημμένῳ ἔκ τε τοῦ ὃν ἔχει ἡ ΑΒ πρὸς τὴν ΒΕ καὶ ἡ ΕΘ πρὸς τὴν ΘΗ. καὶ κοινὸς ἐκκεκρούσθω ὁ τῆς ΑΒ πρὸς *τὴν ΒΕ λόγος· λοιπὸν ἄρα ὁ τῆς ΕΚ πρὸς τὴν ΚΖ λόγος* ὁ αὐτός *ἐστιν τῷ τῆς ΕΘ πρὸς τὴν ΘΗ· παράλληλος ἄρα ἐστὶν ἡ ΘΚ τῇ ΑΓ.*



Εἰς τὸ δεύτερον πόρισμα.

195 β'. *Καταγραφὴ ἡ ΑΒΓΔΕΖΗΘ, ἔστω δὲ παράλληλος ἡ ΑΖ τῇ ΔΒ, ὡς δὲ ἡ ΑΕ πρὸς τὴν ΕΖ, οὕτως ἡ ΓΗ πρὸς τὴν ΗΖ· ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Θ Κ Ζ.*

Ἤχθω διὰ τοῦ Η παρὰ τὴν ΔΕ ἡ ΗΛ, καὶ ἐπιζευχθεῖσα ἡ ΘΚ ἐκβεβλήσθω ἐπὶ τὸ Λ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΑΕ πρὸς τὴν ΕΖ, οὕτως ἡ ΓΗ πρὸς τὴν ΗΖ, ἐναλλάξ ἐστιν ὡς ἡ ΑΕ πρὸς τὴν ΓΗ, οὕτως ἡ ΕΖ πρὸς τὴν ΖΗ. ὡς δὲ ἡ ΑΕ πρὸς τὴν ΓΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΗΛ (διὰ τὸ εἶναι δύο παρὰ δύο, καὶ ἐναλλάξ)· καὶ ὡς ἄρα ἡ ΕΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΗΛ. καὶ ἔστι

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2. *ὡς ἡ ΝΖ* AB, corr. S 3. *ἡ* add. BS 3. 4. *πρὸς τὴν ΑΖ* ABS, corr. *Co* 13. *κοινὸς* S super vs., *κ°* AB, *χ̊* S cod. *Co* 14. *πρὸς* add. S *λοιπὸς Co* 15. *ὁ αὐτός* add. *Co* 16. *παράλληλος Co* pro *λόγος* 18. *β'* add. BS *ἡ ΑΒ ΓΔ ΕΖΗΘ* A, coniunx. BS 20. *τῶν ΘΚΖ* A, distinx. BS 21. *ἐπιζευχθεῖσα Hu* auctore *Co* pro *ἐπεζεύχθω* 22. post *ἐπὶ τὸ Λ* in A rasura est

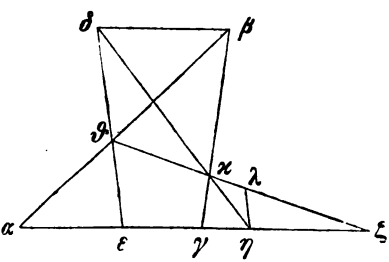
$\alpha\zeta : \zeta\eta = \alpha\delta : \delta\gamma$, e contrario est $\zeta\eta : \alpha\zeta = \delta\gamma : \alpha\delta$, *et* componendo $\alpha\eta : \alpha\zeta = \alpha\gamma : \alpha\delta$, et vicissim $\alpha\eta : \alpha\gamma = \alpha\zeta : \alpha\delta$, *et e contrario* $\alpha\gamma : \alpha\eta = \alpha\delta : \alpha\zeta$, et convertendo $\alpha\gamma : \gamma\eta = \alpha\delta : \delta\zeta$. Sed est [1])

$$\frac{\alpha\delta}{\delta\zeta} = \frac{\alpha\beta}{\beta\lambda} = \frac{\alpha\beta}{\beta\varepsilon} \cdot \frac{\beta\varepsilon}{\beta\lambda} = \frac{\alpha\beta}{\beta\varepsilon} \cdot \frac{\varepsilon\vartheta}{\vartheta\eta} = \frac{\alpha\beta}{\beta\varepsilon} \cdot \frac{\varepsilon\varkappa}{\varkappa\zeta};$$

et *dividendo* tollatur communis proportio $\alpha\beta : \beta\varepsilon$; relinquitur igitur $\varepsilon\varkappa : \varkappa\zeta = \varepsilon\vartheta : \vartheta\eta$; sunt igitur parallelae $\varkappa\vartheta$ $\alpha\gamma$.

In secundum porisma.

II. Figura $\alpha\beta\gamma\delta\varepsilon\zeta\eta\vartheta$, sintque parallelae $\alpha\zeta$ $\delta\beta$, ac sit $\alpha\varepsilon : \varepsilon\zeta = \gamma\eta : \eta\zeta$; dico rectam esse quae per ϑ $\varkappa$ ζ transit. Prop. 128



Ducatur per η rectae $\delta\varepsilon$ parallela $\eta\lambda$, et iuncta $\vartheta\varkappa$ producatur ad λ. Quoniam igitur est $\alpha\varepsilon : \varepsilon\zeta = \gamma\eta : \eta\zeta$, vicissim est

$$\alpha\varepsilon : \gamma\eta = \varepsilon\zeta : \eta\zeta.$$

Sed propter parallelas $\vartheta\delta$ $\eta\lambda$ est

$\eta\lambda : \delta\vartheta = \eta\varkappa : \varkappa\delta$, *et* propter parallelas $\delta\beta$ $\gamma\eta$
$\eta\varkappa : \varkappa\delta = \gamma\eta : \beta\delta$; *ergo etiam*
$\eta\lambda : \delta\vartheta = \gamma\eta : \beta\delta$, *et vicissim*
$\eta\lambda : \gamma\eta = \delta\vartheta : \beta\delta$, *sive* propter parallelas $\delta\beta$ $\alpha\varepsilon$
$= \vartheta\varepsilon : \alpha\varepsilon$. *Ergo e contrario est*
$\alpha\varepsilon : \varepsilon\vartheta = \gamma\eta : \eta\lambda$, *et* vicissim
$\alpha\varepsilon : \gamma\eta = \varepsilon\vartheta : \eta\lambda$.

Ergo etiam (*quia erat* $\alpha\varepsilon : \gamma\eta = \varepsilon\zeta : \eta\zeta$) est
$\varepsilon\zeta : \eta\zeta = \varepsilon\vartheta : \eta\lambda$.

1) Media argumentationis membra hoc loco omissa facile supplentur ex priore demonstratione (p. 867).

PROPOS. 128: vide append.

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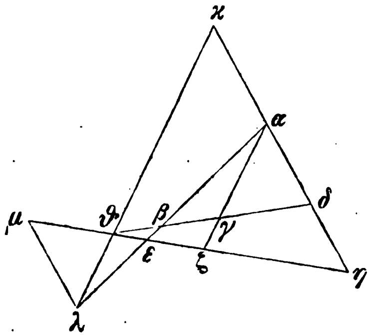
sex octove litterarum 26. καὶ ἐναλλὰξ διὰ τὸ εἶναι δύο παρὰ δύο A[1] in rasura BS, transposuit *Hu* 27. ἐστὶ A[a]BS *παράλληλος ἡ ΕΘ τῇ ΗΛ· εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Θ Λ Ζ,* τουτέστιν ἡ διὰ τῶν Θ Κ Ζ, *ὅπερ*: ~

196 *γ'. Εἰς τρεῖς εὐθείας τὰς ΑΒ ΓΑ ΔΑ διήχθωσαν δύο εὐθεῖαι αἱ ΘΕ ΘΔ· ὅτι ἐστὶν ὡς τὸ ὑπὸ ΘΕ ΗΖ πρὸς τὸ ὑπὸ ΘΗ ΖΕ, οὕτως τὸ ὑπὸ ΘΒ ΔΓ πρὸς τὸ ὑπὸ ΘΔ ΒΓ.*



Ἤχθω διὰ μὲν τοῦ Θ τῇ ΖΓΑ παράλληλος ἡ ΚΛ, καὶ αἱ ΔΑ ΑΒ συμπιπτέτωσαν αὐτῇ κατὰ τὰ Κ Λ σημεῖα, διὰ δὲ τοῦ Λ τῇ ΔΑ παράλληλος ἡ ΛΜ καὶ συμπιπτέτω τῇ ΕΘ ἐπὶ τὸ Μ. ἐπεὶ οὖν ἐστιν ὡς μὲν ἡ ΕΖ πρὸς τὴν ΖΑ, οὕτως ἡ ΕΘ πρὸς τὴν ΘΛ, ὡς δὲ ἡ ΑΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΘΛ πρὸς τὴν ΘΜ (καὶ γὰρ ἡ ΘΚ πρὸς τὴν ΘΗ ἐν παραλλήλῳ), δι' ἴσου ἄρα ἐστὶν ὡς ἡ ΕΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΘΜ· τὸ ἄρα ὑπὸ τῶν ΘΕ ΗΖ ἴσον ἐστὶν τῷ ὑπὸ τῶν ΕΖ ΘΜ. ἄλλο δέ τι τυχὸν τὸ ὑπὸ τῶν ΕΖ ΘΗ· ἔστιν ἄρα ὡς τὸ ὑπὸ τῶν ΕΘ ΗΖ πρὸς τὸ ὑπὸ τῶν ΕΖ ΗΘ, οὕτως τὸ ὑπὸ ΕΖ ΘΜ πρὸς τὸ ὑπὸ ΕΖ ΗΘ, τουτέστιν ἡ ΘΜ πρὸς ΘΗ, τουτέστιν ἡ ΛΘ πρὸς τὴν ΘΚ. κατὰ τὰ αὐτὰ καὶ ὡς ἡ ΚΘ πρὸς τὴν ΘΛ, οὕτως τὸ ὑπὸ ΘΔ ΒΓ πρὸς τὸ ὑπὸ ΘΒ ΓΔ· ἀνάπαλιν ἄρα γίνεται ὡς ἡ ΛΘ πρὸς τὴν ΘΚ, οὕτως τὸ

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2. *Θ Λ Ζ, τουτέστιν ἡ διὰ τῶν Θ Κ Ζ Hu,* ΘΛΖ A(B), ϑ κ λ ζ S (conf. etiam cap. 198 extr.) *ὅπερ* BS, *ο* A 3. *γ'* add. BS 10. 11. *τὰ* ΚΛ A, distinx. BS 12. *τῆι* ΔΑ A² ex *τῆι* ΛΔ 12. 13. *ἡ ΛΜ καὶ*] fortasse *διαχθεῖσα ἡ ΛΜ* 18. 19. *καὶ γὰρ — ἐν παραλλήλῳ* corrupta putant *Co* et *Ge*, at vide Simson. p. 380 sq. 22. *τυχὸν*] forsitan legendum sit *ἔχομεν*; at eadem ratione redit *τυχὸν* infra cap. 204. 205 26. *ὑπὸ* ΘΔΒΓ A, distinx. BS 27. *ἀνάπαλιν* *Co* pro *ἀνάλογον*

Et sunt parallelae $\varepsilon\vartheta$ $\eta\lambda$; recta igitur est quae per puncta ϑ λ ζ*), id est ϑ $\varkappa$ ζ transit, q. e. d.

III. In tres rectas lineas $\alpha\beta$ $\gamma\alpha$ $\delta\alpha$ ducantur duae rectae $\vartheta\varepsilon$ $\vartheta\delta$; dico esse $\vartheta\varepsilon \cdot \eta\zeta : \vartheta\eta \cdot \zeta\varepsilon = \vartheta\beta \cdot \delta\gamma : \vartheta\delta \cdot \beta\gamma$. Prop. 129

Ducatur [1]) per ϑ rectae $\zeta\gamma\alpha$ parallela $\varkappa\lambda$, et *huic* recta $\delta\alpha$ *producta* occurrat in $\varkappa$, itemque recta $\alpha\beta$ in λ, et per λ rectae $\delta\alpha$ parallela *ducatur* $\lambda\mu$, cui $\varepsilon\vartheta$ *producta* occurrat in μ. Quoniam igitur *propter parallelas* $\alpha\zeta$ $\lambda\vartheta$ est

$\varepsilon\zeta : \zeta\alpha = \varepsilon\vartheta : \vartheta\lambda$, et propter parallelas $\alpha\zeta$ $\varkappa\vartheta$ et $\varkappa\eta$ $\mu\lambda$ est $\alpha\zeta : \zeta\eta = \varkappa\vartheta : \vartheta\eta = \lambda\vartheta : \vartheta\mu$, itaque

$\alpha\zeta : \zeta\eta = \vartheta\lambda : \vartheta\mu$, ex aequali igitur est

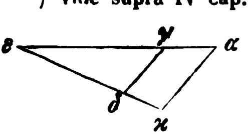
$\varepsilon\zeta : \zeta\eta = \varepsilon\vartheta : \vartheta\mu$;

ergo $\zeta\eta \cdot \varepsilon\vartheta = \varepsilon\zeta \cdot \vartheta\mu$. Sed *fiat proportio ad* aliud rectangulum $\varepsilon\zeta \cdot \vartheta\eta$; est igitur

$$\zeta\eta \cdot \varepsilon\vartheta : \varepsilon\zeta \cdot \vartheta\eta = \varepsilon\zeta \cdot \vartheta\mu : \varepsilon\zeta \cdot \vartheta\eta, \text{ id est}$$
$$= \vartheta\mu : \vartheta\eta, \text{ id est}$$
$$= \lambda\vartheta : \vartheta\varkappa.$$

Eadem ratione [2]) *fit* etiam $\varkappa\vartheta : \vartheta\lambda = \vartheta\delta \cdot \beta\gamma : \vartheta\beta \cdot \gamma\delta$; e contrario igitur fit

*) Vide supra IV cap. 21. Etenim, ut omittamus illum trium circulorum contactum, de quo est libri IV propositio 13, in eadem propositione conversa, id est cap. 21, demonstratio deducitur ad huiusmodi lemma: Si duae parallelae, velut $\alpha\varkappa$ $\gamma\delta$, rectam $\alpha\varepsilon$ in punctis α γ secent, sitque $\alpha\varkappa : \gamma\delta = \alpha\varepsilon : \varepsilon\gamma$, dico puncta $\varkappa$ δ ε in eâdem rectâ esse. Quod illic primum ratione apagogica, tum (p. 212. 213) auxilio parallelogrammi ostenditur. Idem lemma adhibitum esse in VII libri propos. 64 et 118 supra p. 769 adnot. * et 853 adnot. 2 commemoravimus; praeterea conf. infra propos. 130 sq.



PROPOS. 129: Simson p. 380 sqq., Breton p. 221 sq., Chasles p. 75 sq. 82. 87 sq. 101 sq. cet., idem *Aperçu historique* p. 33 sqq. edit. Paris. secundae (p. 31 sqq. versionis German.), Baltzer *Elemente* II p. 365 sqq. edit. IV.

1) Rursus, ut supra ad propos. 127, plures figuras exhibent codices, e quibus una tantummodo (scilicet secunda in cod. et apud Commandinum, quinta apud Gerhardtum) litterarum ordinem $\zeta\gamma\alpha$ in contextu traditum servat. Hanc igitur descripsimus; reliquarum quinque speciem satis accuratam praebet Commandinus. Sunt hi diversi eiusdem propositionis casus, at neutiquam omnes qui fingi possunt; velut septimam figuram a nobis addi necesse fuit in append. ad propos. 139, octavam in append. ad propos. 143.

2) Demonstrat haec singillatim Simsonus p. 381 productâ $\beta\vartheta$ ad ν punctum concursus cum $\lambda\mu$.

ὑπὸ ΘΒ ΓΔ πρὸς τὸ ὑπὸ ΘΔ ΒΓ. ὡς δὲ ἡ ΛΘ πρὸς τὴν ΘΚ, οὕτως ἐδείχθη τὸ ὑπὸ ΕΘ ΗΖ πρὸς τὸ ὑπὸ ΕΖ ΗΘ· καὶ ὡς ἄρα τὸ ὑπὸ ΕΘ ΗΖ πρὸς τὸ ὑπὸ ΕΖ ΗΘ, οὕτως τὸ ὑπὸ ΘΒ ΓΔ πρὸς τὸ ὑπὸ ΘΔ ΒΓ.

197 Διὰ δὲ τοῦ συνημμένου οὕτως. ἐπεὶ τοῦ ὑπὸ ΘΕ ΗΖ πρὸς τὸ ὑπὸ ΘΗ ΖΕ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΘΕ πρὸς τὴν ΕΖ καὶ τοῦ ὃν ἔχει ἡ ΖΗ πρὸς τὴν ΗΘ, καὶ ἔστιν ὡς μὲν ἡ ΘΕ πρὸς τὴν ΕΖ, οὕτως ἡ ΘΛ πρὸς τὴν ΖΛ, ὡς δὲ ἡ ΖΗ πρὸς τὴν ΗΘ, οὕτως ἡ ΖΛ πρὸς τὴν ΘΚ, τὸ ἄρα ὑπὸ ΘΕ ΗΖ πρὸς τὸ ὑπὸ ΘΗ ΕΖ συνῆπται ἔκ τε τοῦ ὃν ἔχει ἡ ΘΛ πρὸς τὴν ΖΛ καὶ τοῦ ὃν ἔχει ἡ ΖΛ πρὸς τὴν ΘΚ. ὁ δὲ συνημμένος ἔκ τε τοῦ τῆς ΘΛ πρὸς τὴν ΖΛ καὶ τοῦ τῆς ΖΛ πρὸς τὴν ΘΚ ὁ αὐτός ἐστιν τῷ τῆς ΘΛ πρὸς τὴν ΘΚ· ἔστιν ἄρα ὡς τὸ ὑπὸ ΘΕ ΗΖ πρὸς τὸ ὑπὸ ΘΗ ΖΕ, οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ. διὰ ταὐτὰ καὶ ὡς τὸ ὑπὸ ΘΔ ΒΓ πρὸς τὸ ὑπὸ ΘΒ ΓΔ, οὕτως ἐστὶν ἡ ΘΚ πρὸς τὴν ΘΛ. καὶ ἀνάπαλίν ἐστιν ὡς τὸ ὑπὸ ΘΒ ΓΔ πρὸς τὸ ὑπὸ ΘΔ ΒΓ, οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ. ἦν δὲ καὶ ὡς τὸ ὑπὸ τῶν ΘΕ ΖΗ πρὸς τὸ ὑπὸ ΘΗ ΖΕ, οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ· καὶ ὡς ἄρα τὸ ὑπὸ τῶν ΘΕ ΖΗ πρὸς τὸ ὑπὸ ΘΗ ΖΕ, οὕτως τὸ ὑπὸ ΘΒ ΓΔ πρὸς τὸ ὑπὸ ΘΔ ΒΓ.

198 δʹ. Καταγραφὴ ἡ ΑΒΓΔΕΖΗΘΚΛ, ἔστω δὲ ὡς τὸ ὑπὸ ΑΖ ΒΓ πρὸς τὸ ὑπὸ ΑΒ ΓΖ, οὕτως τὸ ὑπὸ ΑΖ ΔΕ πρὸς τὸ ὑπὸ ΑΔ ΕΖ· ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Θ Η Ζ σημείων.

Ἐπεί ἐστιν ὡς τὸ ὑπὸ ΑΖ ΒΓ πρὸς τὸ ὑπὸ ΑΒ ΓΖ, οὕτως τὸ ὑπὸ ΑΖ ΔΕ πρὸς τὸ ὑπὸ ΑΔ ΕΖ, ἐναλλάξ ἐστιν

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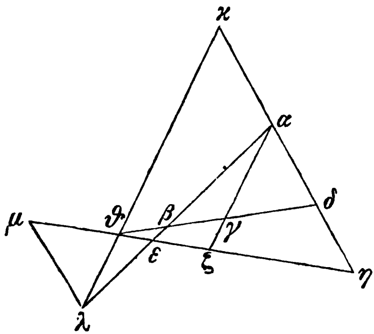
2. 3. πρὸς τὸ ὑπὸ ΕΖΗΘ A, distinx. BS, item posthac in eodem lemmate quaternas litteras coniunctas habet A 3. ὑπὸ ante ΕΖ ΗΘ add. S 7. πρὸς τὴν ΕΖ καὶ τοῦ ὃν ἔχει ἡ ΖΗ bis scripta in ABS, corr. *Co* 16. ταὐτὰ *Hu* pro ταῦτα 18. 19. οὕτως ἡ ΛΘ πρὸς τὴν ΘΚ την δὲ καὶ A(B), corr. S 20. οὕτως ἡ ΘΛ πρὸς τὴν ΘΚ add. *Ge* 20. 21. καὶ ὡς — ὑπὸ τῶν ΘΗ ΖΕ add. *Co* (in quibus τῶν ante ΘΗ ΖΕ del. *Ge*) 23. δʹ add. BS ΑΒΓΔΕΖ ΘΗΙΚΛ A(BS), corr. *Co* 24. ὑπὸ ΑΖΒΓ A, distinx. BS ὑπὸ ΑΒΓΖ A, distinx.

$\vartheta\beta \cdot \gamma\delta : \vartheta\delta \cdot \beta\gamma = \lambda\vartheta : \vartheta\varkappa$; ergo secundum ea quae *modo* demonstrata sunt

$\zeta\eta \cdot \varepsilon\vartheta : \varepsilon\zeta \cdot \vartheta\eta = \vartheta\beta \cdot \gamma\delta : \vartheta\delta \cdot \beta\gamma$.



Per formulam compositae proportionis sic. Quoniam est

$$\frac{\vartheta\varepsilon \cdot \eta\zeta}{\vartheta\eta \cdot \zeta\varepsilon} = \frac{\vartheta\varepsilon}{\zeta\varepsilon} \cdot \frac{\eta\zeta}{\eta\vartheta},$$

estque (*propter parallelas* $\vartheta\lambda$ $\alpha\zeta$) $\vartheta\varepsilon : \zeta\varepsilon = \vartheta\lambda : \zeta\alpha$, et (*propter parallelas* $\alpha\zeta$ $\varkappa\vartheta$) $\eta\zeta : \eta\vartheta = \zeta\alpha : \vartheta\varkappa$, est igitur

$$\frac{\vartheta\varepsilon \cdot \eta\zeta}{\vartheta\eta \cdot \zeta\varepsilon} = \frac{\vartheta\lambda}{\zeta\alpha} \cdot \frac{\zeta\alpha}{\vartheta\varkappa} = \frac{\vartheta\lambda}{\vartheta\varkappa}.$$

Eadem ratione est etiam

$$\frac{\vartheta\delta \cdot \beta\gamma}{\vartheta\beta \cdot \gamma\delta} = \frac{\vartheta\varkappa}{\vartheta\lambda}, \text{ et e contrario } \frac{\vartheta\beta \cdot \gamma\delta}{\vartheta\delta \cdot \beta\gamma} = \frac{\vartheta\lambda}{\vartheta\varkappa};$$

ergo secundum ea quae modo demonstrata sunt

$$\frac{\vartheta\varepsilon \cdot \eta\zeta}{\vartheta\eta \cdot \zeta\varepsilon} = \frac{\vartheta\beta \cdot \gamma\delta}{\vartheta\delta \cdot \beta\gamma}.$$

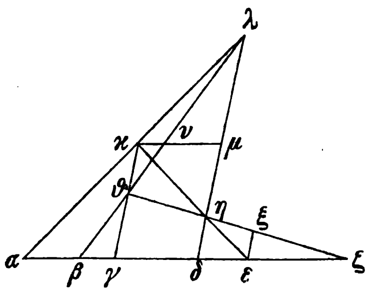
IV. Figura $\alpha\beta\gamma\delta\varepsilon\zeta\eta\vartheta\varkappa\lambda$ *), sit autem $\alpha\zeta \cdot \beta\gamma : \alpha\beta \cdot \gamma\zeta =$ Prop. 130
$\alpha\zeta \cdot \delta\varepsilon : \alpha\delta \cdot \varepsilon\zeta$; dico rectam esse quae per ϑ η ζ transit.

Quoniam est $\alpha\zeta \cdot \beta\gamma : \alpha\beta \cdot \gamma\zeta = \alpha\zeta \cdot \delta\varepsilon : \alpha\delta \cdot \varepsilon\zeta$, vicissim *igitur* est

PROPOS. 130: Simson p. 382 sq., Breton p. 222 sq., Chasles p. 74 sq. 88. 102. 108 sqq., idem *Aperçu historique* p. 36. 376 sqq. (p. 33. 325 sqq. versionis German.).

*) Quattuor punctorum dispositiones, scilicet $\alpha\varepsilon\delta\gamma\beta\zeta$, $\alpha\beta\gamma\delta\varepsilon\zeta$, $\alpha\varepsilon\gamma\delta\beta\zeta$, $\alpha\beta\delta\gamma\varepsilon\zeta$, et octo figuras exhibent codices, quas vide apud Commandinum; quintam dispositionem $\alpha\varepsilon\beta\zeta\gamma\delta$ addit Chasles; nos cum Bretono repetivimus eam tantum figuram, quae secunda est in codicibus; quae quidem una praeter punctorum seriem $\alpha\beta\gamma\delta\varepsilon\zeta$ etiam in altera recta ordinem $\vartheta\eta\zeta$ exhibet.

B, ὑπὸ $\overline{\alpha\beta}$ $\overline{\zeta\gamma}$ S οὕτω A^3BS 25. ὑπὸ $\overline{A\Delta EZ}$ A, distinx. BS, item vs. 28 τῶν $\overline{\Theta HZ}$ A, distinx. BS ὡς τὸ ὑπὸ *ΑΖ ΒΓ* πρὸς τὸ ὑπὸ *ΑΖ ΔΕ*, τουτέστιν ὡς ἡ *ΒΓ* πρὸς τὴν *ΔΕ*, οὕτως τὸ ὑπὸ *ΑΒ ΓΖ* πρὸς τὸ ὑπὸ *ΑΔ ΕΖ*. ἀλλ' ὁ μὲν τῆς *ΒΓ* πρὸς τὴν *ΔΕ* συνῆπται λόγος, ἐὰν διὰ τοῦ *Κ* τῇ *ΑΖ* παράλληλος ἀχθῇ ἡ *ΚΜ*, ἔκ τε τοῦ τῆς *ΒΓ* πρὸς *ΚΝ* καὶ τῆς *ΚΝ* πρὸς *ΚΜ* καὶ ἔτι τοῦ τῆς *ΚΜ* πρὸς *ΔΕ*, ὁ δὲ τοῦ ὑπὸ *ΑΒ ΓΖ* πρὸς τὸ ὑπὸ *ΑΔ ΕΖ* συνῆπται ἔκ τε τοῦ τῆς *ΒΑ* πρὸς *ΑΔ* καὶ τοῦ τῆς *ΓΖ* πρὸς τὴν *ΖΕ*. κοινὸς ἐκκεκρούσθω ὁ τῆς *ΒΑ* πρὸς *ΑΔ* ὁ αὐτὸς ὢν τῷ τῆς *ΝΚ* πρὸς *ΚΜ*· λοιπὸν ἄρα ὁ τῆς *ΓΖ* πρὸς τὴν *ΖΕ* συνῆπται ἔκ τε τοῦ τῆς *ΒΓ* πρὸς τὴν *ΚΝ*, τουτέστιν τοῦ τῆς *ΘΓ* πρὸς τὴν *ΚΘ*, καὶ τοῦ τῆς *ΚΜ* πρὸς τὴν *ΔΕ*, τουτέστιν τοῦ τῆς *ΚΗ* πρὸς τὴν *ΗΕ*· εὐθεῖα ἄρα ἡ διὰ τῶν *Θ Η Ζ*.



Ἐὰν γὰρ διὰ τοῦ *Ε* τῇ *ΘΓ* παράλληλον ἀγάγω τὴν *ΕΞ*, καὶ ἐπιζευχθεῖσα ἡ *ΘΗ* ἐκβληθῇ ἐπὶ τὸ *Ξ*, ὁ μὲν τῆς *ΚΗ* πρὸς τὴν *ΗΕ* λόγος ὁ αὐτός ἐστιν τῷ τῆς *ΚΘ* πρὸς τὴν *ΕΞ*, ὁ δὲ συνημμένος ἔκ τε τοῦ τῆς *ΓΘ* πρὸς τὴν *ΘΚ* καὶ τοῦ τῆς *ΘΚ* πρὸς τὴν *ΕΞ* μεταβάλλεται εἰς τὸν τῆς *ΘΓ* πρὸς *ΕΞ* λόγον, καὶ ὁ τῆς *ΓΖ* πρὸς *ΖΕ* λόγος ὁ αὐτὸς τῷ τῆς *ΓΘ* πρὸς τὴν *ΕΞ*· παραλλήλου οὔσης τῆς *ΓΘ* τῇ *ΕΞ* εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν *Θ Ξ Ζ* (τοῦτο γὰρ φανερόν), ὥστε καὶ ἡ διὰ τῶν *Θ Η Ζ* εὐθεῖά ἐστιν.

199 ε'. Ἐὰν ᾖ καταγραφὴ ἡ *ΑΒΓΔΕΖΗΘ*, γίνεται ὡς ἡ *ΑΔ* πρὸς τὴν *ΔΓ*, οὕτως ἡ *ΑΒ* πρὸς τὴν *ΒΓ*. ἔστω οὖν ὡς ἡ *ΑΔ* πρὸς τὴν *ΔΓ*, οὕτως ἡ *ΑΒ* πρὸς τὴν *ΒΓ*· ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν *Α Η Θ*.

Ἤχθω διὰ τοῦ *Η* τῇ *ΑΔ* παράλληλος ἡ *ΚΛ*. ἐπεὶ

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2. 3. ὑπὸ ΑΒΓΖ πρὸς τὸ ὑπὸ ΑΔΕΖ A, distinx. BS, item vs. 10. 11   5. τοῦ add. BS   7. 8. πρὸς ΚΗ καὶ τῆς ΚΝ A, πρὸς κη καὶ τῆς κη S, corr. B   9. τοῦ τῆς Co pro τὸ τῆς   13. πρὸς τὴν ΔΕ

$$\frac{\alpha\zeta\cdot\beta\gamma}{\alpha\zeta\cdot\delta\varepsilon}=\frac{\beta\gamma}{\delta\varepsilon}=\frac{\alpha\beta\cdot\gamma\zeta}{\alpha\delta\cdot\varepsilon\zeta}=\frac{\alpha\beta}{\alpha\delta}\cdot\frac{\gamma\zeta}{\varepsilon\zeta}.$$

Sed si per $\varkappa$ rectae $\alpha\zeta$ parallela ducatur $\varkappa\mu$, *quae rectam* $\beta\lambda$ *secet in* ν, est

$$\frac{\beta\gamma}{\delta\varepsilon}=\frac{\beta\gamma}{\varkappa\nu}\cdot\frac{\varkappa\nu}{\varkappa\mu}\cdot\frac{\varkappa\mu}{\delta\varepsilon};$$ *est igitur*

$$\frac{\alpha\beta}{\alpha\delta}\cdot\frac{\gamma\zeta}{\varepsilon\zeta}=\frac{\beta\gamma}{\varkappa\nu}\cdot\frac{\varkappa\nu}{\varkappa\mu}\cdot\frac{\varkappa\mu}{\delta\varepsilon}.$$

Dividendo tollatur ab altera parte proportio $\alpha\beta : \alpha\delta$, ab altera quae huic aequalis est $\varkappa\nu : \varkappa\mu$; relinquitur igitur

$$\frac{\gamma\zeta}{\varepsilon\zeta}=\frac{\beta\gamma}{\varkappa\nu}\cdot\frac{\varkappa\mu}{\delta\varepsilon}=\frac{\gamma\vartheta}{\vartheta\varkappa}\cdot\frac{\varkappa\eta}{\eta\varepsilon};$$

recta igitur est quae per $\vartheta\ \eta\ \zeta$ transit.

Etenim si per ε rectae $\vartheta\gamma$ parallelam ducam $\varepsilon\xi$, et iuncta $\vartheta\eta$ producatur ad ξ, est

$$\frac{\varkappa\eta}{\eta\varepsilon}=\frac{\vartheta\varkappa}{\varepsilon\xi},\ \text{et}\ \frac{\gamma\vartheta}{\vartheta\varkappa}\cdot\frac{\vartheta\varkappa}{\varepsilon\xi}=\frac{\gamma\vartheta}{\varepsilon\xi},\ \text{itaque}\ \frac{\gamma\zeta}{\varepsilon\zeta}=\frac{\gamma\vartheta}{\varepsilon\xi}.$$

Et quia $\gamma\vartheta\ \varepsilon\xi$ parallelae sunt, recta igitur est quae per $\vartheta\ \xi\ \zeta$ transit (hoc enim manifestum est[1]); itaque etiam quae per $\vartheta\ \eta\ \zeta$ transit recta est.

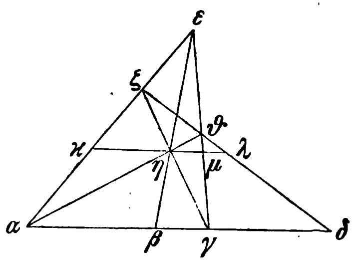
V. Si sit figura $\alpha\beta\gamma\delta\varepsilon\zeta\eta\vartheta$, *et reliqua similiter ac supra* (*propos. 127*) *supponantur*, fit $\alpha\delta : \delta\gamma = \alpha\beta : \beta\gamma$. Iam *vero* Prop. 131 *supponatur* esse $\alpha\delta : \delta\gamma = \alpha\beta : \beta\gamma$; dico rectam esse quae per $\alpha\ \eta\ \vartheta$ transit.

Ducatur per η rectae $\alpha\delta$ parallela $\varkappa\lambda$, *quae rectam* $\varepsilon\gamma$

1) Conf. supra p. 871 adnot. *.

PROPOS. 131: Breton p. 223 sq., Chasles p. 74 sq. 88. 103. 108 sqq., idem *Aperçu historique* p. 36 edit. Parisinae secundae (p. 33 versionis German.), Baltzer *Elemente* II p. 370.

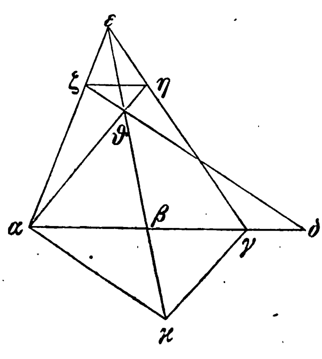
ABS, corr. *Co*   κοινὸς V et super vs. S, κ° ABS   14. τῷ τῆς ηκ S cod. *Co* (recte NK AB), item vs. 16. τὴν κη S   λοιπὸς *Ge*   17. τοῦ add. *Hu*   18. διὰ τῶν ΘΗΚ A(BS), corr. *Co*   19. τῇ ΒΓ παράλληλον ABS, corr. *Co* in Lat. versione   τὴν ΕΞ *Co* pro τὴν ΕΖ   20. ἐπιζευχθεῖσα ἡ *Hu* auctore *Co* pro ἐπιζευχθείσης τῆς   23. μεταβάλλεται *Hu* auctore *Co* pro μεταβαλλόμενος   εἰς τὸ τῆς AB, corr. S   25. πρὸς τὴν ΕΞ *Co* pro πρὸς τὴν ΘΖ   26. τῶν ΘΞΖ A, distinx. BS   27. τῶν ΘΗΖ A, distinx. BS   28. ε' add. BS   31. τῶν ΛΗΘ A, distinx. BS οὖν ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΓ, ἀλλ᾽ ὡς μὲν ἡ ΑΔ πρὸς τὴν ΔΓ, οὕτως ἡ ΚΛ πρὸς τὴν ΛΗ, ὡς δὲ ἡ ΑΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΚΗ πρὸς τὴν ΗΜ, καὶ ὡς ἄρα ἡ ΚΛ πρὸς τὴν ΛΗ, οὕτως ἡ ΚΗ πρὸς τὴν ΗΜ, καὶ λοιπὴ ἡ ΗΛ πρὸς λοιπὴν τὴν ΛΜ ἐστὶν ὡς ἡ ΚΛ πρὸς τὴν ΛΗ, τουτέστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΓ. ἐναλλὰξ ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΗΛ, οὕτως ἡ ΓΔ πρὸς τὴν ΛΜ, τουτέστιν ἡ ΔΘ πρὸς ΘΛ. καὶ ἔστι παράλληλος ἡ ΗΛ τῇ ΑΔ· εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Α Η Θ σημείων· τοῦτο γὰρ φανερόν.



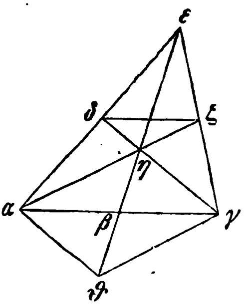
200 ϛʹ. Πάλιν ἐὰν ᾖ καταγραφή, καὶ παράλληλος ἡ ΔΖ τῇ ΒΓ, γίνεται ἴση ἡ ΑΒ τῇ ΒΓ. ἔστω οὖν ἴση· ὅτι παράλληλος.

Ἔστιν δέ· ἐὰν γὰρ ἐπὶ τῆς ΕΒ θῶ τῇ ΗΒ ἴσην τὴν ΒΘ, καὶ ἐπιζεύξω τὰς ΑΘ ΘΓ, γίνεται παραλληλόγραμμον τὸ ΑΘΓΗ, καὶ διὰ τοῦτό ἐστιν ὡς ἡ ΑΔ πρὸς τὴν ΔΕ, οὕτως ἡ ΓΖ πρὸς τὴν ΖΕ (ἑκατέρων γὰρ τῶν εἰρημένων ὁ αὐτός ἐστιν τῷ τῆς ΘΗ πρὸς τὴν ΗΕ λόγος), ὥστε παράλληλός ἐστιν ἡ ΔΖ τῇ ΑΓ.

201 ζʹ. Ἔστω καταγραφή, καὶ τῶν ΔΒ ΒΓ μέση ἀνάλογον ἔστω ἡ ΒΑ· ὅτι παράλληλός ἐστιν ἡ ΖΗ τῇ ΑΓ.



Ἐκβεβλήσθω ἡ ΕΒ, καὶ διὰ τοῦ Α τῇ ΔΖ εὐθεῖα παράλληλος ἤχθω ἡ ΑΚ, καὶ ἐπεζεύχθω ἡ ΓΚ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΓΒ πρὸς τὴν ΒΑ, οὕτως ἡ ΑΒ πρὸς τὴν ΒΔ, ὡς δὲ ἡ ΑΒ πρὸς τὴν ΒΔ, οὕτως ἡ ΚΒ πρὸς τὴν ΒΘ, καὶ ὡς ἄρα ἡ ΓΒ πρὸς τὴν ΒΑ, *secet in* μ. Quoniam igitur est $\alpha\delta : \delta\gamma = \alpha\beta : \beta\gamma$, et $\alpha\delta : \delta\gamma = \varkappa\lambda : \lambda\eta$, et $\alpha\beta : \beta\gamma = \varkappa\eta : \eta\mu$, est igitur $\varkappa\lambda : \lambda\eta = \varkappa\eta : \eta\mu$, et per subtractionem proportionis $\eta\lambda : \lambda\mu = \varkappa\lambda : \lambda\eta$; id est $\alpha\delta : \delta\gamma = \eta\lambda : \lambda\mu$. Vicissim est $\alpha\delta : \eta\lambda = \delta\gamma : \lambda\mu = \delta\vartheta : \vartheta\lambda$. Et sunt parallelae $\eta\lambda$ $\alpha\delta$; recta igitur est quae per puncta α η ϑ transit; hoc enim manifestum est [1]).



VI. Rursus si sit figura $\alpha\beta\gamma\delta\varepsilon\zeta\eta$, et parallelae $\delta\zeta$ $\beta\gamma$, fit $\alpha\beta = \beta\gamma$. Iam *supponatur* esse $\alpha\beta = \beta\gamma$; dico parallelas esse $\delta\zeta$ $\beta\gamma$. Prop. 132

Sunt vero; nam si in *producta* $\varepsilon\beta$ faciam $\beta\vartheta = \eta\beta$, et iungam $\alpha\vartheta$ $\vartheta\gamma$, fit parallelogrammum $\alpha\vartheta\gamma\eta$ *). Et propterea est $\alpha\delta : \delta\varepsilon = \gamma\zeta : \zeta\varepsilon$ (quoniam utraque proportio est $= \vartheta\eta : \eta\varepsilon$), itaque parallelae sunt $\delta\zeta$ $\alpha\gamma$.

VII. Sit figura $\alpha\beta\gamma\delta\varepsilon\zeta\eta\vartheta$, et rectarum $\beta\gamma$ $\beta\delta$ media proportionalis $\alpha\beta$; dico parallelas esse $\zeta\eta$ $\alpha\gamma$. Prop. 133

Producatur $\varepsilon\beta$, et per α rectae $\zeta\delta$ parallela ducatur $\alpha\varkappa$, iungaturque $\gamma\varkappa$. Iam quia est $\beta\gamma : \alpha\beta = \alpha\beta : \beta\delta$, et $\alpha\beta : \beta\delta = \varkappa\beta : \beta\vartheta$ (*in similibus triangulis* $\alpha\beta\varkappa$ $\delta\beta\vartheta$), est igitur $\beta\gamma : \alpha\beta$

1) Conf. supra p. 871, adnot. *.
PROPOS. 132: Simson p. 359, Breton p. 224, Chasles p. 74 sq. 89. 103 sqq., idem *Aperçu historique* l. c.
*) Nimirum quia diametri $\alpha\gamma$ $\vartheta\eta$ sese dimidias secant. Si ad Euclidem refugimus, demonstrandum est esse triangulum $\alpha\beta\eta \cong \gamma\beta\vartheta$, et triangulum $\gamma\beta\eta \cong \alpha\beta\vartheta$ (elem. 1, 4), quo facto reliqua sequuntur ex 1, 27.
PROPOS. 133: Breton p. 224, Chasles p. 74 sq. 89. 104 sqq.

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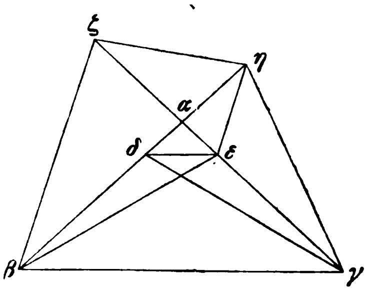
2. 3. *οὕτως ἡ* $\overline{KΛ}$ *πρὸς τὴν* $\overline{ΛM}$ ABS, corr. *Co* 3. *ὡς δὲ ἡ* $\overline{ΛE}$ ABS, corr. *Ge* 5. 6. *ἡ* $\overline{HΛ}$ *πρὸς τὴν* $\overline{ΛM}$ ABS, corr. *Ge* 7—10. *καὶ λοιπὴ — πρὸς τὴν ΛH* del. *Co* 9. 10. *ἐστὶν ὡς ἡ* $\overline{KM}$ *πρὸς τὴν* $\overline{ΛM}$ ABS, corr. *Ge* 11. 12. *πρὸς τὴν* $\overline{ΑΓ}$ *ἀνάλογον ἐστιν — πρὸς τὴν* $\overline{HΔ}$ ABS, corr. *Co* 14. *ἔστι* A[s]BS *τῇ ΑΔ Co* pro *τῆι* $\overline{ΔΘ}$ 15. *τῶν* $\overline{ΑHΘ}$ A, distinx. BS 16. *ϛ'* add. BS 19. *ἐπὶ τῆς EB Hu* pro *τὴν* $\overline{EB}$, del. *Co* 22. *ἑκατερα* AB, *ἑκατέρᾳ* S, corr. *Hu* 23. *λόγος* BS, *λόγον* A 25. *ζ'* add. BS *καὶ Co* pro *κατὰ τῶν* $\overline{AB}$ $\overline{BΓ}$ *μέση* ABS, *τῶν AB BΓ τρίτη Co* (rectius *τῶν ΓB AB τρίτη* Bretonus), corr. *Hu* 26. *ἡ BΑ Hu* pro *ἡ* $\overline{BΔ}$ 28. *ἐκβεβλήσθω ἡ EB Co* pro *ἐκβληθεῖσα ἡ* $\overline{AB}$ 36. *τὴν BΑ Co* pro *τὴν* $\overline{BΔ}$

οὕτως ἡ ΚΒ πρὸς τὴν ΒΘ· παράλληλος ἄρα ἐστὶν ἡ ΑΘ τῇ ΚΓ. ἔστιν οὖν πάλιν ὡς ἡ ΑΖ πρὸς τὴν ΖΕ, οὕτως ἡ ΓΗ πρὸς τὴν ΗΕ (ἑκατέρων γὰρ τῶν εἰρημένων λόγος ὁ αὐτός ἐστιν τῷ τῆς ΚΘ πρὸς τὴν ΘΕ), ὥστε παράλληλός ἐστιν ἡ ΖΗ τῇ ΑΓ.

202 *η΄. Ἔστω βωμίσκος ὁ ΑΒΓΔΕΖΗ, καὶ ἔστω παράλληλος ἡ μὲν ΔΕ τῇ ΒΓ, ἡ δὲ ΕΗ τῇ ΒΖ· ὅτι καὶ ἡ ΔΖ τῇ ΓΗ παράλληλός ἐστιν.*



Ἐπεζεύχθωσαν αἱ ΒΕ ΔΓ ΖΗ· ἴσον ἄρα ἐστὶν τὸ ΔΒΕ τρίγωνον τῷ ΔΓΕ τριγώνῳ. κοινὸν προσκείσθω τὸ ΔΑΕ τρίγωνον· ὅλον ἄρα τὸ ΑΒΕ τρίγωνον ὅλῳ τῷ ΓΔΑ τριγώνῳ ἴσον ἐστίν. πάλιν ἐπεὶ παράλληλός ἐστιν ἡ ΒΖ τῇ ΕΗ, ἴσον ἐστὶν τὸ ΒΖΕ τρίγωνον τῷ ΒΖΗ τριγώνῳ. κοινὸν ἀφῃρήσθω τὸ ΑΒΖ τρίγωνον· λοιπὸν ἄρα τὸ ΑΒΕ τρίγωνον λοιπῷ τῷ ΑΗΖ τριγώνῳ ἴσον ἐστίν. ἀλλὰ τὸ ΑΒΕ τρίγωνον τῷ ΑΓΔ τριγώνῳ ἐστὶν ἴσον· καὶ τὸ ΑΓΔ ἄρα τρίγωνον τῷ ΑΖΗ τριγώνῳ ἴσον ἐστίν. κοινὸν προσκείσθω τὸ ΑΓΗ τρίγωνον· ὅλον ἄρα τὸ ΓΔΗ τρίγωνον ὅλῳ τῷ ΓΖΗ τριγώνῳ ἴσον ἐστίν. καὶ ἔστιν ἐπὶ τῆς αὐτῆς βάσεως τῆς ΓΗ· παράλληλος ἄρα ἐστὶν ἡ ΓΗ τῇ ΔΖ.

203 *θ΄. Ἔστω τρίγωνον τὸ ΑΒΓ, καὶ ἐν αὐτῷ διήχθωσαν αἱ ΑΔ ΑΕ, καὶ τῇ ΒΓ παράλληλος ἤχθω ἡ ΖΗ, καὶ κεκλάσθω ἡ ΖΘΗ, ἔστω δὲ ὡς ἡ ΒΘ πρὸς τὴν ΘΓ, οὕτως ἡ ΔΘ πρὸς τὴν ΘΕ· ὅτι παράλληλός ἐστιν ἡ ΚΛ τῇ ΒΓ.*

Ἐπεὶ γάρ ἐστιν ὡς ἡ ΒΘ πρὸς τὴν ΘΓ, οὕτως ἡ ΔΘ πρὸς τὴν ΘΕ, λοιπὴ ἄρα ἡ ΒΔ πρὸς λοιπὴν τὴν ΓΕ ἐστὶν ὡς ἡ ΔΘ πρὸς τὴν ΘΕ. ὡς δὲ ἡ ΒΔ πρὸς τὴν ΕΓ, οὕτως

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1. ἡ ΑΘ Co pro ἡ ΔΘ   2. πρὸς τὴν ΖΕ Co pro πρὸς τὴν $\overline{ΖΓ}$
3. ἑκατέρων Hu, ἑκατέρᾳ A*BS   4. πρὸς τὴν ΘΕ Co pro πρὸς τὴν

$= \varkappa\beta : \beta\vartheta$; ergo parallelae sunt $\alpha\vartheta$ $\varkappa\gamma$ (*propter similitudinem triangulorum* $\alpha\beta\vartheta$ $\gamma\beta\varkappa$). Iam rursus est $\alpha\zeta : \zeta\varepsilon = \gamma\eta : \eta\varepsilon$ (utraque enim proportio est $= \varkappa\vartheta : \vartheta\varepsilon$), itaque parallelae sunt $\zeta\eta$ $\alpha\gamma$.

VIII. Sit figura arae inaequalibus lateribus exstructae Prop. 134
similis, *quae* βωμίσκος *vocatur* [1]), in eaque $\delta\varepsilon$ parallela rectae $\beta\gamma$, et $\varepsilon\eta$ rectae $\beta\zeta$; dico etiam $\delta\zeta$ rectae $\gamma\eta$ parallelam esse.

Iungantur $\beta\varepsilon$ $\delta\gamma$ $\zeta\eta$; ergo triangulum $\delta\varepsilon\beta$ aequale est triangulo $\delta\varepsilon\gamma$. Commune addatur $\delta\varepsilon\alpha$ triangulum; totum igitur $\alpha\beta\varepsilon$ triangulum toti $\alpha\gamma\delta$ triangulo aequale est. Rursus quia $\beta\zeta$ $\varepsilon\eta$ parallelae sunt, aequalia sunt triangula $\beta\zeta\varepsilon$ $\beta\zeta\eta$. Commune subtrahatur $\beta\zeta\alpha$ triangulum; reliquum igitur $\alpha\beta\varepsilon$ triangulum reliquo $\alpha\eta\zeta$ aequale est. Sed erat triangulum $\alpha\beta\varepsilon$ aequale triangulo $\alpha\gamma\delta$; ergo etiam triangulum $\alpha\gamma\delta$ triangulo $\alpha\eta\zeta$ aequale est. Commune addatur $\alpha\gamma\eta$ triangulum; ergo totum $\gamma\delta\eta$ toti $\gamma\zeta\eta$ aequale est. Et sunt *haec triangula* in eadem basi $\gamma\eta$; ergo $\delta\zeta$ rectae $\gamma\eta$ parallela est.

IX. Sit triangulum $\alpha\beta\gamma$, in eoque ducantur rectae $\alpha\delta$ $\alpha\varepsilon$, Prop. 135
et rectae $\beta\gamma$ parallela ducatur $\zeta\eta$, et *a rectae* $\delta\varepsilon$ *puncto* ϑ ducantur $\vartheta\zeta$ $\vartheta\eta$, sitque $\beta\vartheta : \vartheta\gamma = \delta\vartheta : \vartheta\varepsilon$; dico parallelam esse $\varkappa\lambda$ rectae $\beta\gamma$.

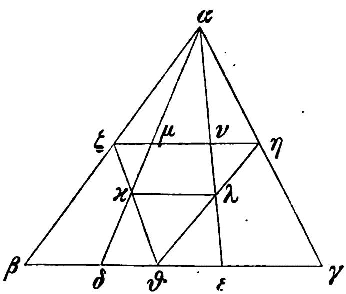
Quoniam est $\beta\vartheta : \vartheta\gamma = \delta\vartheta : \vartheta\varepsilon$, per subtractionem proportionis igitur est $\beta\delta : \varepsilon\gamma = \delta\vartheta : \vartheta\varepsilon$. Sed *propter paralle-*

PROPOS. 134: Breton p. 224 sq., Chasles p. 78. 89. 119 sq., idem *Aperçu historique* p. 36 (p. 34 versionis German.).

1) Distinctius, ut videtur, scriptor dicere potuit "sint duo triangula, inaequali altitudine, $\beta\zeta\gamma$ $\beta\eta\gamma$, sitque $\eta\varepsilon \parallel \zeta\beta$, et $\varepsilon\delta \parallel \gamma\beta$" etc.; sed brevitatis causa, figuram plene constructam intuens, βωμίσκος (vid. ind.) praetulit. Propria quae sit lemmatis ratio, docet Chasles ad porisma XVIII.

PROPOS. 135: Breton p. 225, Chasles p. 78. 89 sq. 108 sqq. 120 sq.

$\overline{B\Theta}$ 5. τῇ *ΑΓ* Bretonus pro τῆι $\overline{ΑΔ}$ 6. η' add. BS ὁ ABS, ἡ *Ge* 17. 18. τῇ *ΒΖ* ἡ *ΕΗ* coni. *Hu* 20. ἀφαιρήσθω A, corr. BS 22. 23. ἐστὶν ἴσον — τῶι $\overline{ΑΖΗ}$ τριγώνωι om. A[1], add. A[2] in marg. (BS) 26. ἐστὶν τῇ *ΓΗ* ἡ *ΔΖ* coni. *Hu* 27. ϑ' add. BS 29. ἡ *ΖΘΗ* *Co* pro ἡ $\overline{ΖΗ}$ 32. λοιπὸν ἄρα A, corr. BS *ἐστὶν ἡ ΖΜ πρὸς ΝΗ· καὶ ὡς ἄρα ἡ ΖΜ πρὸς ΝΗ, οὕτως ἐστὶν ἡ ΔΘ πρὸς τὴν ΘΕ. ἐναλλάξ ἐστιν ὡς ἡ ΖΜ πρὸς τὴν ΔΘ, οὕτως ἡ ΝΗ πρὸς τὴν ΘΕ. ἀλλ' ὡς μὲν ἡ ΖΜ πρὸς τὴν ΔΘ, οὕτως ἐστὶν ἐν παραλλήλῳ ἡ ΖΚ πρὸς τὴν ΚΘ, ὡς δὲ ἡ ΗΝ πρὸς τὴν ΘΕ, οὕτως ἐστὶν ἡ ΗΛ πρὸς τὴν ΛΘ, καὶ ὡς ἄρα ἡ ΖΚ πρὸς τὴν ΚΘ, οὕτως ἐστὶν ἡ ΗΛ πρὸς τὴν ΛΘ· παράλληλος ἄρα ἐστὶν ἡ ΚΛ τῇ ΗΖ, ὥστε καὶ τῇ ΓΒ.*



204 *ι'. Εἰς δύο εὐθείας τὰς ΒΑΕ ΔΑΗ ἀπὸ τοῦ Θ σημείου δύο διήχθωσαν εὐθεῖαι αἱ ΔΘ ΘΕ, ἔστω δὲ ὡς τὸ ὑπὸ τῶν ΔΘ ΒΓ πρὸς τὸ ὑπὸ ΔΓ ΒΘ, οὕτως τὸ ὑπὸ ΘΗ ΖΕ πρὸς τὸ ὑπὸ ΘΕ ΖΗ· ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Γ Α Ζ.*

Ἤχθω διὰ τοῦ Θ τῇ ΓΑ παράλληλος ἡ ΚΛ καὶ συμπιπτέτω ταῖς ΑΒ ΑΔ κατὰ τὰ Κ Λ σημεῖα, καὶ διὰ τοῦ Λ τῇ ΑΔ παράλληλος ἤχθω ἡ ΛΜ, καὶ ἐκβεβλήσθω ἡ ΕΘ ἐπὶ τὸ Μ, διὰ δὲ τοῦ Κ τῇ ΑΒ παράλληλος ἤχθω ἡ ΚΝ, καὶ ἐκβεβλήσθω ἡ ΔΘ ἐπὶ τὸ Ν. ἐπεὶ οὖν διὰ τὰς παραλλήλους γίνεται ὡς ἡ ΔΘ πρὸς τὴν ΘΝ, οὕτως ἡ ΔΓ πρὸς τὴν ΓΒ, τὸ ἄρα ὑπὸ τῶν ΔΘ ΓΒ ἴσον ἐστὶν τῷ ὑπὸ τῶν ΔΓ ΘΝ. ἄλλο δέ τι τυχὸν τὸ ὑπὸ ΔΓ ΒΘ· ἔστιν ἄρα ὡς τὸ ὑπὸ ΔΘ ΒΓ πρὸς τὸ ὑπὸ ΔΓ ΒΘ, οὕτως τὸ ὑπὸ ΓΔ ΘΝ πρὸς τὸ ὑπὸ ΔΓ ΒΘ, τουτέστιν ἡ ΘΝ πρὸς

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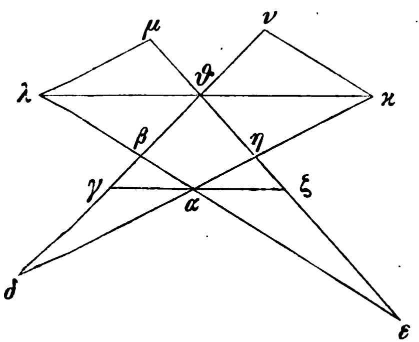
1. *καὶ ὡς — πρὸς ΝΗ* add. Co 8—10. *καὶ ὡς ἄρα — πρὸς τὴν ΛΘ* quater scripta sunt in A, bis in S, semel in V (item B[s]) 13. *ι'* add. BS 14. *διήχθω* A, corr. BS 16. 17. *τὸ ὑπὸ ΘΕΖΗ — τῶν ΓΑΖ* A, distinx. BS 19. *τὰ ΚΛ* A, distinx. BS 22. *ἐκβεβλήσθω* Hu pro *ἐκβληθῆι* 22. 23. *τὰς παραλληλα* (sine acc.) A, *τὰ παράλληλα* B, corr. S 24. 25. *τῶι ὑπὸ τῶν ΔΓΘΗ* A(BS), corr. Co in Lat. versione 25. *τυχὸν*] conf. supra ad p. 870, 22 27. *ὑπὸ ΓΔΘΝ* A, distinx. BS; item posthac in eodem lemmate ac perinde in

las $\zeta\eta$ $\beta\gamma$ est $\beta\delta : \varepsilon\gamma = \zeta\mu : \nu\eta$; ergo etiam $\zeta\mu : \nu\eta = \delta\vartheta : \vartheta\varepsilon$. Vicissim est $\zeta\mu : \delta\vartheta = \nu\eta : \vartheta\varepsilon$. Sed propter parallelas $\zeta\eta$ $\delta\varepsilon$ est $\zeta\mu : \delta\vartheta = \zeta\varkappa : \varkappa\vartheta$, itemque $\nu\eta : \vartheta\varepsilon = \eta\lambda : \lambda\vartheta$; ergo etiam $\zeta\varkappa : \varkappa\vartheta = \eta\lambda : \lambda\vartheta$; ergo recta $\varkappa\lambda$ parallela est rectae $\zeta\eta$, itaque etiam rectae $\beta\gamma$.

X. In duas rectas $\beta\alpha\varepsilon$ $\delta\alpha\eta$ a puncto ϑ ducantur duae Prop. 136
rectae $\vartheta\delta$ $\vartheta\varepsilon$, et *in his puncta* γ ζ *ita sumantur, ut* sit $\delta\vartheta \cdot \beta\gamma : \delta\gamma \cdot \beta\vartheta = \vartheta\eta \cdot \zeta\varepsilon : \vartheta\varepsilon \cdot \zeta\eta$; dico rectam esse quae per γ α ζ transit.



Ducatur per ϑ rectae $\gamma\alpha$ parallela $\varkappa\lambda$, quae cum rectis $\delta\alpha$ $\alpha\beta$ *productis* concurrat in punctis $\varkappa$ λ, et per λ rectae $\delta\alpha$ parallela ducatur $\lambda\mu$, et producatur $\varepsilon\vartheta$ ad μ, per $\varkappa$ autem rectae $\alpha\beta$ parallela ducatur $\varkappa\nu$, et producatur $\delta\vartheta$ ad ν. Iam quia propter parallelas $\vartheta\varkappa$ $\gamma\alpha$ est

$\delta\vartheta : \vartheta\varkappa = \delta\gamma : \gamma\alpha$, *itemque propter binas parallelas* $\gamma\alpha$ $\vartheta\varkappa$ *et* $\beta\alpha$ $\nu\varkappa$

$\vartheta\varkappa : \vartheta\nu = \gamma\alpha : \gamma\beta$, *ex aequali igitur est*[1])

$\delta\vartheta : \vartheta\nu = \delta\gamma : \gamma\beta$;

ergo $\delta\vartheta \cdot \gamma\beta = \delta\gamma \cdot \vartheta\nu$. Sed *fiat proportio ad* aliud rectangulum $\delta\gamma \cdot \beta\vartheta$; est igitur

$\delta\vartheta \cdot \beta\gamma : \delta\gamma \cdot \beta\vartheta = \delta\gamma \cdot \vartheta\nu : \delta\gamma \cdot \beta\vartheta$, id est

$= \vartheta\nu : \beta\vartheta$.

PROPOS. 136 (id est reciproca ad propos. 129): Simson p. 408—411, Breton p. 218 adn. 226 sq., Chasles p. 75 sq. 90. 108 sqq. 122 sq. 124 sq., Baltzer *Elemente* II p. 373.

1) Addita haec secundum Simsonum p. 409.

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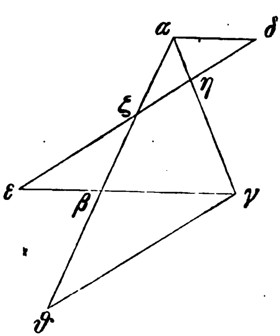
proximis duobus quaternae litterae plerumque coniunctae comparent in A

ΘΒ. ἀλλ' ὡς μὲν τὸ ὑπὸ ΘΔ ΒΓ πρὸς τὸ ὑπὸ ΔΓ ΒΘ, ὑπόκειται τὸ ὑπὸ ΘΗ ΖΕ πρὸς τὸ ὑπὸ ΘΕ ΖΗ, ὡς δὲ ἡ ΘΝ πρὸς ΘΒ, οὕτως ἡ ΚΘ πρὸς ΘΔ, τουτέστιν ἐν παραλλήλῳ ἡ ΗΘ πρὸς τὴν ΘΜ, τουτέστιν τὸ ὑπὸ ΘΗ ΖΕ πρὸς τὸ ὑπὸ ΘΜ ΖΕ· καὶ ὡς ἄρα τὸ ὑπὸ ΘΗ ΖΕ πρὸς τὸ ὑπὸ ΘΕ ΖΗ, οὕτως ἐστὶν τὸ ὑπὸ ΘΗ ΖΕ πρὸς τὸ ὑπὸ ΘΜ ΖΕ· ἴσον ἄρα ἐστὶν τὸ ὑπὸ ΘΕ ΖΗ τῷ ὑπὸ ΘΜ ΖΕ· καὶ ὡς ἄρα ἡ ΘΜ πρὸς τὴν ΘΕ, οὕτως ἡ ΗΖ πρὸς τὴν ΖΕ. συνθέντι καὶ ἐναλλάξ ἐστιν ὡς ἡ ΜΕ πρὸς τὴν ΕΗ, οὕτως ἡ ΘΕ πρὸς τὴν ΕΖ. ἀλλ' ὡς ἡ ΜΕ πρὸς τὴν ΕΗ, οὕτως ἐστὶν ἡ ΛΕ πρὸς τὴν ΕΛ· καὶ ὡς ἄρα ἡ ΛΕ πρὸς τὴν ΕΛ, οὕτως ἡ ΘΕ πρὸς τὴν ΕΖ· παράλληλος ἄρα ἐστὶν ἡ ΛΖ τῇ ΚΛ. ἀλλὰ καὶ ἡ ΓΛ· εὐθεῖα ἄρα ἐστὶν ἡ ΓΛΖ, ὅπερ: ~

Τὰ δὲ πτωτικὰ αὐτοῦ ὁμοίως τοῖς προγεγραμμένοις, ὧν ἐστιν ἀναστρόφιον.

205 ια'. Τρίγωνον τὸ ΑΒΓ, καὶ τῇ ΒΓ παράλληλος ἡ ΑΔ, καὶ διαχθεῖσα ἡ ΔΕ τῇ ΒΓ συμπιπτέτω κατὰ τὸ Ε σημεῖον· ὅτι ἐστὶν ὡς τὸ ὑπὸ ΔΕ ΖΗ πρὸς τὸ ὑπὸ ΕΖ ΗΔ, οὕτως ἡ ΓΒ πρὸς τὴν ΒΕ.



Ἤχθω διὰ τοῦ Γ τῇ ΔΕ παράλληλος ἡ ΓΘ, καὶ ἐκβεβλήσθω ἡ ΑΒ ἐπὶ τὸ Θ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΓΑ πρὸς τὴν ΑΗ, οὕτως ἡ ΓΘ πρὸς τὴν ΖΗ, ὡς δὲ ἡ ΓΑ πρὸς τὴν ΑΗ, οὕτως ἐστὶν ἡ ΕΔ πρὸς τὴν ΔΗ, καὶ ὡς ἄρα ἡ ΕΔ πρὸς τὴν ΔΗ, οὕτως ἐστὶν ἡ ΘΓ πρὸς τὴν ΖΗ· τὸ ἄρα ὑπὸ τῶν ΓΘ ΔΗ ἴσον ἐστὶν τῷ ὑπὸ τῶν ΕΔ ΖΗ. ἄλλο δέ τι τυχὸν τὸ ὑπὸ ΕΖ ΗΔ· ἔστιν ἄρα ὡς τὸ ὑπὸ ΔΕ ΖΗ πρὸς

4. ἡ ΗΘ Co pro ἡ $\overline{\text{ΝΘ}}$ 7. 8. τὸ ὑπὸ — καὶ et 8. ἄρα add. Co

Sed ex hypothesi est $\delta\vartheta \cdot \beta\gamma : \delta\gamma \cdot \beta\vartheta = \vartheta\eta \cdot \zeta\varepsilon : \vartheta\varepsilon \cdot \zeta\eta$, estque *propter parallelas* $\nu\varkappa$ $\lambda\beta$

$\vartheta\nu : \beta\vartheta = \varkappa\vartheta : \vartheta\lambda$, id est propter parallelas $\eta\varkappa$ $\lambda\mu$
$= \eta\vartheta : \vartheta\mu$, id est
$= \vartheta\eta \cdot \zeta\varepsilon : \vartheta\mu \cdot \zeta\varepsilon$; ergo etiam
$\vartheta\eta \cdot \zeta\varepsilon : \vartheta\varepsilon \cdot \zeta\eta = \vartheta\eta \cdot \zeta\varepsilon : \vartheta\mu \cdot \zeta\varepsilon$; itaque
$\vartheta\varepsilon \cdot \zeta\eta = \vartheta\mu \cdot \zeta\varepsilon$; ergo etiam
$\vartheta\mu : \vartheta\varepsilon = \eta\zeta : \zeta\varepsilon$. Componendo est
$\mu\varepsilon : \vartheta\varepsilon = \varepsilon\eta : \varepsilon\zeta$, et vicissim
$\mu\varepsilon : \varepsilon\eta = \vartheta\varepsilon : \varepsilon\zeta$. Sed *propter parallelas* $\lambda\mu$ $\alpha\eta$ est
$\mu\varepsilon : \varepsilon\eta = \lambda\varepsilon : \varepsilon\alpha$; ergo etiam
$\lambda\varepsilon : \varepsilon\alpha = \vartheta\varepsilon : \varepsilon\zeta$;

ergo parallelae sunt $\alpha\zeta$ et $\lambda\vartheta$ *sive* $\lambda\varkappa$. Sed *ex constructione* etiam $\gamma\alpha$ $\lambda\varkappa$ *parallelae sunt;* ergo recta est quae per γ α ζ transit, q. e. d.

Casus huius *lemmatis,* quod est reciprocum *ad lemma III,* similiter se habent ac supra (*propos. 129 adnot. 1*).

XI. *Sit* triangulum $\alpha\beta\gamma$, et rectae $\beta\gamma$ parallela $\alpha\delta$, et ducatur $\delta\varepsilon$, quae *rectas* $\alpha\gamma$ $\alpha\beta$ *secet in* η ζ *ac* cum $\beta\gamma$ *productâ* concurrat in puncto ε; dico esse $\varepsilon\delta \cdot \zeta\eta : \varepsilon\zeta \cdot \eta\delta = \gamma\beta : \beta\varepsilon$. Prop. 137

Ducatur per γ rectae $\delta\varepsilon$ parallela $\gamma\vartheta$, et $\alpha\beta$ producatur ad ϑ. Iam quia *propter parallelas* $\gamma\vartheta$ $\eta\zeta$ est $\gamma\alpha : \alpha\eta = \gamma\vartheta : \zeta\eta$, et *propter parallelas* $\varepsilon\gamma$ $\alpha\delta$ est $\gamma\alpha : \alpha\eta = \varepsilon\delta : \delta\eta$, est igitur etiam $\varepsilon\delta : \delta\eta = \gamma\vartheta : \zeta\eta$, itaque $\gamma\vartheta \cdot \delta\eta = \varepsilon\delta \cdot \zeta\eta$. Sed *fiat proportio ad* aliud rectangulum $\varepsilon\zeta \cdot \eta\delta$; est igitur

PROPOS. 137: Simson p. 411 sq., Breton p. 227, Chasles p. 75 sq. 82. 90. 114 sq. cet., idem *Aperçu historique* p. 34 (p. 31 sq. versionis German.).

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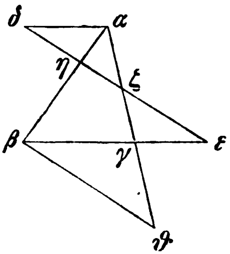
13. *ἀλλὰ καὶ ἡ* $\overline{\Gamma\Lambda}$ ABS, corr. *Co* in Lat. versione 13. 14. *ἡ* $\overline{\Gamma\Lambda Z O}$ *O* A, corr. V (*ἡ* $\overline{\gamma\alpha\zeta}$. *ὅπερ ἔδει* B^{s}S) 17. *ια'*, sed id ante *Τὰ δὲ πτωτικὰ*, add. BS 19. *πρὸς τὸ ὑπὸ* $\overline{\varepsilon\zeta}$ $\overline{\eta\lambda}$ S cod. *Co* (recte $\overline{EZ}$ $\overline{H\Lambda}$ AB), item p. 884, 5



τὸ ὑπὸ *ΔΗ ΕΖ*, οὕτως τὸ ὑπὸ *ΓΘ ΔΗ* πρὸς τὸ ὑπὸ *ΔΗ ΕΖ*, τουτέστιν ἡ *ΓΘ* πρὸς *ΕΖ*, τουτέστιν ἡ *ΓΒ* πρὸς *ΒΕ*. ἔστιν οὖν ὡς τὸ ὑπὸ *ΔΕ ΖΗ* πρὸς τὸ ὑπὸ *ΕΖ ΗΔ*, οὕτως ἡ *ΓΒ* πρὸς *ΒΕ*. τὰ δ᾽ αὐτὰ κἂν ἐπὶ τὰ ἕτερα μέρη ἀχθῇ ἡ *ΑΔ* παράλληλος, καὶ ἀπὸ τοῦ *Δ* ἐκτὸς τοῦ *Γ* ἀχθῇ ἡ *ΔΕ*.

206 ιβ΄. Ἀποδεδειγμένων νῦν τούτων ἔσται δεῖξαι ὅτι, ἐὰν παράλληλοι ὦσιν αἱ *ΑΒ ΓΔ*, καὶ εἰς αὐτὰς ἐμπίπτωσιν εὐθεῖαί τινες αἱ *ΑΔ ΑΖ ΒΓ ΒΖ*, καὶ ἐπιζευχθῶσιν αἱ *ΕΔ ΕΓ*, [ὅτι] γίνεται εὐθεῖα ἡ διὰ τῶν *Η Μ Κ*.

Ἐπεὶ γὰρ τρίγωνον τὸ *ΔΑΖ*, καὶ τῇ *ΔΖ* παράλληλος ἡ *ΑΕ*, καὶ διῆκται ἡ *ΕΓ* συμπίπτουσα τῇ *ΔΖ* κατὰ τὸ *Γ*, διὰ τὸ προγεγραμμένον γίνεται ὡς ἡ *ΔΖ* πρὸς τὴν *ΖΓ*, οὕτως τὸ ὑπὸ *ΓΕ ΗΘ* πρὸς τὸ ὑπὸ *ΓΗ ΘΕ*. πάλιν ἐπεὶ τρίγωνόν ἐστιν τὸ *ΓΒΖ*, καὶ τῇ *ΓΔ* παράλληλος ἦκται ἡ *ΒΕ*, καὶ διῆκται ἡ *ΔΕ* συμπίπτουσα τῇ *ΓΖΔ* κατὰ τὸ *Δ*, γίνεται ὡς ἡ *ΓΖ* πρὸς τὴν *ΖΔ*, οὕτως τὸ ὑπὸ *ΔΕ ΔΚ* πρὸς τὸ ὑπὸ *ΔΚ ΛΕ*· ἀνάπαλιν ἄρα γίνεται ὡς ἡ *ΔΖ* πρὸς τὴν *ΖΓ*, οὕτως τὸ ὑπὸ *ΔΚ ΛΕ* πρὸς τὸ ὑπὸ *ΔΕ ΛΚ*. ἦν δὲ καὶ ὡς ἡ *ΔΖ* πρὸς τὴν *ΖΓ*, οὕτως τὸ ὑπὸ *ΓΕ ΗΘ* πρὸς τὸ ὑπὸ *ΓΗ ΘΕ*· καὶ ὡς ἄρα τὸ ὑπὸ *ΓΕ ΗΘ* πρὸς τὸ ὑπὸ *ΓΗ ΘΕ*, οὕτως ἐστὶν τὸ ὑπὸ *ΔΚ ΛΕ* πρὸς τὸ ὑπὸ *ΔΕ ΚΛ* [ἀνῆκται εἰς τὸ πρὸ ἑνός]. ἐπεὶ οὖν εἰς δύο εὐθείας τὰς *ΓΜΛ ΔΜΘ* δύο εὐθεῖαι διηγμέναι εἰσὶν αἱ *ΕΓ ΕΔ*, καὶ ἔστιν ὡς τὸ ὑπὸ *ΓΕ ΗΘ* πρὸς τὸ ὑπὸ *ΓΗ ΘΕ*, οὕτως τὸ ὑπὸ *ΔΚ ΕΛ* πρὸς τὸ ὑπὸ *ΔΕ ΛΚ*, εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν *Η Μ Κ*· τοῦτο γὰρ προδέδεικται.

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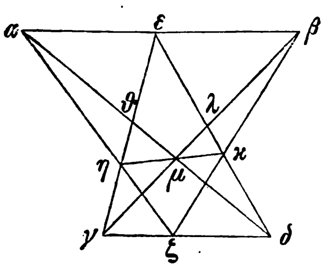
8. 9. *ἐκτὸς ὡς ἐπὶ τὸ Γ διὰ τὴν εὐθεῖαν* ABS, *ἐκτὸς τοῦ Γ ὡς ἐπὶ τὸ Ε ἀχθῇ ἡ ΔΕ Co*, in quibus *ὡς ἐπὶ τὸ Ε* del. *Hu* 10. *ιβ΄* add. BS *νῦν* del. B[1], *οὖν* coni. *Hu* 13. *ὅτι* del. *Hu* (superius *ὅτι* ante *ἐὰν* del. *Ge*) *τῶν ΗΜΚ* A, distinx. BS 18. *τῇ ΓΖ παράλληλος* coni. *Hu* 26. *ἀνῆκται εἰς τὸ πρὸ ἑνός* del. *Hu* (lemma decimum significavit interpolator) 26. 27. *τὰς ΓΜΔ* ABS, corr. *Co* in

$$\varepsilon\delta \cdot \zeta\eta : \varepsilon\zeta \cdot \eta\delta = \gamma\vartheta \cdot \delta\eta : \varepsilon\zeta \cdot \eta\delta,$$ id est
$$= \gamma\vartheta : \varepsilon\zeta,$$ id est *propter parallelas* $\gamma\vartheta$ $\zeta\varepsilon$
$$= \gamma\beta : \beta\varepsilon.$$

Eadem ratione, si ad contrariam partem ducatur $\alpha\delta$ parallela *rectae* $\beta\gamma$, et a δ extra γ ducatur $\delta\varepsilon$, *eique parallela* $\beta\vartheta$, *demonstratur esse* $\varepsilon\delta \cdot \zeta\eta : \varepsilon\zeta \cdot \eta\delta = \beta\gamma : \gamma\varepsilon$.

XII. Iam his demonstratis ostendendum erit, si parallelae Prop. 138 sint $\alpha\beta$ $\gamma\delta$, et in eas incidant quaedam rectae $\alpha\delta$ $\alpha\zeta$ $\beta\gamma$ $\beta\zeta$, *quarum* $\alpha\delta$ $\beta\gamma$ *concurrant in* μ *), et *a quovis rectae* $\alpha\beta$ *puncto inter* α *et* β *sumpto* ducantur $\varepsilon\gamma$ $\varepsilon\delta$, *quarum* $\varepsilon\gamma$ *cum* $\alpha\zeta$ *concurrat in* η *et* $\varepsilon\delta$ *cum* $\beta\zeta$ *in* $\varkappa$, rectam esse quae per η μ $\varkappa$ transit.



Quoniam enim triangulum est $\delta\alpha\zeta$, et rectae $\delta\zeta$ parallela $\alpha\varepsilon$, et ducta est $\varepsilon\gamma$ cum $\delta\zeta$ *productâ* concurrens in γ, propter superius lemma *XI* fit $\delta\zeta : \zeta\gamma = \gamma\varepsilon \cdot \eta\vartheta : \gamma\eta \cdot \vartheta\varepsilon$. Rursus quia est triangulum $\gamma\beta\zeta$, et rectae $\gamma\zeta$ parallela $\varepsilon\beta$, et ducta est $\varepsilon\delta$ cum recta $\gamma\zeta\delta$ concurrens in δ, fit $\gamma\zeta : \zeta\delta = \delta\varepsilon \cdot \varkappa\lambda : \delta\varkappa \cdot \lambda\varepsilon$. E contrario igitur est

$\delta\zeta : \zeta\gamma = \delta\varkappa \cdot \lambda\varepsilon : \delta\varepsilon \cdot \varkappa\lambda$. Sed erat etiam
$\delta\zeta : \zeta\gamma = \gamma\varepsilon \cdot \eta\vartheta : \gamma\eta \cdot \vartheta\varepsilon$; ergo etiam
$\gamma\varepsilon \cdot \eta\vartheta : \gamma\eta \cdot \vartheta\varepsilon = \delta\varkappa \cdot \lambda\varepsilon : \delta\varepsilon \cdot \varkappa\lambda$.

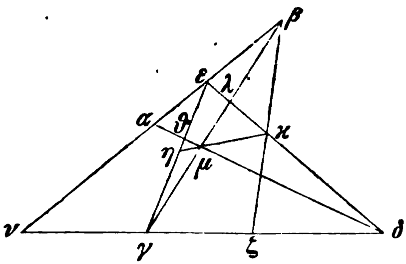
Iam quia in duas rectas $\gamma\mu\lambda$ $\delta\mu\vartheta$ duae rectae $\varepsilon\gamma$ $\varepsilon\delta$ ductae sunt, estque $\gamma\varepsilon \cdot \eta\vartheta : \gamma\eta \cdot \vartheta\varepsilon = \delta\varkappa \cdot \lambda\varepsilon : \delta\varepsilon \cdot \varkappa\lambda$, recta igitur est quae per η μ $\varkappa$ transit; hoc enim supra *lemmate X* demonstratum est.

PROPOS. 138: Simson p. 413 sq., Breton p. 228, Chasles p. 77. 90. 124 sq. 130, idem *Aperçu historique* p. 36 (p. 34 versionis German.), Baltzer *Elemente* II p. 380.

*) Haec addita secundum Simsonum, reliqua a nobis; praeterea totam propositionem alia eaque explicatiore ratione enuntiat Simsonus.

Lat. versione 28. πρὸς τὸ ὑπὸ $\overline{ΓΕ}$ $\overline{ΘΕ}$ ABS, corr. *Co* in Lat. versione 28. 29. οὕτως τὸ ὑπὸ $\overline{ΔΚ}$ $\overline{ΔΛ}$ A, sed corr. pr. manus 30. τῶν $\overline{HMK}$ A, distinx. BS

207 ιγ΄. *Ἀλλὰ δὴ μὴ ἔστωσαν αἱ ΑΒ ΓΔ παράλληλοι, ἀλλὰ συμπιπτέτωσαν κατὰ τὸ Ν· ὅτι πάλιν εὐθεῖά ἐστιν ἡ διὰ τῶν Η Μ Κ.*



Ἐπεὶ εἰς τρεῖς εὐθείας τὰς ΑΝ ΑΖ ΑΔ ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Γ δύο διηγμέναι εἰσὶν αἱ ΓΕ ΓΔ, γίνεται ὡς τὸ ὑπὸ ΓΕ ΗΘ πρὸς τὸ ὑπὸ ΓΗ ΘΕ, οὕτως τὸ ὑπὸ τῶν ΓΝ ΖΔ πρὸς τὸ ὑπὸ τῶν ΝΔ ΓΖ. πάλιν ἐπεὶ ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Δ εἰς τρεῖς εὐθείας τὰς ΒΝ ΒΓ ΒΖ δύο εἰσὶν διηγμέναι αἱ ΔΕ ΔΝ, ἔστιν ὡς τὸ ὑπὸ ΝΓ ΖΔ πρὸς τὸ ὑπὸ ΝΔ ΖΓ, οὕτως τὸ ὑπὸ ΔΚ ΕΛ πρὸς τὸ ὑπὸ ΔΕ ΚΛ. ἀλλ᾽ ὡς τὸ ὑπὸ ΝΓ ΖΔ πρὸς τὸ ὑπὸ ΝΔ ΓΖ, οὕτως ἐδείχθη τὸ ὑπὸ ΓΕ ΗΘ πρὸς τὸ ὑπὸ ΓΗ ΘΕ· καὶ ὡς ἄρα τὸ ὑπὸ ΓΕ ΘΗ πρὸς τὸ ὑπὸ ΓΗ ΘΕ, οὕτως ἐστὶν τὸ ὑπὸ ΔΚ ΕΛ πρὸς τὸ ὑπὸ ΔΕ ΚΛ [ἀπῆκται εἰς ὃ καὶ ἐπὶ τῶν παραλλήλων]. διὰ δὴ τὸ προγεγραμμένον εὐθεῖά ἐστιν ἡ διὰ τῶν Η Μ Κ.

208 ιδ΄. *Ἔστω παράλληλος ἡ ΑΒ τῇ ΓΔ, καὶ διήχθωσαν αἱ ΑΕ ΓΒ, καὶ σημεῖον ἐπὶ τῆς ΒΗ τὸ Ζ, ὥστε εἶναι ὡς τὴν ΔΕ πρὸς τὴν ΕΓ, οὕτως τὸ ὑπὸ ΓΒ ΗΖ πρὸς τὸ ὑπὸ ΖΒ ΓΗ· ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α Ζ Δ.*

Ἤχθω διὰ μὲν τοῦ Δ τῇ ΒΓ παράλληλος ἡ ΔΘ, καὶ ἐκβεβλήσθω ἡ ΑΕ ἐπὶ τὸ Θ, διὰ δὲ τοῦ Θ τῇ ΓΔ παράλληλος ἡ ΘΚ, καὶ ἐκβεβλήσθω ἡ ΒΓ ἐπὶ τὸ Κ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΔΕ πρὸς τὴν ΕΓ, οὕτως τὸ ὑπὸ ΓΒ ΖΗ πρὸς τὸ ὑπὸ ΖΒ ΓΗ, ὡς δὲ ἡ ΔΕ πρὸς τὴν ΕΓ, οὕτως ἐστὶν ἥ τε ΔΘ πρὸς τὴν ΓΗ καὶ τὸ ὑπὸ ΔΘ ΒΖ πρὸς τὸ ὑπὸ

1. ιγ΄ add. BS 2. κατὰ τὸ $\overline{Η}$ ABS, corr. Co 3. τῶν $\overline{ΗΜΚ}$ A, distinx. BS, item vs. 22 7. 8. τοῦ $\overline{Κ}$ — αἱ $\overline{ΓΕ}$ $\overline{ΝΔ}$ ABS, corr. Co 9. 10. ὑπὸ $\overline{ΓΕΗΘ}$ πρὸς τὸ ὑπὸ $\overline{ΓΗΘΕ}$ A, distinx. BS, item vs.

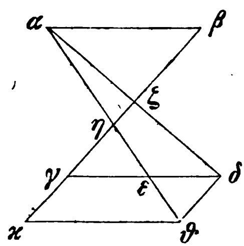
XIII. At ne sint parallelae $\alpha\beta$ $\gamma\delta$, sed convergant in puncto ν; dico rursus rectam esse quae per η μ $\varkappa$ transit. Prop. 139

Quoniam in tres rectas $\alpha\nu$ $\alpha\zeta$ $\alpha\delta$ ab eodem puncto γ duae rectae $\gamma\varepsilon$ $\gamma\delta$ ductae sunt, *propter superius lemma III*[1]) fit $\gamma\varepsilon\cdot\eta\vartheta : \gamma\eta\cdot\vartheta\varepsilon = \gamma\nu\cdot\zeta\delta : \nu\delta\cdot\gamma\zeta$. Rursus quia ab eodem puncto δ in tres rectas $\beta\nu$ $\beta\gamma$ $\beta\zeta$ duae ductae sunt $\delta\varepsilon$ $\delta\nu$, *propter idem lemma* est

$$\nu\gamma\cdot\zeta\delta : \nu\delta\cdot\zeta\gamma = \delta\varkappa\cdot\varepsilon\lambda : \delta\varepsilon\cdot\varkappa\lambda.$$ Sed demonstratum est
$$\nu\gamma\cdot\zeta\delta : \nu\delta\cdot\zeta\gamma = \gamma\varepsilon\cdot\eta\vartheta : \gamma\eta\cdot\vartheta\varepsilon;$$ ergo etiam
$$\gamma\varepsilon\cdot\eta\vartheta : \gamma\eta\cdot\vartheta\varepsilon = \delta\varkappa\cdot\varepsilon\lambda : \delta\varepsilon\cdot\varkappa\lambda.$$

Igitur propter superius *lemma X*[2]) recta est quae per η μ $\varkappa$ transit.

XIV. Sint parallelae $\alpha\beta$ $\gamma\delta$, et ducantur $\alpha\varepsilon$ $\gamma\beta$, et punctum ζ in $\beta\eta$ *ita sumatur*, ut sit $\delta\varepsilon : \varepsilon\gamma = \gamma\beta\cdot\eta\zeta : \zeta\beta\cdot\gamma\eta$; dico rectam esse quae per α ζ δ transit. Prop. 140



Ducatur per δ rectae $\beta\gamma$ parallela $\delta\vartheta$, et producatur $\alpha\varepsilon$ ad ϑ, et per ϑ rectae $\delta\gamma$ parallela ducatur $\vartheta\varkappa$, producaturque $\beta\gamma$ ad $\varkappa$. Iam quia *ex hypothesi* est

$\delta\varepsilon : \varepsilon\gamma = \gamma\beta\cdot\eta\zeta : \zeta\beta\cdot\gamma\eta$, et *propter parallelas* $\delta\vartheta$ $\eta\gamma$ est

$$\delta\varepsilon : \varepsilon\gamma = \delta\vartheta : \gamma\eta = \delta\vartheta\cdot\beta\zeta : \gamma\eta\cdot\beta\zeta,$$

PROPOS. 139: Simson p. 414 sq., Breton p. 228 sq., Chasles p. 77. 91 cet. (ut ad propos. 138).

1) Vide append.

2) Litterae geometricae sic inter se respondent:

lemm. X: Θ B Γ Δ A H Z E
XIII: ε ϑ η γ μ λ $\varkappa$ δ.

PROPOS. 140, sive conversa 137: Simson p. 415 sq., Breton p. 229 sq., Chasles p. 77. 91. 149 sq.

18. 19 12. 13. τῶν $\overline{N\Delta\Gamma Z}$ A, distinx. BS 20. 21. ἀπῆκται — παραλλήλων del. *Hu* 20. ἀνῆκται *Ge* 21. εἴσο καὶ ABS, forsitan εἰς τὸ δέκατον voluerit interpolator 23. ιδ′ add. BS 24. ἐπὶ BS, ἐπεὶ A τῆς $\overline{ZH}$ AS cod. *Co*, τῆς $\overline{\eta\zeta}$ B, corr. *Co* 26. τῶν $\overline{AZ\Delta}$ A, distinx. BS 28. ἐκβληθῆ A(B), ἐκβληθήτω SV, corr. *Ge* 31. τὸ ὑπὸ $\overline{B\Gamma}$ $\overline{ZH}$ ABS, corr. *Co* 31. ἐστὶν del. *Hu*

τῶν ΓΗ ΒΖ, ἴσον ἄρα ἐστὶν τὸ ὑπὸ τῶν ΒΓ ΖΗ τῷ ὑπὸ ΔΘ ΒΖ· ἀνάλογον ἄρα ἐστὶν ὡς ἡ ΓΒ πρὸς τὴν ΒΖ, οὕτως ἡ ΔΘ πρὸς τὴν ΗΖ, *τουτέστιν ὡς ἡ ΓΚ πρὸς τὴν ΗΖ· καὶ ὅλη ἄρα ἡ ΚΒ πρὸς ὅλην τὴν ΒΗ ἐστὶν ὡς ἡ ΚΓ πρὸς ΖΗ, τουτέστιν ὡς ἡ ΔΘ πρὸς ΖΗ. ἀλλʼ ὡς ἡ ΚΒ πρὸς ΒΗ ἐν παραλλήλῳ, οὕτως ἐστὶν ἡ ΘΑ πρὸς ΑΗ καὶ ἡ ΔΘ πρὸς ΖΗ. καὶ εἰσὶν παράλληλοι αἱ ΔΘ ΖΗ· εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Α Ζ Δ σημείων.*

209 ιε΄. *Τούτου προτεθεωρημένου ἔστω παράλληλος ἡ ΑΒ τῇ ΓΔ, καὶ εἰς αὐτὰς ἐμπιπτέτωσαν εὐθεῖαι αἱ ΑΖ ΖΒ ΓΕ ΕΔ, καὶ ἐπεζεύχθωσαν αἱ ΒΓ ΗΚ· ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α Μ Δ.*

Ἐπεζεύχθω ἡ ΔΜ καὶ *ἐκβεβλήσθω ἐπὶ τὸ Θ. ἐπεὶ οὖν τριγώνου τοῦ ΒΓΖ [ἐκτὸς] ἀπὸ τῆς κορυφῆς τοῦ Β σημείου τῇ ΓΔ παράλληλος ἦκται ἡ ΒΕ, καὶ διῆκται ἡ ΔΕ, γίνεται ὡς ἡ ΓΖ πρὸς ΖΔ, οὕτως τὸ ὑπὸ ΔΕ ΚΑ πρὸς τὸ ὑπὸ ΕΑ ΚΔ. ὡς δὲ τὸ ὑπὸ ΔΕ ΚΑ πρὸς τὸ ὑπὸ ΔΚ ΑΕ, οὕτως ἐστὶν τὸ ὑπὸ ΓΗ ΘΕ πρὸς τὸ ὑπὸ ΓΕ ΗΘ (εἰς τρεῖς* γὰρ *εὐθείας τὰς ΓΑ ΔΘ ΗΚ δύο εἰσὶν διηγμέναι ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Ε αἱ ΕΓ ΕΔ)· καὶ ὡς ἄρα ἡ ΔΖ πρὸς ΖΓ, οὕτως ἐστὶν τὸ ὑπὸ ΓΕ ΗΘ πρὸς τὸ ὑπὸ ΓΗ ΘΕ. καὶ ἔστιν εὐθεῖα ἡ διὰ τῶν Θ Μ Δ· διὰ*

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3. *πρὸς τὴν ΗΖ* add. *Hu* coll. vs. 5 (brevius scribi poterat *οὕτως ἡ ΔΘ, τουτέστιν ἡ ΓΚ, πρὸς τὴν ΗΖ*) 4. *καὶ ὅλῃ* A, corr. BS 7. *εὐθειαι* (sine acc.) A(B), corr. S 8. *τῶν ΑΖΔ* A[3] ex *τῶν ΑΖ**, distinx. BS 9. *ιε΄* add. BS 11. *ἐπεζεύχθω* A, corr. BS 12. *διὰ τῶν ΗΜΚ* A(BS), corr. *Co* 13. *ἡ λμ* S cod. *Co* (recte *ἡ ΔΜ* AB) *καὶ* add. *Co* *ἐπὶ τὸ Κ* ABS, corr. *Co* 14. *ἐκτὸς* del. *Hu* auctore Simsono 15. *διῆκται ἡ ΔΒ* AB, *διῆκται ἡ βδ* S, *ducitur ED Co*, corr. *Hu* 16. *πρὸς ΖΔ Co* (in Lat. versione) pro *πρὸς ΖΓ* 17. 18. *πρὸς τὸ ὑπὸ ΔΚΑΒ* A(BS), *πρὸς τὸ ὑπὸ ΕΑ ΚΑ Co*, corr. *Hu* 19. *γὰρ* add. *Hu* auctore *Co* *τὰς ΓΑΔΘΗΚ* A, distinx. BS 22. *καὶ ἔστιν* cet.] immo *εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Α Θ Δ διὰ τὸ προγεγραμμένον. καὶ ἔστιν εὐθεῖα ἡ διὰ τῶν Θ Μ Δ· εὐθεῖα ἄρα καὶ ἡ διὰ τῶν Α Μ Δ* (vel *ὥστε καὶ ἡ διὰ — ἐστὶν εὐθεῖα*) *διὰ τῶν ΗΜΚ* A(BS), corr. *Hu* (*διὰ τῶν Δ Μ Θ Co*)

est igitur $\gamma\beta \cdot \eta\zeta = \delta\vartheta \cdot \beta\zeta$; itaque per proportionem est

$\gamma\beta : \beta\zeta = \delta\vartheta : \eta\zeta$, id est

$= \gamma\varkappa : \eta\zeta$; ergo etiam tota ad totam

$\varkappa\beta : \beta\eta = \gamma\varkappa : \eta\zeta = \delta\vartheta : \eta\zeta$.

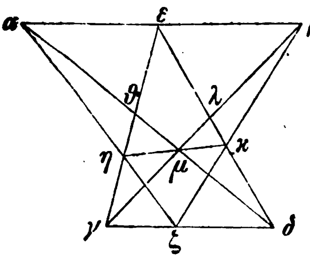
Sed inter parallelas $\alpha\beta$ $\varkappa\vartheta$ est $\varkappa\eta : \eta\beta = \vartheta\eta : \eta\alpha$, *ideoque componendo*

$\varkappa\beta : \beta\eta = \vartheta\alpha : \alpha\eta$. *Sed erat* $\varkappa\beta : \beta\eta = \delta\vartheta : \zeta\eta$; *ergo*

$\vartheta\alpha : \alpha\eta = \delta\vartheta : \zeta\eta$.

Et sunt parallelae $\delta\vartheta$ $\zeta\eta$; recta igitur est quae per α ζ δ transit[1]).

XV. Hoc demonstrato sint parallelae $\alpha\beta$ $\gamma\delta$, inque eas incidant rectae $\alpha\zeta$ $\zeta\beta$ $\gamma\varepsilon$ $\varepsilon\delta$, et iungantur $\beta\gamma$ $\eta\varkappa$; dico rectam esse quae per α μ δ transit[2]). Prop. 141



Iungatur $\delta\mu$ producaturque ad ϑ *punctum concursûs cum* $\gamma\varepsilon$. Iam quia a vertice β trianguli $\beta\gamma\zeta$ rectae $\gamma\delta$ parallela ducta est $\beta\varepsilon$, et *inter parallelas* ducta $\delta\varepsilon$, *propter lemma XI* fit

$\gamma\zeta : \zeta\delta = \delta\varepsilon \cdot \varkappa\lambda : \varepsilon\lambda \cdot \varkappa\delta$.

Sed, quia in tres rectas $\gamma\lambda$ $\delta\vartheta$ $\eta\varkappa$ (*id est* $\mu\gamma$ $\mu\eta$ $\mu\vartheta$) ab eodem puncto ε ductae sunt $\varepsilon\gamma$ $\varepsilon\delta$, *propter lemma III* est

$\delta\varepsilon \cdot \varkappa\lambda : \varepsilon\lambda \cdot \varkappa\delta = \gamma\eta \cdot \vartheta\varepsilon : \gamma\varepsilon \cdot \eta\vartheta$ *);

ergo etiam

$\delta\zeta : \zeta\gamma = \gamma\varepsilon \cdot \eta\vartheta : \gamma\eta \cdot \vartheta\varepsilon$;

ergo propter superius *lemma recta est quae per* α ϑ δ *transit.*

1) Conf. supra p. 871 adnot. *.

PROPOS. 141: Simson p. 416 sq., Breton p. 230, Chasles p. 77. 91 sq. 141, idem *Aperçu historique* p. 36 (p. 34 versionis German.).

2) Explicatius Simson p. 416: "sit $\alpha\beta$ parallela rectae $\gamma\delta$, et a punctis α β inflectantur ad $\gamma\delta$ rectae $\alpha\zeta$ $\beta\zeta$; a punctis vero γ δ ad $\alpha\beta$ inflectantur $\gamma\varepsilon$ $\delta\varepsilon$, sitque η intersectio ipsarum $\alpha\zeta$ $\gamma\varepsilon$, et $\varkappa$ intersectio reliquarum $\beta\zeta$ $\delta\varepsilon$, et ducatur $\beta\gamma$, quae occurrat iunctae $\eta\varkappa$ in μ; erunt α μ δ puncta in recta linea".

*) Vide append.

τὸ προγεγραμμένον ἄρα καὶ ἡ διὰ τῶν Α Μ Δ ἐστὶν εὐθεῖα.

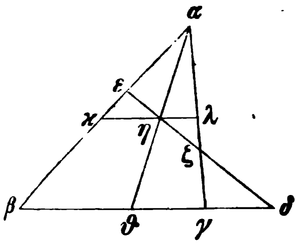
210 ιϛʹ. *Εἰς δύο εὐθείας τὰς ΑΒ ΑΓ ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Δ δύο διήχθωσαν αἱ ΔΒ ΔΕ, καὶ ἐπ᾽ αὐτῶν εἰλήφθω σημεῖα τὰ Η Θ, ἔστω δὲ ὡς τὸ ὑπὸ ΕΗ ΖΔ πρὸς τὸ ὑπὸ ΔΕ ΗΖ, οὕτως τὸ ὑπὸ ΒΘ ΓΔ πρὸς τὸ ὑπὸ ΒΔ ΓΘ· ὅτι εὐθεῖά ἐστιν ἡ διὰ τῶν Α Η Θ.*

Ἤχθω διὰ τοῦ Η τῇ ΒΔ παράλληλος ἡ ΚΛ. ἐπεὶ οὖν ἐστιν ὡς τὸ ὑπὸ ΕΗ ΖΔ πρὸς τὸ ὑπὸ ΔΕ ΖΗ, οὕτως τὸ ὑπὸ ΒΘ ΓΔ πρὸς τὸ ὑπὸ ΒΔ ΓΘ, ἀλλὰ ὁ τοῦ *ὑπὸ ΕΗ ΖΔ πρὸς τὸ ὑπὸ ΔΕ ΗΖ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΗΕ πρὸς ΕΔ, τουτέστιν ἡ ΚΗ πρὸς ΒΔ, καὶ ἐξ οὗ ὃν ἔχει ἡ ΔΖ πρὸς ΖΗ, τουτέστιν ἡ ΓΔ πρὸς τὴν ΗΛ, ὁ δὲ τοῦ ὑπὸ ΒΘ ΓΔ πρὸς τὸ ὑπὸ ΒΔ ΓΘ συνῆπται λόγος ἔκ τε τοῦ ὃν ἔχει ἡ ΘΒ πρὸς ΒΔ καὶ ἐξ οὗ ὃν ἔχει ἡ ΔΓ πρὸς ΓΘ, καὶ ὁ* ἔκ τε τοῦ *τῆς ΚΗ ἄρα πρὸς ΒΔ καὶ τοῦ τῆς ΔΓ πρὸς ΗΛ ὁ αὐτός ἐστιν τῷ συνημμένῳ ἔκ τε τοῦ τῆς ΒΘ πρὸς ΒΔ καὶ τοῦ τῆς ΔΓ πρὸς ΓΘ. ὁ δὲ τῆς ΚΗ πρὸς ΒΔ συνῆπται ἔκ τε τοῦ τῆς ΚΗ πρὸς ΒΘ καὶ τοῦ τῆς ΒΘ πρὸς ΒΔ· ὁ ἄρα συνημμένος ἔκ τε τοῦ τῆς ΚΗ πρὸς ΒΘ καὶ τοῦ τῆς ΒΘ πρὸς ΒΔ καὶ ἔτι τοῦ τῆς ΔΓ πρὸς ΗΛ ὁ αὐτός ἐστιν τῷ συνημμένῳ ἔκ τε τοῦ τῆς ΒΘ πρὸς ΒΔ καὶ τοῦ τῆς ΔΓ πρὸς ΓΘ. κοινὸς ἐκκεκρούσθω ὁ τῆς ΒΘ πρὸς ΒΔ λόγος· λοιπὸς ἄρα ὁ συνημμένος ἔκ τε τοῦ τῆς ΚΗ πρὸς ΒΘ καὶ τοῦ τῆς ΔΓ πρὸς ΗΛ ὁ αὐτός ἐστιν τῷ τῆς ΔΓ πρὸς τὴν ΓΘ, τουτέστιν τῷ συνημμένῳ ἔκ τε τοῦ τῆς ΔΓ πρὸς τὴν ΗΛ καὶ τοῦ τῆς ΗΛ πρὸς τὴν ΘΓ. καὶ πάλιν κοινὸς ἐκκεκρούσθω ὁ τῆς ΔΓ πρὸς τὴν ΗΛ λόγος· λοιπὸς ἄρα ὁ τῆς ΚΗ πρὸς τὴν ΒΘ λόγος ὁ αὐτός ἐστιν τῷ τῆς ΗΛ πρὸς τὴν ΘΓ. καὶ ἐναλλάξ ἐστιν ὡς ἡ ΚΗ πρὸς τὴν ΗΛ, οὕτως ἡ ΒΘ πρὸς τὴν*

1. *τῶν* ΑΜΔ A, distinx. BS 3. *ιϛʹ* add. BS 4. *διήχθη* A, corr. BS 5. *τὰ* ΗΘ A, distinx. BS *δὲ* Hu pro *δὴ* 7. *τῶν* ΑΗΘ A, distinx. BS 10. *ὁ* add. BS, *τοῦ* Ge 16. *ἡ ΔΓ πρὸς ΓΕ* ABS, corr. Co in Lat. versione *ἔκ τε τοῦ* add. Hu (nec tamen persanatus locus esse videtur, nisi *καὶ ὁ συνημμένος ἄρα ἔκ τε τοῦ τῆς*

Et *ex constructione* recta est quae per ϑ μ δ transit; ergo etiam recta est quae per α μ δ transit.

XVI. In duas rectas $\alpha\beta$ $\alpha\gamma$ ab eodem puncto δ ducantur duae rectae $\delta\beta$ $\delta\varepsilon$, et in his sumantur duo puncta ϑ η, sit autem $\varepsilon\eta \cdot \zeta\delta : \delta\varepsilon \cdot \eta\zeta = \beta\vartheta \cdot \gamma\delta : \beta\delta \cdot \gamma\vartheta$; dico rectam esse quae per α η ϑ transit. Prop. 142



Ducatur [1]) per η rectae $\beta\delta$ parallela $\kappa\lambda$. Iam quia est $\varepsilon\eta \cdot \zeta\delta : \delta\varepsilon \cdot \eta\zeta = \beta\vartheta \cdot \gamma\delta : \beta\delta \cdot \gamma\vartheta$, ac per formulam compositae proportionis

$$\frac{\varepsilon\eta \cdot \zeta\delta}{\delta\varepsilon \cdot \eta\zeta} = \frac{\eta\varepsilon}{\varepsilon\delta} \cdot \frac{\delta\zeta}{\zeta\eta} = \frac{\kappa\eta}{\beta\delta} \cdot \frac{\gamma\delta}{\eta\lambda},$$

itemque

$$\frac{\beta\vartheta \cdot \gamma\delta}{\beta\delta \cdot \gamma\vartheta} = \frac{\beta\vartheta}{\beta\delta} \cdot \frac{\delta\gamma}{\gamma\vartheta},$$ ergo etiam est

$$\frac{\kappa\eta}{\beta\delta} \cdot \frac{\gamma\delta}{\eta\lambda} = \frac{\beta\vartheta}{\beta\delta} \cdot \frac{\delta\gamma}{\gamma\vartheta}.$$ Sed est

$$\frac{\kappa\eta}{\beta\delta} = \frac{\kappa\eta}{\beta\vartheta} \cdot \frac{\beta\vartheta}{\beta\delta};$$ ergo $$\frac{\kappa\eta}{\beta\vartheta} \cdot \frac{\beta\vartheta}{\beta\delta} \cdot \frac{\gamma\delta}{\eta\lambda} = \frac{\beta\vartheta}{\beta\delta} \cdot \frac{\delta\gamma}{\gamma\vartheta}.$$

Dividendo tollatur communis proportio $\beta\vartheta : \beta\delta$; relinquitur igitur

$$\frac{\kappa\eta}{\beta\vartheta} \cdot \frac{\gamma\delta}{\eta\lambda} = \frac{\delta\gamma}{\gamma\vartheta} = \frac{\delta\gamma}{\eta\lambda} \cdot \frac{\eta\lambda}{\gamma\vartheta}.$$

Et rursus tollatur communis proportio $\delta\gamma : \eta\lambda$; relinquitur igitur $\kappa\eta : \beta\vartheta = \eta\lambda : \gamma\vartheta$. Et vicissim est $\kappa\eta : \eta\lambda = \beta\vartheta : \vartheta\gamma$,

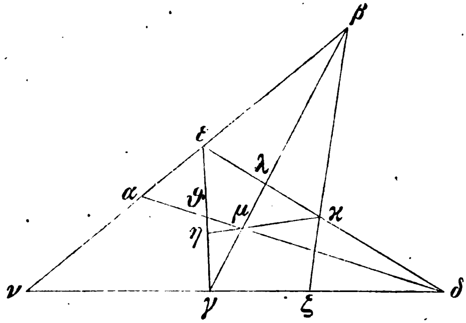
PROPOS. 142 (id est propos. 136 aliter demonstrata): Simson p. 409—411, Breton p. 230 sq., Chasles p. 76. 92. 142 sq. 150 cet., Baltzer *Elemente* II p. 373.

1) Rursus ex plurimis, quae fingi possunt figuris, unam tantum adscripsimus; duas exhibet codex, scilicet hanc ipsam et alteram cum punctorum in basi dispositione β δ γ ϑ, quae cum ad lemma XVII valeat, repetita est a nobis in appendice ad propos. 143; tertiam addit Commandinus cum dispositione β ϑ δ γ; quarta supra est in lemm. X, quod litteris convenienter mutatis dat seriem ϑ β γ δ. Conf. etiam infra propos. 144 cum append.

KH πρὸς BΔ cet. scripseris) 18. πρὸς $\overline{ΘΔ}$ καὶ τοῦ τῆς $\overline{ΑΓ}$ ABS, corr. *Co* 23. κοινὸς BS super vs., κ° ABS, item vs. 28 24. ὁ τῆς $\overline{ΘB}$ AB, corr. S

ΘΓ, καὶ εἰσὶν αἱ ΚΛ ΒΓ παράλληλοι· εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Λ Η Θ σημείων.

211 *ιζ΄. Ἀλλὰ δὴ μὴ ἔστω παράλληλος ἡ ΑΒ τῇ ΓΔ, ἀλλὰ συμπιπτέτω κατὰ τὸ Ν.*



Ἐπεὶ οὖν ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Δ εἰς τρεῖς εὐθείας τὰς ΒΝ ΒΓ ΒΖ δύο εὐθεῖαι διηγμέναι εἰσὶν αἱ ΔΕ ΔΝ, ἔστιν ὡς τὸ ὑπὸ ΝΔ ΓΖ πρὸς τὸ ὑπὸ ΝΓ ΔΖ, οὕτως τὸ ὑπὸ ΔΕ ΚΛ πρὸς τὸ ὑπὸ ΕΛ ΚΔ. ὡς δὲ τὸ ὑπὸ ΕΔ ΚΛ πρὸς τὸ ὑπὸ ΕΛ ΚΔ, οὕτως ἐστὶν τὸ ὑπὸ ΕΘ ΓΗ πρὸς τὸ ὑπὸ ΕΓ ΘΗ (πάλιν γὰρ εἰς τρεῖς τὰς ΓΛ ΔΘ ΗΚ ἀπὸ τοῦ αὐτοῦ σημείου τοῦ Ε δύο ἠγμέναι εἰσὶν αἱ ΕΓ ΕΔ)· καὶ ὡς ἄρα τὸ ὑπὸ ΕΘ ΓΗ πρὸς τὸ ὑπὸ ΕΓ ΘΗ, οὕτως τὸ ὑπὸ ΝΔ ΓΖ πρὸς τὸ ὑπὸ ΝΓ ΖΔ· διὰ δὴ τὸ προγεγραμμένον εὐθεῖά ἐστιν ἡ διὰ τῶν Α Θ Δ· καὶ ἡ διὰ τῶν Α Μ Δ ἄρα εὐθεῖά ἐστιν.

212 *ιη΄. Τρίγωνον τὸ ΑΒΓ, καὶ τῇ ΒΓ παράλληλος ἤχθω ἡ ΑΔ, καὶ διήχθωσαν αἱ ΔΕ ΖΗ, ἔστω δὲ ὡς τὸ ἀπὸ ΕΒ πρὸς τὸ ὑπὸ ΕΓΒ, οὕτως ἡ ΒΗ πρὸς τὴν ΗΓ· ὅτι, ἐὰν ἐπιζευχθῇ ἡ ΒΔ, γίνεται εὐθεῖα ἡ διὰ τῶν Θ Κ Γ.*

Ἐπεί ἐστιν ὡς τὸ ἀπὸ τῆς ΕΒ πρὸς τὸ ὑπὸ ΕΓΒ, οὕτως ἡ ΒΗ πρὸς ΗΓ, κοινὸς [ἄρα] προσκείσθω ὁ τῆς ΓΕ πρὸς ΕΒ λόγος ὁ αὐτὸς ὢν τῷ τοῦ ὑπὸ ΕΓΒ πρὸς τὸ ὑπὸ

2. *τῶν* ΑΗΘ A, distinx. BS 3. *ιζ΄* BS, ΙΗ A[1] in marg. 7. 8. *τὰς βη* B[s]S cod. *Co* (recte *τὰς* ΒΝ A) 16. 17. *τὸ ὑπὸ εϑ γν* S cod. *Co* (recte *τὸ ὑπὸ* ΕΘ ΓΗ AB) 17. *πρὸς τὸ ὑπὸ* ΕΓ ΘΝ ABS, corr. *Co*, item vs. 19. 20 19. *ἄρα τὸ ὑπὸ εϑ γν* S cod. *Co* (recte AB, ut supra) 20. *τὸ ὑπὸ* ΝΛ ΓΖ *πρὸς* bis scripta in A ΔΖ (ante *διὰ*) *Co* *δὴ* add. *Ge* 21. 22. *τῶν* ΛΘΔ — *τῶν* ΛΜΔ A, distinx. BS 23. *ιη΄* add. BS 24. *ὡς τὰ ἀπὸ* AB, corr. S 26. *τῶν* ΘΚΓ A, distinx. BS, item p. 894, 12 28. *κοινὸν* AB[1], corr. B[c]S *ἄρα* del. *Hu*

suntque parallelae $\varkappa\lambda$ $\beta\gamma$; recta igitur est quae per puncta α η ϑ transit[1]).

XVII. At ne sint parallelae $\alpha\beta$ $\gamma\delta$, sed convergant in puncto ν (*ceteris ut in lemmate XV manentibus*). Prop. 143

Iam quia ab eodem puncto δ in tres rectas $\beta\nu$ $\beta\gamma$ $\beta\zeta$ duae rectae $\delta\varepsilon$ $\delta\nu$ ductae sunt, *propter lemma III* est

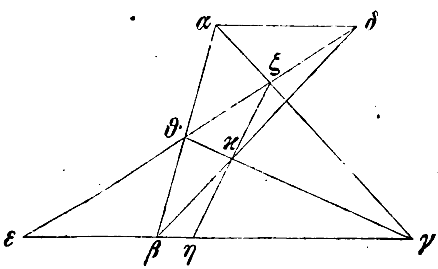
$\nu\delta \cdot \gamma\zeta : \nu\gamma \cdot \delta\zeta = \delta\varepsilon \cdot \varkappa\lambda : \varepsilon\lambda \cdot \varkappa\delta$ *). Sed rursus, quia in tres rectas $\gamma\lambda$ $\delta\vartheta$ $\eta\varkappa$ (*id est* $\mu\lambda$ $\mu\delta$ $\mu\varkappa$) ab eodem puncto ε duae $\varepsilon\gamma$ $\varepsilon\delta$ ductae sunt, est

$\varepsilon\delta \cdot \varkappa\lambda : \varepsilon\lambda \cdot \varkappa\delta = \varepsilon\vartheta \cdot \gamma\eta : \varepsilon\gamma \cdot \vartheta\eta$ **); ergo etiam

$\varepsilon\vartheta \cdot \gamma\eta : \varepsilon\gamma \cdot \vartheta\eta = \nu\delta \cdot \gamma\zeta : \nu\gamma \cdot \delta\zeta$.

Iam propter superius *lemma* recta est quae per α ϑ δ transit**); ergo etiam recta est quae per α μ δ transit.

XVIII. *Sit* triangulum $\alpha\beta\gamma$, et rectae $\beta\gamma$ parallela ducatur $\alpha\delta$, et ducatur *utcunque* $\delta\varepsilon$, *quae rectis* $\alpha\beta$ $\alpha\gamma$ *occurrat in* ϑ ζ; *sit autem in* $\beta\gamma$ *punctum* η, quod faciat $\varepsilon\beta^2 : \varepsilon\gamma \cdot \gamma\beta = \beta\eta : \eta\gamma$, *et iungatur* $\zeta\eta$, *cui occurrat* iuncta $\beta\delta$ *in* $\varkappa$***); dico rectam esse quae per ϑ $\varkappa$ γ transit. Prop. 144



Quoniam est $\varepsilon\beta^2 : \varepsilon\gamma \cdot \gamma\beta = \beta\eta : \eta\gamma$, utraque proportio multiplicetur per $\frac{\gamma\varepsilon}{\varepsilon\beta}$, *vel potius*, quod ad idem redit, per $\frac{\varepsilon\gamma \cdot \gamma\beta}{\varepsilon\beta \cdot \beta\gamma}$; est igitur

1) Demonstrationem sic fere explet Simson p. 411: Quoniam est $\varkappa\eta : \eta\lambda = \beta\vartheta : \vartheta\gamma$, componendo erit $\varkappa\lambda : \lambda\eta = \beta\gamma : \gamma\vartheta$. Sed est $\alpha\lambda : \lambda\varkappa = \alpha\gamma : \gamma\beta$; igitur ex aequali $\alpha\lambda : \lambda\eta = \alpha\gamma : \gamma\vartheta$. Et parallelae sunt $\lambda\eta$ $\gamma\vartheta$; ergo (propter lemma p. 871 adnot. *) in recta linea sunt α η ϑ puncta.

PROPOS. 143: Simson p. 417 sq., Breton p. 231 sq., Chasles p. 77. 92. 141, idem *Aperçu historique* p. 36 (p. 34 versionis German.).

*) Vide casum secundum in append. ad propos. 139.

**) Vide append.

PROPOS. 144: Simson p. 426 sq., Breton p. 232 sq., Chasles p. 79. 92 sq. 148 sq.

***) Sic auctore Simsono enuntiationem distinctiorem reddidimus.

ΕΒΓ· δι᾽ ἴσου ἄρα ὁ τοῦ ἀπὸ ΕΒ πρὸς τὸ ὑπὸ ΕΒΓ λόγος, τουτέστιν ὁ τῆς ΕΒ πρὸς τὴν ΒΓ, ὁ αὐτός ἐστιν τῷ συνημμένῳ ἔκ τε τοῦ τῆς ΒΗ πρὸς ΗΓ καὶ τοῦ τοῦ ὑπὸ ΕΓΒ πρὸς τὸ ὑπὸ ΕΒΓ, ὅς ἐστιν ὁ αὐτὸς τῷ τῆς ΕΓ πρὸς ΕΒ· ὥστε ὁ τοῦ ἀπὸ ΕΒ πρὸς τὸ ὑπὸ ΕΒΓ συνῆπται ἔκ τε τοῦ ὃν ἔχει ἡ ΒΗ πρὸς ΗΓ καὶ τοῦ ὃν ἔχει ἡ ΕΓ πρὸς ΕΒ, ὅς ἐστιν ὁ αὐτὸς τῷ τοῦ ὑπὸ ΕΓ ΒΗ πρὸς τὸ ὑπὸ ΕΒ ΓΗ. ὡς δὲ ἡ ΕΒ πρὸς τὴν ΒΓ, οὕτως ἐστὶν διὰ τὸ προγεγραμμένον λῆμμα τὸ ὑπὸ ΔΖ ΘΕ πρὸς τὸ ὑπὸ ΔΕ ΖΘ· καὶ ὡς ἄρα τὸ ὑπὸ ΓΕ ΒΗ πρὸς τὸ ὑπὸ ΓΗ ΕΒ, οὕτως ἐστὶν τὸ ὑπὸ ΔΖ ΘΕ πρὸς τὸ ὑπὸ ΔΕ ΖΘ· εὐθεῖα ἄρα ἐστὶν ἡ διὰ τῶν Θ Κ Γ· τοῦτο γὰρ ἐν τοῖς πτωτικοῖς τῶν ἀναστροφίων.

213 ιθ΄. *Εἰς τρεῖς εὐθείας τὰς ΑΒ ΑΓ ΑΔ ἀπό τινος σημείου τοῦ Ε δύο διήχθωσαν αἱ ΕΖ ΕΒ, ἔστω δὲ ὡς ἡ ΕΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΘΕ πρὸς τὴν ΘΗ· ὅτι γίνεται καὶ ὡς ἡ ΒΕ πρὸς τὴν ΒΓ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΓ.*

Ἤχθω διὰ τοῦ Η τῇ ΒΕ παράλληλος ἡ ΛΚ. ἐπεὶ οὖν ἐστιν ὡς ἡ ΕΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΕΘ πρὸς τὴν ΘΗ, ἀλλ᾽ ὡς μὲν ἡ ΕΖ πρὸς τὴν ΖΗ, οὕτως ἡ ΕΒ πρὸς τὴν ΗΚ, ὡς δὲ ἡ ΕΘ πρὸς τὴν ΘΗ, οὕτως [ἐστὶν] ἡ ΔΕ πρὸς τὴν ΗΛ, καὶ ὡς ἄρα ἡ ΒΕ πρὸς τὴν ΗΚ, οὕτως ἐστὶν ἡ ΔΕ πρὸς τὴν ΗΛ. ἐναλλάξ ἐστιν ὡς ἡ ΕΒ πρὸς τὴν ΕΔ, οὕτως ἡ ΚΗ πρὸς τὴν ΗΛ. ὡς δὲ ἡ ΚΗ πρὸς τὴν ΗΛ, οὕτως ἐστὶν ἡ ΒΓ πρὸς τὴν ΓΔ· καὶ ὡς ἄρα ἡ ΒΕ πρὸς τὴν ΕΔ, οὕτως ἡ ΒΓ πρὸς τὴν ΓΔ. ἐναλλάξ ἐστιν ὡς ἡ ΕΒ πρὸς τὴν ΒΓ, οὕτως ἡ ΕΔ πρὸς τὴν ΔΓ.

Τὰ δὲ πτωτικὰ ὁμοίως.

214 κ΄. *Ἔστω δύο τρίγωνα τὰ ΑΒΓ ΔΕΖ ἴσας ἔχοντα τὰς Α Δ γωνίας· ὅτι ἐστὶν ὡς τὸ ὑπὸ ΒΑΓ πρὸς τὸ ὑπὸ ΕΔΖ, οὕτως τὸ ΑΒΓ τρίγωνον πρὸς τὸ ΔΕΖ τρίγωνον.*

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1. *δι᾽ ἴσου* — 5. *συνῆπται*] vide append. 3. *καὶ τῶι τοῦ* ABS, *τῶι* del. *Ge*, corr. *Hu* 5. *τοῦ ἀπὸ Hu* pro *ἀπὸ τοῦ* *συνῆκται* A, corr. BS 9. 10. *τὸ ὑπὸ ΔΕ ΖΘ πρὸς τὸ ὑπὸ ΔΖ ΘΕ* ABS, corr. Simsonus p. 427, item vs. 11 10. *ἄρα τὸ ὑπὸ ΓΕΒΗ* A, distinx. BS *πρὸς τὸ ὑπὸ ΓΗ ΘΒ* ABS, corr. *Co* in Lat. versione 14. *ιθ΄*

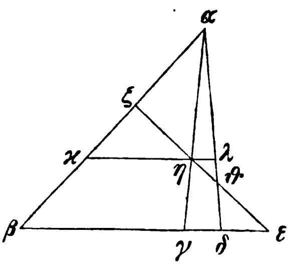
$$\frac{\varepsilon\beta^2}{\varepsilon\beta\cdot\beta\gamma} = \frac{\beta\eta}{\eta\gamma}\cdot\frac{\varepsilon\gamma\cdot\gamma\beta}{\varepsilon\beta\cdot\beta\gamma}, \text{ id est } \frac{\varepsilon\beta}{\beta\gamma} = \frac{\beta\eta}{\eta\gamma}\cdot\frac{\varepsilon\gamma}{\varepsilon\beta} = \frac{\beta\eta\cdot\varepsilon\gamma}{\eta\gamma\cdot\varepsilon\beta}.$$

Sed propter superius lemma *XI* est

$$\frac{\varepsilon\beta}{\beta\gamma} = \frac{\delta\zeta\cdot\vartheta\varepsilon}{\delta\varepsilon\cdot\zeta\vartheta}; \text{ ergo etiam } \frac{\beta\eta\cdot\varepsilon\gamma}{\eta\gamma\cdot\varepsilon\beta} = \frac{\delta\zeta\cdot\vartheta\varepsilon}{\delta\varepsilon\cdot\zeta\vartheta}.$$

Sed in duas rectas $\varkappa\beta$ $\varkappa\zeta$ *ab eodem puncto* ε *ductae sunt* $\varepsilon\beta\eta$ $\varepsilon\zeta\delta$, *et in his sumpta puncta* γ ϑ, *quae faciant* (*ut modo demonstratum est*) $\varepsilon\gamma\cdot\beta\eta : \varepsilon\beta\cdot\eta\gamma = \varepsilon\vartheta\cdot\zeta\delta : \varepsilon\delta\cdot\zeta\vartheta$; ergo propter ea quae inter casus reciprocorum demonstrata sunt recta est quae per ϑ $\varkappa$ γ transit[1]).

XIX. In tres rectas $\alpha\beta$ $\alpha\gamma$ $\alpha\delta$ a quodam puncto ε duae ducantur $\varepsilon\zeta$ $\varepsilon\beta$, sitque $\varepsilon\zeta : \zeta\eta = \vartheta\varepsilon : \vartheta\eta$; dico esse etiam $\varepsilon\beta : \beta\gamma = \varepsilon\delta : \delta\gamma$. Prop. 145



Ducatur per η rectae $\beta\varepsilon$ parallela $\varkappa\lambda$. Iam quia est

$\varepsilon\zeta : \zeta\eta = \varepsilon\vartheta : \vartheta\eta$, et *propter parallelas* $\beta\varepsilon$ $\varkappa\eta$

$\varepsilon\zeta : \zeta\eta = \varepsilon\beta : \varkappa\eta$, et *propter parallelas* $\eta\lambda$ $\delta\varepsilon$

$\varepsilon\vartheta : \vartheta\eta = \varepsilon\delta : \eta\lambda$, est etiam

$\varepsilon\beta : \varkappa\eta = \varepsilon\delta : \eta\lambda$, *et* vicissim

$\varepsilon\beta : \varepsilon\delta = \varkappa\eta : \eta\lambda$.

Sed *propter parallelas* $\varkappa\lambda$ $\beta\delta$ est $\varkappa\eta : \eta\lambda = \beta\gamma : \gamma\delta$; ergo

$\varepsilon\beta : \varepsilon\delta = \beta\gamma : \gamma\delta$, *et* vicissim

$\varepsilon\beta : \beta\gamma = \varepsilon\delta : \delta\gamma$.

Alii autem casus similiter *demonstrantur*.

XX. Sint duo triangula $\alpha\beta\gamma$ $\delta\varepsilon\zeta$ aequalibus angulis α δ; dico esse $\beta\alpha\cdot\alpha\gamma : \varepsilon\delta\cdot\delta\zeta = \Delta\ \alpha\beta\gamma : \Delta\ \delta\varepsilon\zeta$. Prop. 146

1) Vide append.

PROPOS. 145: Simson p. 513 sq., Breton p. 233, Chasles p. 77. 93. 210 sq. 277. 320.

PROPOS. 146: Simson p. 515 sq., Breton p. 233 sq., Chasles p. 77. 93. 247. 295. 307.

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add. BS 18. ἤχϑη AB, corr. S 21. ἐστὶν del. *Hu* 29. κ′ add. BS *ΔEZ*] *E* puncto notatum in A 29. 30. τὰς $\overline{ΑΔ}$ A, distinx. BS 31. πρὸς τὸ $\overline{ΕΔΖ}$ ABS, corr. V