

Book I of the Elements ΣΤΟΙΧΕΙΩΝ Α Öğelerin Birinci Kitabı

Euclid ΕΥΚΛΕΙΔΟΣ Öklid

September 29, 2016

Recovered from T_EX files with pdf version dated May 30, 2012

Edited to agree with the version of September 27, 2011

Contents

	2.2	Analiz	12	3.23	45
	2.3	Dil	13	3.24	46
				3.25	48
				3.26	49
				3.27	53
				3.28	54
				3.29	55
				3.30	57
				3.31	57
				3.32	58
				3.33	60
				3.34	61
				3.35	62
				3.36	63
				3.37	64
				3.38	65
				3.39	66
				3.40	67
				3.41	68
				3.42	69
				3.43	70
				3.44	71
				3.45	73
				3.46	75
				3.47	76
				3.48	78
	3		14		
	3.1	19		
	3.2	20		
	3.3	21		
	3.4	22		
	3.5	24		
	3.6	26		
	3.7	27		
	3.8	29		
	3.9	30		
	3.10	31		
	3.11	32		
	3.12	33		
	3.13	34		
	3.14	35		
	3.15	37		
	3.16	38		
	3.17	39		
	3.18	40		
	3.19	40		
	3.20	41		
	3.21	42		
	3.22	44		
1	Introduction		4		
1.1	Layout		4		
1.2	Text		4		
1.3	Analysis		4		
1.4	Language		5		
1.4.1	Writing		6		
1.4.2	Nouns		6		
1.4.3	The definite article		7		
1.4.4	Prepositions		8		
1.4.5	Verbs		9		
1.5	Translation		10		
2	Giriş		12		
2.1	Sayfa düzeni ve Metin		12		

List of Tables

1.1	Greek text, coded for \LaTeX	4
1.2	The Greek alphabet	6
1.3	Declension of Greek nouns	7
1.4	The Greek article	8
1.5	Nesting of Greek adjective phrases	9
1.6	Greek prepositions	10
2.1	Yunan alfabesi	13

DRAFT

Chapter 1

Introduction

1.1 Layout

Book I of Euclid’s *Elements* is presented here in three parallel columns: the original Greek text in the middle column, an English translation to its left, and a Turkish translation to its right.

Euclid’s *Elements* consist of 13 books, each divided into **propositions**. Some books also have **definitions**, and Book I has also **postulates** and **common notions**. In the presentation here, the Greek text of each sentence of each proposition is broken into units so that

1. each unit will fit on one line,
2. the unit as such has a role in the sentence,
3. the units, kept in the same order, make sense when translated into English.

Each proposition of the *Elements* is accompanied by

1.2 Text

We receive Euclid’s text through various filters. The *Elements* are supposed to have been composed around 300 B.C.E. Heiberg’s text (published in 1883) is based mainly on a manuscript in the Vatican written the tenth century C.E., closer to our time than to Euclid’s time. Knorr [8] argues that Euclid’s original intent may be better reflected in some Arabic translations from the eighth and ninth centuries. (The argument is summarized in [9].) Nonetheless, we shall just use the Heiberg text.

More precisely, for convenience, we take the Greek text in our underlying \LaTeX file from the \LaTeX files of Richard Fitzpatrick, who has published his own parallel English translation.¹ (In the underlying \LaTeX file, the enunciation of Proposition I.1 in Greek reads as in Table 1.1.) Fitzpatrick reports that his Greek text is that of Heiberg,

a picture of points and lines, with most points (and some lines) labelled with letters. This picture is the **lettered diagram**. We place the diagram for each proposition *after* the words. According to Reviel Netz [12, p. 35, n. 55], this is where the diagram appeared in the original scroll, presumably so that one would know how far to unroll the scroll in order to read the proposition. The end of a proposition is not to be considered as an undignified position. Indeed, Netz judges the diagram to be a *metonym* for the proposition: something associated with the proposition that is used to stand for the proposition. (Today the *enunciation* of a proposition—see §1.3 below—would appear to be the common metonym.)

but he gives it without Heiberg’s *apparatus criticus*. Also his method of transcription is unclear. There is at least one mistake in his text ($\tau\rho\delta\varsigma$ for $\pi\rho\delta\varsigma$ near the beginning of I.5). We shall correct such mistakes, if we find them, although we shall not look for them systematically.

In the process of translating, we have made use of a printout of the Greek text of Myungsun Ryu.² We do not have a \LaTeX file for this text; only pdf. The text is said to be taken from the *Perseus Digital Library*.

We also refer to images of Heiberg’s original text [1], which are available as pdf files from the Wilbour Hall website³ and from European Cultural Heritage Online (ECHO).⁴ In preparing the files from the latter source for printing, we have trimmed the black borders by means of a program called `briss`.⁵

>E*ρ*’i t[~]hc doje’ishc e>uje’iac peperasm’enhc tr’igwnon >is’opleuron sust’hasajai.

Table 1.1: Greek text, coded for \LaTeX

1.3 Analysis

¹<http://farside.ph.utexas.edu/euclid.html>

²<http://en.wikipedia.org/wiki/File:Euclid-Elements.pdf>

³<http://www.wilbourhall.org/>

⁴<http://echo.mpiwg-berlin.mpg.de/home/>

⁵<http://briss.sourceforge.net/>

Each proposition of the *Elements* can be understood as being a **problem** or a **theorem**. Writing around 320 C.E., Pappus of Alexandria [17, pp. 564–567] describes the distinction:

Those who favor a more technical terminology in geometrical research use

- **problem** (πρόβλημα) to mean a [proposition⁶] in which it is proposed to do or construct [something]; and
- **theorem** (θεώρημα), a [proposition] in which the consequences and necessary implications of certain hypotheses are investigated;

but among the ancients some described them all as problems, some as theorems.

In short, a problem proposes something to *do*; a theorem proposes something to *see*. (The Greek for *theorem* means more generally ‘that which is looked at’ and is related to the verb θεάομαι ‘look at’; from this also comes θέατρον ‘theater’.)

Be it a problem or a theorem, a proposition—or more precisely the *text* of a proposition—can be analyzed into as many as six parts. The Green Lion edition [3, p. xxiii] of Heath’s translation of Euclid describes this analysis as found in Proclus’s *Commentary on the First Book of Euclid’s Elements* [14, p. 159]. In the fifth century C.E., Proclus⁷ writes:

Every problem and every theorem that is furnished with all its parts should contain the following elements:

- 1) an **enunciation** (πρότασις),
- 2) an **exposition** (ἔκθεσις),
- 3) a **specification** (διορισμός),
- 4) a **construction** (κατασκευή),
- 5) a **proof** (ἀπόδειξις), and
- 6) a **conclusion** (συμπέρασμα).

Of these, the enunciation states what is given and what is being sought from it, for a perfect enunciation consists of both these parts. The exposition takes separately what is given and prepares it in advance for use in the investigation. The specification takes separately the thing that is sought and makes clear precisely what it is. The construction adds what is lacking in the given for finding what is sought. The proof draws the proposed inference by reasoning scientifically from the propositions that have been admitted. The conclusion reverts to the enunciation, confirming what has been proved.

So many are the parts of a problem or a theorem. The most essential ones, and those which are always present, are enunciation, proof, and conclusion.

1.4 Language

The Greek language that we have begun discussing is the language of Euclid: *ancient* Greek. This language belongs to the so-called Indo-European family of languages. English also belongs to this family, but Turkish does not.

Alternative translations are:

- for ἔκθεσις, *setting out*, and
- for διορισμός, *definition of goal* [12, p. 10].

Heiberg’s analysis of the text of the *Elements* into paragraphs does not correspond exactly to the analysis of Proclus; but Netz uses the analysis of Proclus in his *Shaping of Deduction in Greek Mathematics* [12], and we shall use it also, according to the following understanding:

1. The *enunciation* of a proposition is a general statement, without reference to the lettered diagram. The statement is about some subject, perhaps a straight line or a triangle.

2. In the *exposition*, that subject is identified in the diagram by means of letters; the existence of the subject is established by means of a third-person imperative verb.

3. (a) The *specification* of a *problem* says what will be done with the subject, and it begins with the words δεῖ δὴ. Here δεῖ is an impersonal verb with the meaning of ‘it is necessary to’ or ‘it is required to’ or simply ‘one must’; while δὴ is a ‘temporal particle’ with the root meaning of ‘at this or that point’ [10]. That which is necessary is expressed by a clause with an infinitive verb. In translating, we may use the English form ‘It is necessary for *A* to be *B*.’

(b) The specification of a *theorem* says what will be proved about the subject, and it begins with the words λέγω ὅτι ‘I say that’. The same expression may also appear in a problem, in an additional specification at the head of the proof, after the construction.

4. In the *construction*, if it is present, the second word is often γάρ, a ‘confirmatory adverb and causal conjunction’ [16, ¶2803, p. 637]. We translate it as ‘for’, at the beginning of the sentence; but again, γάρ itself is the second word, because it is *postpositive*: it simply never appears at the beginning of a sentence.

5. Then the *proof* often begins with the particle ἐπεὶ ‘because, since’. The ἐπεὶ (or other words) may be followed by οὖν, a ‘confirmatory or inferential’ postpositive particle [16, ¶2955, p. 664].

6. The *conclusion* repeats the enunciation, usually with the addition of the postpositive particle ἄρα ‘therefore’. Then, after the repeated enunciation, the conclusion ends with one of the clauses:

(a) ὅπερ ἔδει ποιῆσαι ‘just what it was necessary to do’ (in problems); Heiberg translates this into Latin as *quod oportebat fieri*, although *quod erat faciendum* or QEF is also used;

(b) ὅπερ ἔδει δεῖξαι ‘just what it was necessary to show’ (in theorems): in Latin, *quod erat demonstrandum*, or QED.

However, in some ways, Turkish is closer to Greek than English is. Modern scientific terminology, in English or Turkish, often has its origins in Greek.

⁶Ivor Thomas [17, p. 567] uses *inquiry* here in his translation; but there is *no* word in the Greek original corresponding to this or to *proposition*.

⁷Proclus was born in Byzantium (that is, Constantinople, now

İstanbul), but his parents were from Lycia (Likya), and he was educated first in Xanthus. He moved to Alexandria, then Athens, to study philosophy [14, p. xxxix].

capital	minuscule	transliteration	name
A	α	a	alpha
B	β	b	beta
Γ	γ	g	gamma
Δ	δ	d	delta
E	ε	e	epsilon
Z	ζ	z	zeta
H	η	ê	eta
Θ	θ	th	theta
I	ι	i	iota
K	κ	k	kappa
Λ	λ	l	lambda
M	μ	m	mu
N	ν	n	nu
Ξ	ξ	x	xi
O	ο	o	omicron
Π	π	p	pi
P	ρ	r	rho
Σ	σ, ς	s	sigma
T	τ	t	tau
Υ	υ	y, u	upsilon
Φ	φ	ph	phi
X	χ	ch	chi
Ψ	ψ	ps	psi
Ω	ω	ô	omega

Table 1.2: The Greek alphabet

1.4.1 Writing

The Greek alphabet, in Table 1.2, is the source for the Latin alphabet (which is used by English and Turkish), and it is a source for much scientific symbolism. The vowels of the Greek alphabet are α, ε, η, ι, ο, υ, and ω, where η is a long ε, and ω is a long ο; the other vowels (α, ι, υ) can be long or short. Some vowels may be given tonal accents (acute, circumflex, grave). An initial vowel takes either a rough-breathing mark (as in ἄ) or a smooth-breathing mark (ᾰ): the former mark is transliterated by a preceding h, and the latter can be ignored, as in ὑπερβολή *hyperbolê hyperbola*, ὀρθογώνιον *orthogônon rectangle*. Likewise, ῥ is transliterated as rh, as in ῥόμβος *rhombos rhombus*. A long vowel may have an iota subscript (α, η, ω), especially

in case-endings of nouns. Of the two forms of minuscule sigma, the ς appears at the ends of words; elsewhere, σ appears, as in βάσις *basis base*.

In increasing strength, the Greek punctuation marks are [, · .], corresponding to our [, ; .]. (The Greek question-mark is like our semicolon, but it does not appear in Euclid.)

Euclid himself will have used only the capital letters; the minuscules were developed around the ninth century [16, ¶2, p. 8]. The accent marks were supposedly invented around 200 B.C.E., because the pronunciation of the accents was dying out [16, ¶161, p. 38].

1.4.2 Nouns

As in Turkish, so in Greek, a single noun or verb can appear in many different forms. The general analysis is the same: the noun or verb can be analyzed as STEM + ENDING (*gönde + ek*).⁸

Like a Turkish noun, a Greek noun changes to show distinctions of *case* and *number*. Unlike a Turkish noun, a Greek noun does not take a separate ending (such as *-ler*) for the plural number; rather, each case-ending has a singular form and a plural form. (There is also a dual form, but this is rarely seen, although the distinction between the dual and the plural number occurs for example

in ἐκάτερος/ἕκαστος ‘either/each’.)

Unlike a Turkish noun, a Greek noun has one of three *genders*: masculine, feminine, or neuter. We can use this notion to distinguish nouns that are *substantives* from nouns that are *adjectives*. A substantive always keeps the same gender, whereas an adjective *agrees* with its associated noun in case, number, and gender.⁹ (Turkish does not show such agreement.)

The Greek cases, with their rough counterparts in Turkish, are as follows:

1. nominative (the dictionary form),

and neuter the inanimate, then the distinction between animate and inanimate is shown in *who/which*. Agreement of adjective with noun in English is seen in the demonstratives: *this word/these words*.

⁸The stem may be further analyzable as ROOT + CHARACTERISTIC.

⁹English retains the notion of gender only in its personal pronouns: *he, she, it*. If masculine and feminine are together the *an-*

2. genitive (*-in hâli* or *-den hâli*),
3. dative (*-e hâli* or *-le hâli*¹⁰ or *-de hâli*),
4. accusative (*-i hâli*),
5. vocative (usually the same as the nominative, and anyway it is not needed in mathematics, so we shall ignore it below).

The accusative case is the case of the direct object of a verb. Turkish assigns the ending *-i* only to *definite* direct objects; otherwise, the nominative is used. However, for a neuter Greek noun, the accusative case is always the same as the nominative.¹¹

A Greek noun is of the *vowel declension* or the *consonant declension*, depending on its stem. Within the vowel

declension, there is a further distinction between the $\bar{\alpha}$ - or *first* declension and the *o*- or *second* declension. Then the consonant declension is the *third* declension. The spelling of the case of a noun depends on declension and gender. Turkish might be said to have four declensions; but the variations in the case-endings in Turkish are determined by the simple rules of vowel harmony, so that it may be more accurate to say that Turkish has only one declension. Some variations in the Greek endings are due to something like vowel harmony, but the rules are much more complicated. Some examples are in Table 1.3.

The meanings of the Greek cases are refined by means of *prepositions*, discussed below.

		1st feminine	1st feminine	2nd masculine	2nd neuter	3rd neuter
singular	nominative	γραμμή	γωνία	κύκλος	τρίγωνον	μέρος
	genitive	γραμμής	γωνίας	κύκλου	τρίγωνου	μέρους
	dative	γραμμῇ	γωνίᾳ	κύκλῳ	τρίγωνῳ	μέρει
	accusative	γραμμήν	γωνίαν	κύκλον	τρίγωνον	μέρος
plural	nominative	γραμμαί	γωνίαι	κύκλοι	τρίγωνα	μέρη
	genitive	γραμμῶν	γωνίων	κύκλων	τρίγωνων	μέρων
	dative	γραμμαῖς	γωνίαις	κύκλοις	τρίγωνοις	μέρεσι
	accusative	γραμμάς	γωνίας	κύκλους	τρίγωνα	μέρη
		<i>line</i>	<i>angle</i>	<i>circle</i>	<i>triangle</i>	<i>part</i>

Table 1.3: Declension of Greek nouns

1.4.3 The definite article

Greek has a definite article, corresponding somewhat to the English *the*. Whereas *the* has only one form, the Greek article, like an adjective, shows distinctions of gender, number, and case, with forms as in Table 1.4.

Euclid may use (a case-form of) τό Α σημείον ‘the A point’ or ἡ ΑΒ εὐθεία [γραμμῇ] ‘the AB straight [line]’. Here the letters A and AB come between the article and the noun, in what Smyth calls *attributive* position [16, ¶1154]. Then A itself is not a point, and AB is not a line; the point and the line are seen in a diagram, *labelled* with the indicated letters. However, Euclid may omit the noun, speaking of τό Α ‘the A’ or ἡ ΑΒ ‘the AB’.

Sometimes (as in Proposition 3) a single letter may denote a straight line; but then the letter takes the feminine article, as in ἡ Γ ‘the Γ’, since γραμμῇ ‘line’ is feminine. Netz [12, 3.2.3, p.113] suggests that Euclid uses the neuter σημείον rather than the feminine στιγμή for ‘point’ so that points and lines will have different genders. (See Proposition 43 for a related example.)

In general, an adjective may be given an article and used as a substantive. (Compare ‘The best is the enemy of the good’, attributed to Voltaire in the French form *Le mieux est l’ennemi du bien*.¹²) The adjective need not even have the article. Euclid usually (but not always) says *straight* instead of *straight line*, and *right* instead of *right angle*. In our translation, we use STRAIGHT and RIGHT

when the substantives *straight line* and *right angle* are to be understood.

Euclid may also refer (as in Proposition 5) to κοινή ἡ ΒΓ ‘the ΒΓ, which is common’. Here the adjective κοινή ‘common’ would appear to be in *predicate* position [16, ¶1168]. In this position, the adjective serves not to distinguish the straight line in question from other straight lines, but to express its relation to other parts of the diagram (in this case, that it is the base of two different triangles).

Similarly, Euclid may use the adjective ὅλος *whole* in predicate position, as in Proposition 4: ὅλον τὸ ΑΒΓ τρίγωνον ἐπὶ ὅλον τὸ ΔΕΖ τρίγωνον ἐφαρμόσει ‘the ΑΒΓ triangle, as a whole, to the ΔΕΖ triangle, as a whole, will apply’. Smyth’s examples of adjective position include:

attributive: τὸ ὅλον στράτευμα *the whole army*;

predicate: ὅλον τὸ στράτευμα *the army as a whole*.

The distinction here may be that the whole army may have attributes of a person, as in ‘The whole army is hungry’; but the army as a whole does not (as a whole, it is not a person). The distinction is subtle, and in the example from Euclid, Heath just gives the translation ‘the whole triangle’.

In Proposition 5, Euclid refers to ἡ ὑπὸ ΑΒΓ γωνία, which perhaps stands for ἡ περιεχομένη ὑπὸ τῆς ΑΒΓ γραμμῆς γωνία ‘the contained-by-the-ΑΒΓ-line angle’ or

¹⁰One source, Özkırmılı [15, p. 155], does indeed treat *-le* as one of the *durum* or *hâl ekleri*.

¹¹English nouns retain a sort of genitive case, in the possessive forms: *man/man’s/men/men’s*. There are further case-distinctions in pronouns: *he/his/him, she/her, they/their/them*.

¹²<http://en.wikiquote.org/wiki/Voltaire>, accessed July 8, 2011.

¹³This is an elaboration of an observation by Netz [12, 3.2.1, p. 105; 4.2.1.1, pp. 133-4].

	m.	f.	n.
nom.	ὁ	ἡ	τό
gen.	τοῦ	τῆς	τοῦ
dat.	τῷ	τῇ	τῷ
acc.	τόν	τήν	τό
nom.	οἱ	αἱ	τά
gen.	τῶν	τῶν	τῶν
dat.	τοῖς	ταῖς	τοῖς
acc.	τούς	τάς	τά

Table 1.4: The Greek article

ἡ περιεχομένη ὑπὸ τῶν AB, BG εὐθειῶν γραμμῶν γωνία ‘the bounded-by-the-AB-BG-straight-lines angle’.¹³ In the same proposition, the form γωνία ἡ ὑπὸ ABΓ appears (actually γωνία ἡ ὑπὸ BZΓ), with no obvious distinction in meaning. (Each position of [ἡ] ὑπὸ ABΓ is called attribu-

tive by Smyth.) For short, Euclid may say just ἡ ὑπὸ ABΓ for the angle, without using γωνία.

The nesting of adjectives between article and noun can be repeated. An extreme example is the phrase from the enunciation of Proposition 47 analyzed in Table 1.5.

1.4.4 Prepositions

In the example in Table 1.5, the preposition ἀπό appears. This is used only before nouns in the genitive case. It usually has the sense of the English preposition *from*, as in the first postulate, or in the construction of Proposition 1, where straight lines are drawn *from* the point Γ to A and B. In Table 1.5 then, the sense of the Greek is not exactly that the square sits *on* the side, but that it arises *from* the side.

Euclid uses various prepositions, which, when used before nouns in various cases, have meanings roughly as in Table 1.6. Details follow.

When its object is in the accusative case, the preposition ἐπί has the sense of the English preposition *to*, as again in the first postulate, or in the construction of Proposition 1, where straight lines are drawn *from* Γ *to* A and B.

The prepositional phrase ἐπὶ τὰ αὐτὰ μέρη ‘to the same parts’ is used several times, as for example in the fifth postulate and Proposition 7. The object of the preposition ἐπὶ is again in the accusative case, but is plural. It would appear that, as in English, so in Greek, ‘parts’ can have the sense of the singular ‘region’. More precisely in this case, the meaning of ‘parts’ would appear to be ‘side [of a straight line]’; and one might translate the phrase ἐπὶ τὰ αὐτὰ μέρη by ‘on the same side’ (as Heath does).¹⁴ The more general sense of ‘part’ is used in the fifth common notion.

The object of the preposition ἐπί may also be in the genitive case. Then ἐπί has the sense of *on*, as yet again in the construction of Proposition 1, where a triangle is constructed *on* the straight line AB.

The preposition πρὸς is used in the set phrase πρὸς ὀρθὰς [γωνίας] *at right angles*, where the noun phrase ὀρθὰς [γωνία] *right [angle]* is a plural accusative. Also in the definitions of angle and circle, πρὸς is used with the accusative, in a sense normally expressed in English by ‘to’. In every other case in Euclid’s Book I, πρὸς is used with

the dative case and also has the sense of *at* or *on* as for example in Proposition 2, where a straight line is to be placed *at* a given point.

There is a set phrase, used in Propositions 14, 23, 24, 31, 42, 45, and 46, in which πρὸς appears twice: πρὸς τῇ εὐθείᾳ καὶ τῷ πρὸς αὐτῇ σημείῳ ‘*at* the straight [line] and [*at*] the point *on* it’. (It is assumed here that the *first* occurrence of πρὸς takes two objects, both STRAIGHT and *point*. It is unlikely that *point* is un-governed, since according to Smyth [16, ¶1534], in prose, ‘the dative of place (chiefly *place where*) is used only of proper names’.)

The preposition διά is used with the accusative case to give *explanations*. The explanation might be a clause whose verb is an infinitive and whose subject is in the accusative case itself; then the whole clause is given the accusative case by being preceded by the neuter accusative article τό.¹⁵ The first example is in Proposition 4: διά τὸ ἴσην εἶναι τὴν AB τῇ ΔE ‘because AB is equal to ΔE’.

The preposition διά is also used with the genitive case, with the sense of *through* as in speaking of a straight line *through* a point. This use of διά always occurs in a set phrase as in the enunciation of Proposition 31, where the straight line through the point is also parallel to some other straight line.

The preposition κατὰ is used in Book I always with a name or a word for a *point* in the accusative case. This point may be where two straight lines meet, as in Proposition 27, or where a straight line is bisected, as in Proposition 10. The set phrase κατὰ κορυφήν ‘at a head’ occurs for example in the enunciation of Proposition 15 to describe angles that are ‘vertically opposite’ or simply *vertical*.

The preposition μετὰ, used with the genitive case, means *with*. It occurs in Book I only in Proposition 43, only with the names of triangles, only in the sentence τὸ AΕΚ τρίγωνον μετὰ τοῦ ΚΗΓ ἴσον ἐστὶ τῷ ΑΘΚ τρίγωνῳ μετὰ τοῦ ΚΖΓ ‘Triangle AΕΚ, with [triangle] ΚΗΓ, is equal to triangle ΑΘΚ with [triangle] ΚΖΓ’.

¹⁴According to Netz [12, 3.2.2, p. 112], ‘parts’ means ‘direction’ in this phrase, and only in this phrase.

¹⁵It may however be pointed out that the article τό could also be

in the nominative case. However, prepositions are never followed by a case that is unambiguously nominative.

τὸ ἀπὸ τῆς τῆν ὀρθὴν γωνίαν ὑποτεينوῦσης πλευρᾶς τετράγωνον
 ἀπὸ τῆς τῆν ὀρθὴν γωνίαν ὑποτεينوῦσης πλευρᾶς
 τῆν ὀρθὴν γωνίαν

the right angle
 on the side subtending the right angle
 the square on the side subtending the right angle

Table 1.5: Nesting of Greek adjective phrases

The preposition *παρά* is used in Book I only in Proposition 44, with the name of a straight line in the genitive case; and then the preposition has the sense of *along*: a parallelogram is to be constructed, one of whose sides is set *along* the original straight line so that they coincide.

The adjective *παράλληλος* ‘parallel’, used frequently starting with Proposition 27, seems to result from *παρά* + *ἀλλήλων* ‘alongside one another’. Here *ἀλλήλων* is the reciprocal pronoun ‘one another’, never used in the singular or nominative; it seems to result from *ἄλλος* ‘another’. The dative plural *ἀλλήλοις* occurs frequently, as in Proposition 1, where circles cut *one another*, and two straight lines are equal *to one another*.

The preposition *ὑπό* is used in naming angles by letters, as in *ἡ ὑπὸ ABΓ γωνία* ‘the angle ABΓ’. Possibly such a phrase arises from a longer phrase, as in Proposition 4, *ἡ γωνία ἡ ὑπὸ τῶν εὐθειῶν περιεχομένη* ‘the angle that is contained *by* the [two] sides [elsewhere indicated]’. Here *ὑπό* precedes the agent of a passive verb, and the noun for the agent is in the genitive case. There is a similar use in the enunciation of Proposition 9: *ἡ ὑπὸ BAΓ γωνία δίχα τέμνεται ὑπὸ τῆς AZ εὐθείας* ‘The angle BAΓ is bisected *by* the [straight line] AZ’.

The preposition *ὑπό* is also used with nouns in the accusative case. It may then have the meaning of *under*, as in Proposition 5. More commonly it just precedes ob-

jects of the verb *ὑποτείνω* ‘stretch under’, used in English in the Latinate form *subtend*. The subject of this verb will be the side of a triangle, and the object will be the opposite angle.

The preposition *ἐν* ‘in’ is used only with the dative, frequently in the phrase *ἐν ταῖς αὐταῖς παραλλήλοις* ‘in the same parallels’, starting with Proposition 35. It is used in Proposition 42 and later with reference to parallelograms *in* a given angle. Finally, in Proposition 47 (the so-called Pythagorean Theorem), there is a general reference to a situation *in* right-angled triangles.

The preposition *ἐξ* ‘from’ is used with the genitive case. In Proposition 7, in the set phrase *ἐξ ἀρχῆς* ‘from the beginning’, that is, *original*. Beyond this, *ἐξ* appears only in the problematic definitions of straight line and plane surface, in the set phrase *ἐξ ἰσοῦ*: ‘from equality’ or, as Heath has it, ‘evenly’.

The preposition *περὶ* ‘about’ is used only in Propositions 43 and 44, only with the accusative, only with reference to figures arranged *about* the diameter of a parallelogram.

Greek has a few other prepositions: *σύν*, *ἀντί*, *πρό*, *ἀμφί*, and *ὑπέρ*; but these are not used in Book I. Any of the prepositions may be used also as a *prefix* in a noun or verb.

1.4.5 Verbs

A *verb* may show distinctions of *person*, *number*, *voice*, *tense*, *mood (mode)*, and *aspect*. Names for the forms that occur in Euclid are:

1. *mood*: indicative, imperative, or subjunctive;
2. *aspect*: continuous, perfect, or aorist;
3. *number*: singular or plural;
4. *voice*: active or passive;
5. *person*: first or third;
6. *tense*: past, present, or future.

(In other Greek writing there are also a *second* person, a *dual* number, and an *optative* mood. One speaks of a *middle* voice, but this usually has the same form as the passive.) Euclid also uses *verbal nouns*, namely *infinitives* (verbal substantives) and *participles* (verbal adjectives).

Suppose the utterance of a sentence involves three things: the *speaker* of the sentence, the *act* described by

the sentence, and the *performer* of the act. If only for the sake of remembering the six verb features above, one can make associations as follows:

1. mood: speaker
2. aspect: act
3. number: performer
4. voice: performer–act
5. person: speaker–performer
6. tense: act–speaker.

First-person verbs are rare in Euclid. As noted above, *λέγω* ‘I say’ is used at the beginning of specifications of theorems, and a few other places. Also, *δείξομεν* ‘we shall show’ is used a few times. The other verbs are in the third person.

Of the 48 propositions of Book I, 14 have enunciations of the form *Ἐάν* + SUBJUNCTIVE.

Often in sentences of the logical form ‘If *A*, then *B*’, Euclid will express ‘If *A*’ as a *genitive absolute*, a noun and participle in the genitive case. We use the corresponding absolute construction in English.

	genitive	dative	accusative
ἀπό	from		
διά	through [a point]		owing to
ἐν		in	
ἐξ	from [the beginning]		
ἐπι	on		to
κατά			at [a point]
μετά	with		
παρά	along [a straight line]		
περί			about
πρός		at/on	at [right angles]
ὑπό	by		under

Table 1.6: Greek prepositions

1.5 Translation

The Perseus website,¹⁶ with its Word Study Tool, is useful for parsing. However, in the work of interpreting the Greek, we also consult print resources, such as Smyth's *Greek Grammar* [16], the *Greek-English Lexicon* of Liddell, Scott, and Jones [10], the *Pocket Oxford Classical Greek Dictionary* [11], and Heath's translation of the *Elements* [3, 2].

There are online lessons on reading Euclid in Greek.¹⁷

In translating Euclid into English, Heath seems to stay as close to Euclid as possible, under the requirement that the translation still read well *as English*. There may be subtle ways in which Heath imposes modern ways of thinking that are foreign to Euclid.

The English translation here tries to stay even closer to Euclid than Heath does. The purpose of the translation is to elucidate the original Greek. This means the translation may not read so well as English. In particular, word order may be odd. Simple declarative sentences in English normally have the order SUBJECT-VERB-OBJECT (or SUBJECT-COPULA-PREDICATE). When Euclid uses another order, say SUBJECT-OBJECT-VERB (or SUBJECT-PREDICATE-COPULA), the translation *may* follow him. There is a precedent for such variations in English order, albeit from a few centuries ago. For example, there is the rendition by George Chapman (1559?–1634) of Homer's *Iliad* [13]. Chapman begins his version of Homer thus:

Achilles' banefull wrath resound, O Goddess,
that imposd
Infinite sorrowes on the Greekes, and many
brave soules losd
From breasts Heroique—sent them farre, to
that invisible cave
That no light comforts; and their lims to dogs
and vultures gave.
To all which Jove's will gave effect; from whom
first strife begunne
Betwixt Atrides, king of men, and Thetis' god-

¹⁶<http://www.perseus.tufts.edu/hopper/collection?collection=Perseus%3Acorpus%3Aperseus%2Cwork%2CEuclid%2C%20Elements>

¹⁷<http://www.du.edu/~etuttle/classics/nugreek/contents.htm>

¹⁸The Gospel According to St Matthew, 6:19: 'Lay not up for yourselves treasures upon earth, where moth and rust doth corrupt, and where thieves break through and steal'.

¹⁹Text taken from <http://www.gutenberg.org/files/205/205-h/>

like Sonne.

The word order SUBJECT-PREDICATE-COPULA is seen also in the lines of Sir Walter Raleigh (1554?–1618), quoted approvingly by Henry David Thoreau (1817–62) [18]:

But men labor under a mistake. The better part of the man is soon plowed into the soil for compost. By a seeming fate, commonly called necessity, they are employed, as it says in an old book, laying up treasures which moth and rust will corrupt and thieves break through and steal.¹⁸ It is a fool's life, as they will find when they get to the end of it, if not before. It is said that Deucalion and Pyrrha created men by throwing stones over their heads behind them:—

*“Inde genus durum sumus, experient-
sque laborum,
Et documenta damus qua simus origine
nati.”*

Or, as Raleigh rhymes it in his sonorous way,—

*“From thence our kind hard-hearted is,
enduring pain and care,
Approving that our bodies of a stony
nature are.”*

So much for a blind obedience to a blundering oracle, throwing the stones over their heads behind them, and not seeing where they fell.¹⁹

More examples:

The man recovered of the bite,
The dog it was that died.²⁰

Whose woods these are I think I know.
His house is in the village though;
He will not see me stopping here
To watch his woods fill up with snow.²¹

205-h.htm, July 6, 2011.

²⁰The last lines of 'An Elegy on the Death of a Mad Dog' by Oliver Goldsmith (1728–1774) (http://www.poetry-archival.com/g/an_elegy_on_the_death_of_a_mad_dog.html, accessed July 12, 2011).

²¹The first stanza of 'Stopping by Woods on a Snowy Evening' by Robert Frost (<http://www.poetryfoundation.org/poem/171621>, accessed July 12, 2011).

DRAFT

Chapter 2

Giriş

2.1 Sayfa düzeni ve Metin

Öklid'in *Öğelerinin* birinci kitabı, burada üç sütun halinde sunuluyor: orta sütunda orijinal Yunanca metin, onun solunda bir İngilizce çevirisi ve sağında bir Türkçe çevirisi yer alıyor.

Öklid'in *Öğeleri*, her biri **önermelere** bölünmüş olan 13 kitaptan oluşur. Bazı kitaplarda **tanımlar** da vardır. Birinci kitap ayrıca **postülatları** ve **genel kavramları** da içerir. Yunanca metnin her önermesinin her cümlesi öyle birimlere bölünmüştür ki

1. her birim bir satıra sığar,
2. birimler cümle içinde bir rol oynarlar
3. İngilizceye çevirirken birimlerin sırasını korumak anlamlı olur.

2.2 Analiz

Öğelerin her önermesi bir **problem** veya bir **teorem** olarak anlaşılabilir. M.S. 320 civarında yazan İskenderiyeli Pappus bu ayrımı tarif ediyor [17, pp. 564–567] :

Geometrik araştırmada daha teknik terimleri tercih edenler

- **problem** (πρόβλημα) terimini içinde [birşey] yapılması veya inşa edilmesi önerilen [bir önerme] anlamında; ve
- **teorem** (θεώρημα) terimini içinde belirli bir hipotezin sonuçlarının ve gerekliliklerinin incelendiği [bir önerme] anlamında;

kullanırlar ama antiklerin bazıları bunların tümünü problem, bazıları da teorem olarak tarif etmiştir.

Kısaca, bir problem birşey *yapmayı* önerir; bir teorem birşeyi *görmeyi*. (Yunancada *Teorem* kelimesi daha genel olarak 'bakılmış olan' anlamındadır ve θεωρεῖν 'bak' fiilyle ilgilidir; burdan ayrıca θέατρον 'theater' kelimesi de türemiştir.)

İster bir problem, ister bir teorem olsun, bir önerme—ya da daha tam anlamıyla bir önermenin *metni* —altı parçaya kadar ayrılıp analiz edilebilir. Öklid'in Heath çevirisinin The Green Lion baskısı [3, p. xxiii] bu analizi Proclus'un *Commentary on the First Book of Euclid's Elements* [14, p. 159] kitabında bulunan haliyle tarif eder. M.S., beşinci yüzyılda Proclus¹ şöyle yazmıştır:

¹Proclus Bizans (yani, Konstantinapolis, şimdi İstanbul) doğumludur, ama aslında Likyalıdır, ve ilk eğitimini Ksantos'ta almıştır.

Öğelerin her önermesinin yanında, çoğu noktanın (ve bazı çizgilerin) harflerle isimlendirildiği, bir çizgi ve noktalar resmi yer alır. Bu resim **harfli diagramdır**. Her önermede diagramı kelimelerin *sonuna* yerleştiriyoruz. Reviel Netz'e göre orijinal ruloda diagram burada yer alırdı ve böylece okuyan önermeyi okumak için ruloyu ne kadar açması gerektiğini bilirdi [12, p. 35, n. 55].

Öklid'in yazdıklarının çeşitli süzgeçlerden geçmiş haline ulaşabiliyoruz. Öğelerin M. Ö. 300 civarında yazılmış olması gerekir. Bizim kullandığımız 1883'te yayınlanan Heiberg versiyonu onuncu yüzyılda Vatikan'da yazılan bir elyazmasına dayanmaktadır.

Bütün parçalarıyla donatılmış her problem ve teorem aşağıdaki öğeleri içermelidir:

- 1) bir **ilan** (πρότασις),
- 2) bir **açıklama** (ἐκθεσις),
- 3) bir **belirtme** (διορισμός),
- 4) bir **hazırlama** (κατασκευή),
- 5) bir **gösteri** (ἀπόδειξις), and
- 6) bir **bitirme** (συμπέρασμα).

Bunlardan, ilan, verileni ve bundan ne sonuç elde edileceğini belirtir çünkü mükemmel bir ilan bu iki parçanın ikisini de içerir. Açıklama, verileni ayrıca ele alır ve bunu daha sonra incelemede kullanılmak üzere hazırlar. Belirtme, elde edilecek sonucu ele alır ve onun ne olduğunu kesin bir şekilde açıklar. Hazırlama, elde edilecek sonuca ulaşmak için verilecek neyin eksik olduğunu söyler. Gösteri, önerilen çıkarımı kabul edilen önermelerden bilimsel akıl yürütmeye oluşturur. Bitirme, ilana geri dönerek ispatlanmış olanı onaylar.

Bir problem veya teoremin parçaları arasında en önemli olanları, her zaman bulunan, ilan, gösteri ve bitirmedir.

Biz de Proclus'un analizini aşağıdaki anlamıyla kullanacağız:

1. *İlan*, bir önermenin, harfli diagrama gönderme yapmayan, genel beyanıdır. Bu beyan, bir doğru veya üçgen

Felsefe öğrenmek için İskenderiye'ye ve sonra da Atina'ya gitmiştir. [14, p. xxxix].

gibi bir nesne hakkındadır.

2. *Açıklamada*, bu nesne diagramla harfler aracılığıyla özdeşleştirilir. Bu nesnenin varlığı üçüncü tekil emir kipinde bir fiil ile oluşturulur.

3. (a) *Belirtme*, bir *problemde*, nesne ile ilgili ne yapılacağını söyler ve $\delta\epsilon\iota$ $\delta\eta$ kelimeleriyle başlar. Burada $\delta\epsilon\iota$, 'gereklidir', $\delta\eta$ ise 'şimdi' anlamındadır.

(b) Bir *teoremden* belirtme, nesneyle ilgili neyin ispatlanacağını söyler ve 'İddia ediyorum ki' anlamına gelen $\lambda\epsilon\gamma\omega$ $\delta\tau\iota$ kelimeleriyle başlar. Aynı ifade, bir *problemde* de belirtmeye ek olarak, gösterinin başında, hazırlamanın sonunda görülebilir.

4. *Hazırlamada*, eğer varsa, ikinci kelime $\gamma\acute{\alpha}\rho$, onaylayıcı bir zarf ve sebep belirten bir bağlaçtır. Bu kelimeyi cümlelerin birinci kelimesi 'çünkü' olarak çeviriyoruz.

5. *Gösteri* genellikle $\epsilon\pi\epsilon\iota$ 'çünkü, olduğundan' ilgeciyle başlar.

6. *Bitirme*, ilanı tekrarlar ve genellikle 'dolayısıyla' ilgecini içerir. Tekrarlanan ilandan sonra bitirme aşağıdaki iki kılıptan biriyle sonlanır:

(a) $\acute{\omicron}\pi\epsilon\rho$ $\acute{\epsilon}\delta\epsilon\iota$ $\pi\omicron\iota\tilde{\eta}\sigma\alpha\iota$ 'yapılması gereken tam buydu' (problemlerde);

(b) $\acute{\omicron}\pi\epsilon\rho$ $\acute{\epsilon}\delta\epsilon\iota$ $\delta\epsilon\iota\tilde{\xi}\alpha\iota$ 'gösterilmesi gereken tam buydu' (teoremlerde): Latince, *quod erat demonstrandum*, veya QED.

2.3 Dil

Öklid'in kullandığı dil: *Antik Yunan*cadır. Bu dil Hint-Avrupa dilleri ailesindedir. İngilizce de bu ailedendir ancak Türkçe değildir. Fakat bazı yönlerden Türkçe, Yunan-

caya, İngilizceden daha yakındır. İngilizce ve Türkçenin günümüz bilimsel terminolojisinin kökleri genellikle Yunan-

büyük	küçük	okunuş	isim
A	α	a	alfa
B	β	b	beta
Γ	γ	g	gamma
Δ	δ	d	delta
E	ϵ	e	epsilon
Z	ζ	z (ds)	zeta
H	η	ê (uzun e)	eta
Θ	θ	th	theta
I	ι	i	iota (yota)
K	κ	k	kappa
Λ	λ	l	lambda
M	μ	m	mü
N	ν	n	nü
Ξ	ξ	ks	ksi
O	o	o (kisa)	omikron
Π	π	p	pi
P	ρ	r	rho (ro)
Σ	σ, ς	s	sigma
T	τ	t	tau
Υ	υ	y, ü	üpsilon
Φ	φ	f	phi
X	χ	h (kh)	khi
Ψ	ψ	ps	psi
Ω	ω	ô (uzun o)	omega

Table 2.1: Yunan alfabesi

Chapter 3

'Definitions'

Boundaries¹

[1] A point is
[that] whose part is nothing.²

[2] A line,
length without breadth.

[3] Of a line,
the extremities are points.

[4] A straight line is
whatever [line] evenly
with the points of itself
lies.

[5] A surface is
what has length and breadth only.

[6] Of a surface,
the boundaries are lines.

[7] A plane surface is
what [surface] evenly
with the points of itself
lies.

[8] A plane angle is,
...³
in a plane,
two lines taking hold of one another,
and not lying on a STRAIGHT,
to one another
the inclination of the lines.

[9] Whenever the lines containing the
angle
be straight,
rectilineal is called the angle.

[10] Whenever
a STRAIGHT,
standing on a STRAIGHT,
the adjacent angles

Όροι

Σημεῖόν ἐστιν,
οὗ μέρος οὐθέν.

Γραμμὴ δὲ
μῆκος ἀπλατές.

Γραμμῆς δὲ
πέρατα σημεῖα.

Εὐθεῖα γραμμὴ ἐστίν,
ἣτις ἐξ ἴσου
τοῖς ἐφ' ἑαυτῆς σημεῖοις
κεῖται.

Ἐπιφάνεια δὲ ἐστίν,
ὃ μῆκος καὶ πλάτος μόνον ἔχει.

Ἐπιφανείας δὲ
πέρατα γραμμαί.

Ἐπίπεδος ἐπιφάνειά ἐστίν,
ἣτις ἐξ ἴσου
ταῖς ἐφ' ἑαυτῆς εὐθείαις
κεῖται.

Ἐπίπεδος δὲ γωνία ἐστίν
ἢ
ἐν ἐπιπέδῳ
δύο γραμμῶν ἀπτομένων ἀλλήλων
καὶ μὴ ἐπ' εὐθείας κειμένων
πρὸς ἀλλήλας
τῶν γραμμῶν κλίσις.

Όταν δὲ αἱ περιέχουσαι τὴν γωνίαν
γραμμαί
εὐθεῖαι ᾧσιν,
εὐθύγραμμος καλεῖται ἡ γωνία.

Όταν δὲ
εὐθεῖα
ἐπ' εὐθεῖαν σταθεῖσα
τὰς ἐφεξῆς γωνίας

Sınırlar

Bir nokta,
parçası hiçbir şey olandır.

Bir çizgi,
ensiz uzunluktur.

Bir çizginin
uçlarındakiler, noktaldır.

Bir doğru,
üzerindeki noktalara hizalı uzanan bir
çizgidir.

Bir yüzey,
sadece eni ve boyu olandır.

Bir yüzeyin
uçlarındakiler, çizgilerdir.

Bir düzlem,
üzerindeki doğruların noktalarıyla
hizalı uzanan bir yüzeydir.

Bir düzlem açısı,
bir düzlemde
kesişen ve aynı doğru üzerinde uzan-
mayan
iki çizginin birbirine göre eğikliğidir.

Ve açığı içeren çizgiler
bire doğru olduğu zaman
düzkenar, denir açığı .

Bir doğru
başka bir doğrunun üzerine yerleşip
birbirine eşit bitişik açılar oluşturu-
duğunda,

¹The usual translation is 'definitions', but what follow are not really definitions in the modern sense.

²Presumably subject and predicate are inverted here, so the sense

is that of 'A point is that of which nothing is a part.'

³There is no way to put 'the' here to parallel the Greek.

equal to one another make,
right
either of the equal angles is,
and
the STRAIGHT that has been stood
is called perpendicular
to that on which it has been stood.⁴

[11] An obtuse angle is
that [which is] greater than a RIGHT.

[12] Acute,
that less than a RIGHT.

[13] A boundary is
whis is a limit of something.

[14] A figure is
what is contained by some boundary
or boundaries.⁵

[15] A circle is
a plane figure
contained by one line
[which is called the circumference]
to which,
from one point
of those lying inside of the figure
all STRAIGHTS falling
[to the circumference of the circle]
are equal to one another.

[16] A⁶ center of the circle
the point is called.

[17] A diameter of the circle is
some STRAIGHT
drawn through the center
and bounded
to either parts
by the circumference of the circle,
which also bisects the circle.

[18] A semicircle is
the figure contained
by the diameter
and the circumference taken off by it.
A center of the semicircle [is] the same
which is also of the circle.

[19] Rectilineal figures are⁷
those contained by STRAIGHTS,
triangles, by three,
quadrilaterals, by four,
polygons,⁸ by more than four
STRAIGHTS contained.

ἴσας ἀλλήλαις ποιῆ,
ὀρθῇ
ἑκατέρα τῶν ἴσων γωνιῶν ἐστὶ,
καὶ
ἢ ἐφ' ἐστῆκυῖα εὐθεῖα
κάθετος καλεῖται,
ἐφ' ἣν ἐφέστηκεν.

Ἄμβλεῖα γωνία ἐστὶν
ἢ μείζων ὀρθῆς.

Ὁξεῖα δὲ
ἢ ἐλάσσων ὀρθῆς.

Ὅρος ἐστίν, ὃ τινός ἐστι πέρας.

Σχῆμά ἐστι
τὸ ὑπὸ τινος ἢ τινῶν ὁρῶν πε-
ριεχόμενον.

Κύκλος ἐστὶ
σχῆμα ἐπίπεδον
ὑπὸ μιᾶς γραμμῆς περιεχόμενον
[ἢ καλεῖται περιφέρεια],
πρὸς ἣν
ἀφ' ἑνὸς σημείου
τῶν ἐντὸς τοῦ σχήματος κειμένων
πᾶσαι αἱ προσπίπτουσαι εὐθεῖαι
[πρὸς τὴν τοῦ κύκλου περιφέρειαν]
ἴσαι ἀλλήλαις εἰσίν.

Κέντρον δὲ τοῦ κύκλου
τὸ σημεῖον καλεῖται.

Διάμετρος δὲ τοῦ κύκλου ἐστὶν
εὐθεῖα τις
διὰ τοῦ κέντρου ἠγμένη
καὶ περατουμένη
ἐφ' ἑκάτερα τὰ μέρη
ὑπὸ τῆς τοῦ κύκλου περιφέρειας,
ἣτις καὶ δίχα τέμνει τὸν κύκλον.

Ἡμικύκλιον δὲ ἐστὶ
τὸ περιεχόμενον σχῆμα
ὑπὸ τε τῆς διαμέτρου
καὶ τῆς ἀπολαμβανομένης ὑπ' αὐτῆς πε-
ριφέρειας.
κέντρον δὲ τοῦ ἡμικυκλίου τὸ αὐτό,
ὃ καὶ τοῦ κύκλου ἐστίν.

Σχήματα εὐθύγραμμά ἐστι
τὰ ὑπὸ εὐθειῶν περιεχόμενα,
τρίπλευρα μὲν τὰ ὑπὸ τριῶν,
τετράπλευρα δὲ τὰ ὑπὸ τεσσάρων,
πολύπλευρα δὲ τὰ ὑπὸ πλείονων ἢ τεσ-
σάρων

eşit açılarn her birine dik aç,ı,
ve diğ erinin üzerinde duran doğruya
da;
üzerinde durduđu doğruya bir dik
doğ ru denir.

Bir geniş aç,ı,
büyük olandır bir dik aç,ıdan.

Bir dar aç,ı,
küçük olandır bir dik aç,ıdan.

Bir *sınır*,
bir şeyin ucunda olandır.

Bir figür,
bir sınır tarafından veya sınırlarla
içerilendir.

Bir daire,
bir çizgiye iç erilen
[bu çizgiye ç ember denir]
bir figürdür öyle ki
figürün iç erisindeki
noktaların birinden
ç izgi üzerine gelen
tüm doğrular,
birbirine eş ittir;

Ve o noktaya, dairenin merkezi denir.

Bir dairenin bir ç apı,
dairenin merkezinden geç ip
her iki tarafta da
dairenin çevresinde
sınırlanan
bir doğ rudur
ve böyle bir doğ ru, daireyi ikiye böler.

Bir yarıdaire,
bir ç ap
ve onun kestiđ i bir çevrece
iç erilen figürdür, ve yarıdairenin
merkezi, o dairenin merkeziyle
aynıdır.

Düzkenar figürler,
doğ rularca iç erilenlerdir. *Üçkenar*
figürler üç, *dörtkenar* figür-
ler dört ve *çokkenar* figürler
ise dörtten daha fazla doğ ruca
iç erilenlerdir.

⁴This definition is quoted in Proposition 12.

⁵In Greek what is repeated is not 'boundary' but 'some'.

⁶None of the terms defined in this section is preceded by a defi-
nite article. In particular, what is being defined here is not *the* center

of a circle, but *a* center. However, it is easy to show that the center
of a given circle is unique; also, in Proposition III.1, Euclid finds *the*
center of a given circle.

[20] There being trilateral figures, an equilateral triangle is that having three sides equal, isosceles, having only two sides equal, scalene, having three unequal sides.	εὐθειῶν περιεχόμενα. ὦν δὲ τριπλευρῶν σχημάτων ἰσόπλευρον μὲν τρίγωνόν ἐστι τὸ τὰς τρεῖς ἴσας ἔχον πλευράς, ἰσοσκελὲς δὲ τὸ τὰς δύο μόνας ἴσας ἔχον πλευράς, σκαληνὸν δὲ τὸ τὰς τρεῖς ἀνίσους ἔχον πλευράς.	Üçkenar figürlerden bir eşkenar üçgen, üç kenarı eşit olan, ikizkenar, eşit iki kenarı olan çeşitkenar, üç kenarı eşit olmayandır.
[21] Yet of trilateral figures, a right-angled triangle is that having a right angle, obtuse-angled, having an obtuse angle, acute-angled, having three acute angles.	Ἔτι δὲ τῶν τριπλευρῶν σχημάτων ὀρθογώνιον μὲν τρίγωνόν ἐστι τὸ ἔχον ὀρθήν γωνίαν, ἀμβλυγώνιον δὲ τὸ ἔχον ἀμβλεῖαν γωνίαν, ὀξυγώνιον δὲ τὸ τὰς τρεῖς ὀξείας ἔχον γωνίας.	Ayrıca, üçkenar figürlerden, bir dik üçgen, bir dik açısı olan, geniş açılı, bir geniş açısı olan, dar açılı, üç açısı dar açı olandır.
[22] Of quadrilateral figures, a square is what is equilateral and right-angled, an oblong, right-angled, but not equilateral, a rhombus, equilateral, but not right-angled, rhomboid, having opposite sides and angles equal, which is neither equilateral nor right-angled; and let quadrilaterals other than these be called trapezia.	Τῶν δὲ τετραπλευρῶν σχημάτων τετράγωνον μὲν ἐστίν, ὃ ἰσόπλευρόν τε ἐστὶ καὶ ὀρθογώνιον, ἑτερόμηκες δέ, ὃ ὀρθογώνιον μὲν, οὐκ ἰσόπλευρον δέ, ῥόμβος δέ, ὃ ἰσόπλευρον μὲν, οὐκ ὀρθογώνιον δέ, ῥομβοειδὲς δὲ τὸ τὰς ἀπεναντίον πλευράς τε καὶ γωνίας ἴσας ἀλλήλαις ἔχον, ὃ οὔτε ἰσόπλευρόν ἐστιν οὔτε ὀρθογώνιον· τὰ δὲ παρὰ ταῦτα τετράπλευρα τραπέζια καλεῖσθω.	Dörtkenar figürlerden bir kare, hem eşit kenar hem de dik-açılı olan, bir dikdörtgen, dik-açılı olan ama eşit kenar olmayan, bir eşkenar dörtgen, eşit kenar olan ama dik-açılı olmayan, bir paralelkenar karşılıklı kenar ve açıları eşit olan ama eşit kenar ve dik-açılı olmayandır. Ve bunların dışında kalan dörtkenarlara yamuk denilsin.
[23] Parallels are STRAIGHTS, whichever, being in the same plane, and extended to infinity to either parts, to neither [parts] fall together with one another.	Παράλληλοι εἰσὶν εὐθεῖαι, αἵτινες ἐν τῷ αὐτῷ ἐπιπέδῳ οὔσαι καὶ ἐκβαλλόμεναι εἰς ἄπειρον ἐφ' ἑκάτερα τὰ μέρη ἐπὶ μηδέτερα συμπύπτουσιν ἀλλήλαις.	Paraleller, aynı düzlemde bulunan ve her iki yönde de sınırsızca uzatıldıklarında hiçbir noktada kesişmeyen doğrulardır.

⁷As in Turkish, so in Greek, a plural subject can take a singular verb, when the subject is of the neuter gender in Greek, or names inanimate objects in Turkish.

⁸To maintain the parallelism of the Greek, we could (like Heath) use 'trilateral', 'quadrilateral', and 'multilateral' instead of 'triangle', 'quadrilateral', and 'polygon'. Today, triangles and quadrilaterals are polygons. For Euclid, they are not: you never call a triangle a polygon, because you can give the more precise information that it is a triangle.

Postulates

Postulates

Let it have been postulated from any point to any point a straight line to draw.

Also, a bounded STRAIGHT continuously in a straight to extend.

Also, to any center and distance a circle to draw.

Also, all right angles equal to one another to be.

Also, if in two straight lines falling the interior angles to the same parts less than two RIGHTS make, the two STRAIGHTS, extended to infinity, fall together, to which parts are the less than two RIGHTS.

Αιτήματα

Ἦιτήσθω ἀπὸ παντὸς σημείου ἐπὶ πᾶν σημεῖον εὐθεῖαν γραμμὴν ἀγαγεῖν.

Καὶ πεπερασμένην εὐθεῖαν κατὰ τὸ συνεχές ἐπ' εὐθείας ἐκβαλεῖν.

Καὶ παντὶ κέντρῳ καὶ διαστήματι κύκλον γράψεσθαι.

Καὶ πάσας τὰς ὀρθὰς γωνίας ἴσας ἀλλήλαις εἶναι.

Καὶ ἐὰν εἰς δύο εὐθείας εὐθεῖα ἐμπίπτουσα τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη γωνίας δύο ὀρθῶν ἐλάσσονας ποιῇ, ἐκβαλλομένας τὰς δύο εὐθείας ἐπ' ἄπειρον συμπίπτειν, ἐφ' ἃ μέρη εἰσὶν αἱ τῶν δύο ὀρθῶν ἐλάσσονες.

Postulatlar

Herhangi bir noktadan herhangi bir noktaya bir doğru çizilmesi.

Sonlu bir doğrunun kesiksiz şekilde sonlu uzatılması.

Her merkez ve uzunluk için bir daire çizilmesi.

Bütün dik açılardan bir birine eşit olduğu.

İki doğruyu kesen bir doğrunun aynı tarafta oluşturduğu iç açılar iki dik açıdan küçükse, bu iki doğrunun, sınırsızca uzatıldıklarında açılardan iki dik açıdan küçük olduğu tarafta kesişeceği.

Common Notions

Common notions

Equals to the same
also to one another are equal.

Also, if to equals
equals be added,
the wholes are equal.

Also, if from equals
equals be taken away,
the remainders are equal.

Also things applying to one another
are equal to one another.

Also, the whole
than the part is greater.

Κοινὰ ἔννοιαι

Τὰ τῶ αὐτῶ ἴσα
καὶ ἀλλήλοισ ἐστὶν ἴσα.

Καὶ ἐὰν ἴσοις
ἴσα προστεθῆ,
τὰ ὅλα ἐστὶν ἴσα.

αὶ ἐὰν ἀπὸ ἴσων
ἴσα ἀφαιρεθῆ,
τὰ καταλειπόμενά ἐστὶν ἴσα.

Καὶ τὰ ἐφαρμόζοντα ἐπ' ἀλλήλα
ἴσα ἀλλήλοισ ἐστὶν.

Καὶ τὸ ὅλον
τοῦ μέρους μείζον [ἐστὶν].

Genel Kavramlar

Aynı şeye eşitler
birbirlerine de eşittir.

Eğer eşitlere
eşitler eklenirse,
elde edilenler de eşittir.

Eğer eşitlerden
eşitler çıkartılırsa,
kalanlar eşittir.

Birbiriyle çakışan şeyler
birbirine eşittir.

Bütün,
parçadan büyüktür.

DRAFT

3.1

On
the¹ given bounded STRAIGHT
for² an equilateral triangle
to be constructed.

Let be³
the given bounded STRAIGHT
AB.

It is necessary then
on the STRAIGHT AB
for an equilateral triangle
to be constructed.⁴

To center A
at distance AB
suppose a circle has been drawn,
[namely] BΓΔ,
and moreover,
to center B
at distance BA
suppose a circle has been drawn,
[namely] AΓΕ,
and from the point Γ,
where the circles cut one another,
to the points A and B,
suppose there⁵ have been joined
the STRAIGHTS ΓΑ and ΓΒ.

And since the point A
is the center of the circle ΓΔΒ,
equal is ΑΓ to ΑΒ;
moreover,
since the point B
is the center of the circle ΓΑΕ,
equal is ΒΓ to ΒΑ.

Ἐπι
τῆς δοθείσης εὐθείας πεπερασμένης
τρίγωνον ἰσόπλευρον
συστήσασθαι.

Ἔστω
ἡ δοθεῖσα εὐθεῖα πεπερασμένη
ἡ ΑΒ.

Δεῖ δὴ
ἐπὶ τῆς ΑΒ εὐθείας
τρίγωνον ἰσόπλευρον
συστήσασθαι.

Κέντρῳ μὲν τῷ Α
διαστήματι δὲ τῷ ΑΒ
κύκλος γεγράφθω
ὁ ΒΓΔ,
καὶ πάλιν
κέντρῳ μὲν τῷ Β
διαστήματι δὲ τῷ ΒΑ
κύκλος γεγράφθω
ὁ ΑΓΕ,
καὶ ἀπὸ τοῦ Γ σημείου,
καθ' ὃ τέμνουσιν ἀλλήλους οἱ κύκλοι,
ἐπὶ τὰ Α, Β σημεία
ἐπεζεύχθωσαν
εὐθεῖαι αἱ ΓΑ, ΓΒ.

Καὶ ἐπει τὸ Α σημεῖον
κέντρον ἐστὶ τοῦ ΓΔΒ κύκλου,
ἴση ἐστὶν ἡ ΑΓ τῆ ΑΒ·
πάλιν,
ἐπει τὸ Β σημεῖον
κέντρον ἐστὶ τοῦ ΓΑΕ κύκλου,
ἴση ἐστὶν ἡ ΒΓ τῆ ΒΑ.

Verilmiş sınırlanmış doğruya
eşkenar üçgen
inşa edilmesi.

Verilmiş
sınırlanmış doğru
AB olsun.

Şimdi gereklidir
AB doğrusuna
eşkenar üçgenin
inşa edilmesi.

A merkezine,
AB uzaklığında olan
çember çizilmiş olsun,
BΓΔ,
ve yine
B merkezine,
BA uzaklığında olan
çember çizilmiş olsun,
ΑΓΕ,
çemberlerin kesiştiği
Γ noktasından
A, B noktalarına
ΓΑ, ΓΒ doğruları birleştirilmiş olsun.

A noktası ΓΔΒ çemberinin merkezi
olduğu için,
ΑΓ, ΑΒ doğrusuna eşittir.
Yine
B noktası ΓΑΕ çemberinin merkezi
olduğu için,
ΒΓ, ΒΑ doğrusuna eşittir.

¹Heath's translation has the indefinite article 'a' here, in accordance with modern mathematical practice. However, Euclid does use the Greek *definite* article here, just as in the *exposition* (see §1.3). In particular, he uses the definite article as a *generic* article, which 'makes a single object the representative of the entire class' [16, ¶1123, p. 288]. English too has a generic use of the definite article, 'to indicate the class or kind of objects, as in the well-known aphorism: *The child is the father of the man*' [6, p. 76]. (However, the enormous *Cambridge Grammar* does not discuss the generic article in the obvious place [7, 5.6.1, pp. 568–71]. By the way, the 'well-known aphorism' is by Wordsworth; see http://en.wikisource.org/wiki/Ode:_Intimations_of_Immortality_from_Recollections_of_Early_Childhood [accessed July 27, 2011].) See note 1 to Proposition 9 below.

²The Greek form of the enunciation here is an infinitive clause, and the subject of such a clause is generally in the accusative case [16, ¶1972, p. 438]. In English, an infinitive clause with expressed subject (as here) is always preceded by 'for' [7, 14.1.3, p. 1178]. Normally such a clause, in Greek or English, does not stand by itself as a complete sentence; here evidently it is expected to. Note that the Greek infinitive is thought to be originally a noun in the dative case [16, ¶1969, p. 438]; the English infinitive with 'to' would seem to be formed similarly.

³We follow Euclid in putting the verb (a third-person imperative) first; but a smoother translation of the exposition here would be, 'Let the given finite straight line be AB.' Heath's version is, 'Let AB be the given finite straight line.' By the argument of Netz [12, pp. 43–4], this would appear to be a misleading translation, if not a mistranslation. Euclid's expression ἡ ΑΒ, 'the AB', must be understood as an abbreviation of ἡ εὐθεῖα γραμμὴ ἡ ΑΒ or ἡ ΑΒ εὐθεῖα γραμμή, 'the

straight line AB'. In Proposition XIII.4, Euclid says, Ἔστω εὐθεῖα ἡ ΑΒ, which Heath translates as 'Let AB be a straight line'; but then this suggests the expansion 'Let the straight line AB be a straight line', which does not make much sense. Netz's translation is, 'Let there be a straight line, [namely] AB.' The argument is that Euclid does *not* use words to establish a correlation between letters like A and B and points. The correlation has already been established in the diagram that is before us. By saying, Ἔστω εὐθεῖα ἡ ΑΒ, Euclid is simply calling our attention to a part of the diagram. Now, in the present proposition, Heath's translation of the exposition is expanded to, 'Let the straight line AB be the given finite straight line', which does seem to make sense, at least if it can be expanded further to 'Let the finite straight line AB be the given finite straight line.' But, unlike AB, the given finite straight line was already mentioned in the enunciation, so it is less misleading to name this first in the exposition.

⁴Slightly less literally, 'It is necessary that on the STRAIGHT AB, an equilateral triangle be constructed.'

⁵Instead of 'suppose there have been joined', we could write 'let there have been joined'. However, each of these translations of a Greek *third-person* imperative begins with a *second-person* imperative (because there is no *third-person* imperative form in English, except in some fixed forms like 'God bless you'). The logical subject of the verb 'have been joined' is 'the STRAIGHT AB'; since this comes after the verb, it would appear to be an *extraposed subject* in the sense of the *Cambridge Grammar of the English Language* [7, 2.16, p. 67]. Then the grammatical subject of 'have been joined' is 'there', used as a *dummy*; but it will not always be appropriate to use a dummy in such situations [7, 16.63, p. 1402–3].

And ΓA was shown equal to AB ;
therefore either of ΓA and ΓB to AB
is equal.

But equals to the same
are also equal to one another;
therefore also ΓA is equal to ΓB .
Therefore the three ΓA , AB , and ΓB
are equal to one another.

Equilateral therefore
is triangle $AB\Gamma$.

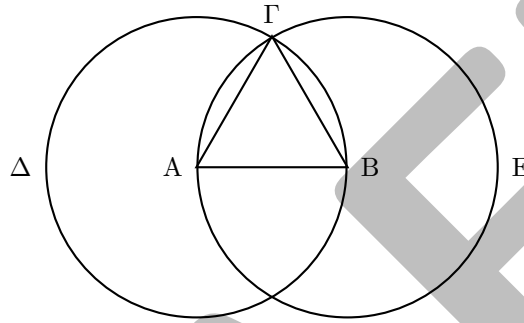
Also, it has been constructed
on the given bounded STRAIGHT
 AB ;
—just what it was necessary to do.

ἐδείχθη δὲ καὶ ἡ ΓA τῆς AB ἴση·
ἐκατέρα ἄρα τῶν ΓA , ΓB τῆς AB
ἐστὶν ἴση.
τὰ δὲ τῶ αὐτῶ ἴσα
καὶ ἀλλήλοις ἐστὶν ἴσα·
καὶ ἡ ΓA ἄρα τῆς ΓB ἐστὶν ἴση·
αἱ τρεῖς ἄρα αἱ ΓA , AB , ΓB
ἴσαι ἀλλήλαις εἰσίν.

Ἰσόπλευρον ἄρα
ἐστὶ τὸ $AB\Gamma$ τρίγωνον.
καὶ συνέσταται
ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης
τῆς AB .⁶
ὅπερ ἔδει ποιῆσαι.

Ve ΓA doğrusunun, AB doğrusuna eşit
olduğu gösterilmişti.
O zaman ΓA , ΓB doğrularının her biri
 AB doğrusuna eşittir.
Ama aynı şeye eşit olanlar
birbirine eşittir.
O zaman ΓA , ΓB doğrusuna eşittir.
O zaman o üç doğru, ΓA , AB , ΓB ,
birbirine eşittir.

Eşkenardır dolayısıyla,
 $AB\Gamma$ üçgeni
ve inşa edilmiştir
verilmiş sınırlanmış,
 AB doğrusuna;
— yapılması gereken tam buydu.



3.2

At the given point,
equal to the given STRAIGHT,
for a STRAIGHT to be placed.

Let be
the given point A ,
and the given STRAIGHT, $B\Gamma$.

It is necessary then
at the point A
equal to the given STRAIGHT $B\Gamma$
for a STRAIGHT to be placed.

For, suppose there has been joined
from the point A to the point B
a STRAIGHT, AB ,
and there has been constructed on it
an equilateral triangle, ΔAB ,
and suppose there have been extended
on a STRAIGHT¹ with ΔA and ΔB
the STRAIGHTS AE and BZ ,
and to the center B
at distance $B\Gamma$
suppose a circle has been drawn,
 $\Gamma H\Theta$,
and again to the center Δ
at distance ΔH
suppose a circle has been drawn,

Πρὸς τῷ δοθέντι σημείῳ
τῆς δοθείσης εὐθείας ἴσην
εὐθεῖαν θέσθαι.

Ἐστω
τὸ μὲν δοθὲν σημεῖον τὸ A ,
ἡ δὲ δοθείσα εὐθεῖα ἡ $B\Gamma$.

δεῖ δὴ
πρὸς τῷ A σημείῳ
τῆς δοθείσης εὐθείας τῆς $B\Gamma$ ἴσην
εὐθεῖαν θέσθαι.

Ἐπεζεύχθω γὰρ
ἀπὸ τοῦ A σημείου ἐπὶ τὸ B σημεῖον
εὐθεῖα ἡ AB ,
καὶ συνεστάτω ἐπ' αὐτῆς
τρίγωνον ἰσόπλευρον τὸ ΔAB ,
καὶ ἐκβεβλήσθωσαν
ἐπ' εὐθείας ταῖς ΔA , ΔB
εὐθεῖαι αἱ AE , BZ ,
καὶ κέντρῳ μὲν τῷ B
διαστήματι δὲ τῷ $B\Gamma$
κύκλος γεγράφθω
ὁ $\Gamma H\Theta$,
καὶ πάλιν κέντρῳ τῷ Δ
καὶ διαστήματι τῷ ΔH
κύκλος γεγράφθω

Verilmiş noktaya
verilmiş doğruya eşit olan
bir doğrunun konulması.

Verilmiş nokta A olsun,
verilmiş doğru $B\Gamma$.

Gereklidir
 A noktasına,
 $B\Gamma$ doğrusuna eşit olan
bir doğrunun konulması.

Çünkü, birleştirilmiş olsun
 A noktasından B noktasına,
 AB doğrusu,
ve bu doğru üzerine inşa edilmiş olsun
eşkenar üçgen ΔAB ,
ve uzatılmış olsun,
 ΔA , ΔB doğrularından
 AE , BZ doğruları
ve B merkezine,
 $B\Gamma$ uzaklığında,
çizilmiş olsun,
 $\Gamma H\Theta$ çemberi ve yine Δ merkezine,
 ΔH uzaklığında
çizilmiş olsun,
 HKA çemberi .

⁶Normally Heiberg puts a semicolon at this position. Perhaps he has a period here only because he has bracketed the following words (omitted here): 'Therefore, on a given bounded STRAIGHT,

an equilateral triangle has been constructed.' According to Heiberg, these words are found, not in the manuscripts of Euclid, but in Proclus's commentary [14, p. 210] alone.

HKΛ.

Since then the point B is the center of ΓΗΘ,

BΓ is equal to BH.

Moreover,

since the point Δ is the center of the circle KHA,

equal is ΔA to ΔH;

of these, the [part] ΔA to ΔB is equal.

Therefore the remainder AA to the remainder BH is equal.

But BΓ was shown equal to BH.

Therefore either of AA and BΓ to BH is equal.

But equals to the same

also are equal to one another.

And therefore AA is equal to BΓ.

Therefore at the given point A equal to the given STRAIGHT BΓ the STRAIGHT AA is laid down; —just what it was necessary to do.

ὁ HKΛ.

Ἐπεὶ οὖν τὸ B σημεῖον κέντρον ἐστὶ τοῦ ΓΗΘ,

ἴση ἐστὶν ἡ BΓ τῆς BH.

πάλιν,

ἐπεὶ τὸ Δ σημεῖον κέντρον ἐστὶ τοῦ HKΛ κύκλου,

ἴση ἐστὶν ἡ ΔA τῆς ΔH,

ὣν ἡ ΔA τῆς ΔB

ἴση ἐστὶν.

λοιπὴ ἄρα ἡ AA

λοιπῆς τῆς BH

ἐστὶν ἴση.

ἐδείχθη δὲ καὶ ἡ BΓ τῆς BH ἴση·

ἐκατέρω ἄρα τῶν AA, BΓ τῆς BH

ἐστὶν ἴση.

τὰ δὲ τῶ αὐτῶ ἴσα

καὶ ἀλλήλοις ἐστὶν ἴσα·

καὶ ἡ AA ἄρα τῆς BΓ ἐστὶν ἴση.

Πρὸς ἄρα τῶ δοθέντι σημείῳ τῶ A τῆς δοθείσης εὐθείας τῆς BΓ ἴση εὐθεῖα κείται ἡ AA· ὅπερ ἔδει ποιῆσαι.

B noktası ΓΗΘ çemberinin merkezi olduđu için,

BΓ, BH doğrusuna eşittir.

Yine,

Δ noktası HKΛ çemberinin merkezi olduđu için,

ΔA, ΔH doğrusuna eşittir,

ve (birincinin) ΔA parçası,

(ikincinin) ΔB parçasına eşittir.

Dolayısıyla AA kalanı,

BH kalanına

eşittir.

Ve BΓ doğrusunun, BH doğrusuna eşit olduđu gösterilmişti.

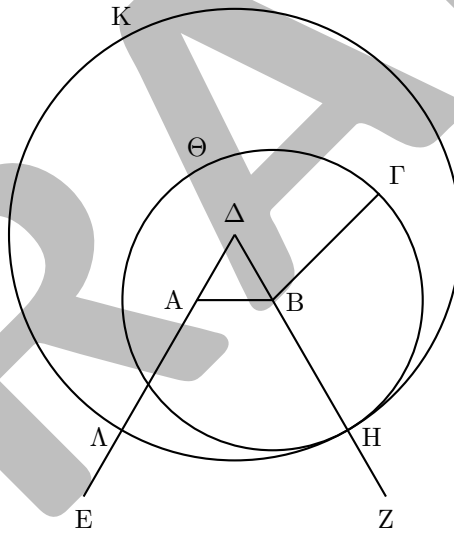
Dolayısıyla AA, BΓ doğrularının her biri BH doğrusuna eşittir.

Ama aynı şeye eşit olanlar birbirine eşittir.

Ve dolayısıyla AA da, BΓ doğrusuna eşittir.

Dolayısıyla verilmiş A noktasına verilmiş BΓ doğrusuna eşit olan AA doğrusu konulmuştur;

— yapılması gereken tam buydu.



3·3

Two unequal STRAIGHTS being given, from the greater, equal to the less, a STRAIGHT to take away.

Let be

the two given unequal STRAIGHTS AB and Γ,¹

of which let the greater be AB.

It is necessary then

from the greater, AB,

Δύο δοθεισῶν εὐθειῶν ἀνίσων ἀπὸ τῆς μείζονος τῆς ἐλάσσονος ἴσην εὐθεῖαν ἀφελεῖν.

Ἐστωσαν

αἱ δοθεῖσαι δύο εὐθεῖαι ἀνισοί

αἱ AB, Γ,

ὣν μείζων ἔστω ἡ AB·

δεῖ δὴ

ἀπὸ τῆς μείζονος τῆς AB

İki eşit olmayan doğru verilmiş ise, daha büyükten daha küçüğe eşit olan bir doğru kesmek.

İki verilmiş doğru

AB, Γ

olsunlar;

daha büyüğü AB olsun.

Gereklidir

daha büyük olan AB doğrusundan

¹The phrase ἐπ' εὐθείας will recur a number of times. The adjective, which is feminine here, appears to be a genitive singular, though it could be accusative plural.

¹Since Γ is given the feminine gender in the Greek, this is a sign that Γ is indeed a line and not a point. See the Introduction.

equal to the less, Γ ,
to take away a STRAIGHT.

Let there be laid down
at the point A,
equal to the line Γ ,
 $A\Delta$;
and to center A
at distance $A\Delta$
suppose circle ΔEZ has been drawn.

And since the point A
is the center of the circle ΔEZ ,
equal is AE to $A\Delta$.
But Γ to $A\Delta$ is equal.
Therefore either of AE and Γ
is equal to $A\Delta$;
and so AE is equal to Γ .

Therefore, two unequal STRAIGHTS
being given, AB and Γ ,
from the greater, AB,
an equal to the less, Γ ,
has been taken away, [namely] AE;
—just what it was necessary to do.

τῆ ἐλάσσονι τῆ Γ ἴσην
εὐθειᾶν ἀφελεῖν.

Κεῖσθω
πρὸς τῷ A σημείῳ
τῆ Γ εὐθείᾳ ἴση
ἡ $A\Delta$.
καὶ κέντρῳ μὲν τῷ A
διαστήματι δὲ τῷ $A\Delta$
κύκλος γεγράφθω ὁ ΔEZ .

Καὶ ἐπεὶ τὸ A σημεῖον
κέντρον ἐστὶ τοῦ ΔEZ κύκλου,
ἴση ἐστὶν ἡ AE τῆ $A\Delta$.
ἀλλὰ καὶ ἡ Γ τῆ $A\Delta$ ἐστὶν ἴση.
ἐκατέρα ἄρα τῶν AE, Γ
τῆ $A\Delta$ ἐστὶν ἴση.
ὥστε καὶ ἡ AE τῆ Γ ἐστὶν ἴση.

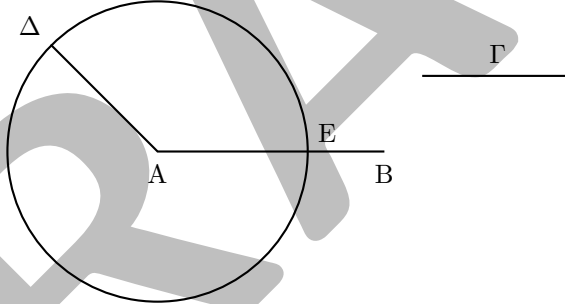
Δύο ἄρα δοθεισῶν εὐθειῶν ἀνίσων τῶν
AB, Γ
ἀπὸ τῆς μείζονος τῆς AB
τῆ ἐλάσσονι τῆ Γ ἴση
ἀφῆρηται ἡ AE.
ὅπερ ἔδει ποιῆσαι.

daha küçük olan Γ doğrusuna eşit olan
bir doğru kesmek.

Konulsun
A noktasına
 Γ doğrusuna eşit olan
 $A\Delta$ doğrusu.
Ve A merkezine
 $A\Delta$ uzaklığında olan
 ΔEZ çemberi çizilmiş olsun.

Ve A noktası
 ΔEZ çemberinin merkezi olduğu için,
AE, $A\Delta$ doğrusuna eşittir.
Ama Γ , $A\Delta$ doğrusuna eşittir.
Dolayısıyla AE, Γ doğrularının her
biri
 $A\Delta$ doğrusuna eşittir.
Sonuç olarak,
AE, Γ doğrusuna eşittir.

Dolayısıyla iki eşit olmayan AB, Γ
doğrusu verilmiş ise,
daha büyük olan AB doğrusundan
daha küçük olan Γ doğrusuna eşit olan
AE doğrusu kesilmişti;
—just what it was necessary to do.



3.4

If two triangles
two sides
to two sides
have equal,¹
either [side] to either,²
and angle to angle have equal,
—that which is by the equal
STRAIGHTS³
contained,
also⁴ base to base
they will have equal,
and the triangle to the triangle
will be equal,
and the remaining angles
to the remaining angles
will be equal,
either to either,

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δυοῖ πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκατέρα
καὶ τὴν γωνίαν τῆ γωνία ἴσην ἔχη
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην,
καὶ τὴν βάσιν τῆ βάσει
ἴσην ἔξει,
καὶ τὸ τρίγωνον τῷ τριγώνῳ
ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἐκατέρα ἐκατέρα,
ὅψ' ἂς αἱ ἴσαι πλευραὶ ὑποτείνουσιν.

Eğer iki üçgende
iki kenar
iki kenara
eşit olursa
(her biri birine)
ve açı açıya eşit olursa
(yani, eşit doğrular tarafından
içerilen),
hem taban tabana
eşit olacak,
hem üçgen üçgene
eşit olacak,
hem de geriye kalan açılar
geriye kalan açılara
eşit olacak,
her biri birine,
(yani) eşit kenarları görenler.

—those that the equal sides subtend.

Let be
two triangles $AB\Gamma$ and ΔEZ ,
the two sides AB and AF
to the two sides ΔE and ΔZ
having equal,
either to either,
 AB to ΔE and AF to ΔZ ,
and angle BAG
to $E\Delta Z$
equal.

I say that
the base $B\Gamma$ is equal to the base EZ ,
and triangle $AB\Gamma$
will be equal to triangle ΔEZ ,
and the remaining angles
to the remaining angles
will be equal,
either to either,
those that equal sides subtend,
[namely] $AB\Gamma$ to ΔEZ ,
and AGB to ΔZE .

For, there being applied
triangle $AB\Gamma$
to triangle ΔEZ ,
and there being placed
the point A on the point Δ ,
and the STRAIGHT AB on ΔE ,
also the point B will apply⁵ to E ,
by the equality of AB to ΔE .
Then, AB applying to ΔE ,
also STRAIGHT AG will apply to ΔZ ,
by the equality
of angle BAG to $E\Delta Z$.
Hence the point Γ to the point Z
will apply,
by the equality, again, of AG to ΔZ .
But B had applied to E ;
Hence the base $B\Gamma$ to the base EZ
will apply.
For if,
 B applying to E ,
and Γ to Z ,
the base $B\Gamma$ will not apply to EZ ,
two STRAIGHTS will enclose a space,
which is impossible.
Therefore will apply
base $B\Gamma$ to EZ
and will be equal to it.
Hence triangle $AB\Gamma$ as a whole

Ἐστω
δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ
τὰς δύο πλευρὰς τὰς AB , AG
ταῖς δυοὶ πλευραῖς ταῖς ΔE , ΔZ
ἴσας ἔχοντα
ἑκατέραν ἑκατέρᾳ
τὴν μὲν AB τῇ ΔE τὴν δὲ AG τῇ ΔZ
καὶ γωνίαν τὴν ὑπὸ BAG
γωνία τῇ ὑπὸ $E\Delta Z$
ἴσην.

λέγω, ὅτι
καὶ βάσις ἡ $B\Gamma$ βάσει τῇ EZ ἴση ἐστίν,
καὶ τὸ $AB\Gamma$ τρίγωνον
τῷ ΔEZ τριγώνῳ ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἑκατέρα ἑκατέρᾳ,
ὅψ' ἂς αἱ ἴσαι πλευραὶ ὑποτείνουσιν,
ἡ μὲν ὑπὸ $AB\Gamma$ τῇ ὑπὸ ΔEZ ,
ἡ δὲ ὑπὸ AGB τῇ ὑπὸ ΔZE .

Ἐφαρμοζομένου γὰρ
τοῦ $AB\Gamma$ τριγώνου
ἐπὶ τὸ ΔEZ τρίγωνον
καὶ τιθεμένου
τοῦ μὲν A σημείου ἐπὶ τὸ Δ σημεῖον
τῆς δὲ AB εὐθείας ἐπὶ τὴν ΔE ,
ἐφαρμόσει καὶ τὸ B σημεῖον ἐπὶ τὸ E
διὰ τὸ ἴσην εἶναι τὴν AB τῇ ΔE .
ἐφαρμοσάσης δὲ τῆς AB ἐπὶ τὴν ΔE
ἐφαρμόσει καὶ ἡ AG εὐθεῖα ἐπὶ τὴν ΔZ
διὰ τὸ ἴσην εἶναι
τὴν ὑπὸ BAG γωνίαν τῇ ὑπὸ $E\Delta Z$.
ὥστε καὶ τὸ Γ σημεῖον ἐπὶ τὸ Z σημεῖον
ἐφαρμόσει
διὰ τὸ ἴσην πάλιν εἶναι τὴν AG τῇ ΔZ .
ἀλλὰ μὴν καὶ τὸ B ἐπὶ τὸ E ἐφηρμόκει·
ὥστε βάσις ἡ $B\Gamma$ ἐπὶ βάσιν τὴν EZ
ἐφαρμόσει.
εἰ γὰρ
τοῦ μὲν B ἐπὶ τὸ E ἐφαρμόσαντος
τοῦ δὲ Γ ἐπὶ τὸ Z
ἡ $B\Gamma$ βάσις ἐπὶ τὴν EZ οὐκ ἐφαρμόσει,
δύο εὐθεῖαι χωρίον περιέξουσιν·
ὅπερ ἐστὶν ἀδύνατον.
ἐφαρμόσει ἄρα
ἡ $B\Gamma$ βάσις ἐπὶ τὴν EZ
καὶ ἴση αὐτῇ ἔσται·
ὥστε καὶ ὅλον τὸ $AB\Gamma$ τρίγωνον

Verilmiş olsun,
 $AB\Gamma$ ve ΔEZ (adlarında) iki üçgen,
iki kenarı AB , AG
 ΔE , ΔZ iki kenarıma
eşit olan
her biri birine,
(şöyle ki) AB , ΔE kenarıma ve AG , ΔZ
kenarıma,
ve BAG (tarafından içerilen) açısı
 $E\Delta Z$ açısına
eşit olan.

İddia ediyorum ki,
 $B\Gamma$ tabanı eşittir EZ tabanıma,
ve $AB\Gamma$ üçgeni
eşit olacak ΔEZ üçgenine,
ve geriye kalan açılar eşit olacak geriye
kalan açılara,
her biri birine,
(şöyle ki) eşit kenarları görenler;
 $AB\Gamma$, ΔEZ açısına,
 AGB , ΔZE açısına.

Çünkü, üstüne koyulursa
 $AB\Gamma$ üçgeni
 ΔEZ üçgeninin,
ve yerleştirilirse
 A noktası Δ noktasına,
ve AB doğrusu ΔE doğrusuna,
o zaman B noktası yerleşecek E nok-
tasına,
 AB doğrusunun ΔE doğrusuna eşitliği
sayesinde.
Böylece, AB doğrusunu yerleştirilince
 ΔE doğrusuna,
 AG doğrusu üstüne gelecek ΔZ
doğrusunun,
 BAG açısının eşitliği sayesinde,
 $E\Delta Z$ açısına.
Dolayısıyla, Γ noktası yerleşecek Z
noktasına,
eşitliği sayesinde, yine, AG doğrusu-
nun ΔZ doğrusuna.
Ama B konuldu E noktasına;
Dolayısıyla, $B\Gamma$ tabanı üstüne gelecek
 EZ tabanıma.
Çünkü eğer, konulunca B , E nok-
tasına,
ve Γ , Z noktasına,
 $B\Gamma$ tabanı yerleşmeyecekse EZ ta-
banına,

¹More smoothly, 'If two triangles have two sides equal to two sides'.

²That is, 'respectively'. We could translate the Greek also as 'each to each'; but the Greek *ἑκατέρος* has the dual number, as opposed to *ἕκαστος* 'each'. The English form 'either' is a remnant of the dual number.

³It appears that for Euclid, things are never simply *equal*; they are equal to something. Here the equal STRAIGHTS containing the angle are not equal to one another; they are separately equal to the two STRAIGHTS in the other triangle.

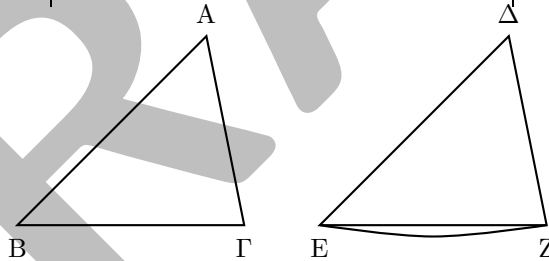
⁴Here Euclid's *καί* has a different meaning from the earlier instance; now it shows the transition to the conclusion of the enunciation. In fact the conclusion has the form *καί...καί...καί...*. This general form might be translated as 'Both...and...and...' The word *both* properly refers to two things, but the Oxford English Dictionary cites an example from Chaucer (1386) where it refers to three things: 'Both heaven and earth and sea'. The word *both* seems to have entered English late, from Old Norse; it supplanted the earlier *wordbo*.

to triangle ΔEZ as a whole
will apply
and will be equal to it,
and the remaining angles
to the remaining angles
will apply,
and be equal to them,
 $AB\Gamma$ to ΔEZ
and AGB to ΔZE .

If, therefore, two triangles
two sides
to two sides
have equal,
either to either,
and angle to angle have equal,
—that which is by the equal
STRAIGHTS
contained,
also base to base
they will have equal,
and the triangle to the triangle
will be equal,
and the remaining angles
to the remaining angles
will be equal,
either to either,
—those that the equal sides subtend;
—just what it was necessary to show.

ἐπὶ ὅλον τὸ ΔEZ τριγώνων
ἐφαρμοῖσει
καὶ ἴσον αὐτῷ ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ἐπὶ τὰς λοιπὰς γωνίας
ἐφαρμοῖσουσι
καὶ ἴσαι αὐταῖς ἔσονται,
ἢ μὲν ὑπὸ $AB\Gamma$ τῇ ὑπὸ ΔEZ
ἢ δὲ ὑπὸ AGB τῇ ὑπὸ ΔZE .

Ἐὰν ἄρα δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δύο πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκατέρα
καὶ τὴν γωνίαν τῇ γωνίᾳ ἴσην ἔχη
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην,
καὶ τὴν βάσιν τῇ βάσει
ἴσην ἔξει,
καὶ τὸ τρίγωνον τῷ τριγώνῳ
ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἐκατέρα ἐκατέρα,
ὅφ' ἂς αἱ ἴσαι πλευραὶ ὑποτείνουσιν·
ὅπερ ἔδει δεῖξαι.



iki doğru çevreleyecek bir alan,
imkansız olan.
Bu yüzden $B\Gamma$ tabanı çakışacak EZ
tabanıyla
ve eşit olacak ona.
Dolayısıyla $AB\Gamma$ üçgeninin tamamı
üstüne gelecek ΔEZ üçgeninin
tamamına,
ve eşit olacak ona,
ve geriye kalan açılar üstüne gelecekler
geriye kalan açılardan,
ve eşit olacaklar onlara;
 $AB\Gamma$, ΔEZ açısına
ve AGB , ΔZE açısına.

Dolayısıyla, eğer,
iki üçgenin, varsa iki kenarı eşit olan
iki kenara,
her bir (kenar) birine,
ve varsa açığına eşit açısı,
(yani) eşit doğrularca içerilen,
hem tabana eşit tabanları olacak,
hem üçgen eşit olacak üçgene,
hem de geriye kalan açılar eşit olacak
geriye kalan açılardan,
her biri birine,
(yani) eşit kenarları görenler;
— gösterilmesi gereken tam buydu.

3.5

In¹ isosceles triangles,
the angles at the base
are equal to one another,
and,
the equal STRAIGHTS being extended,
the angles under the base
will be equal to one another.

Let there be
an isosceles triangle, $AB\Gamma$
having equal
side AB to side AG ,
and suppose have been extended
on a STRAIGHT with AB and AG

Τῶν ἰσοσκελῶν τριγώνων
αἱ πρὸς τῇ βάσει γωνίαι
ἴσαι ἀλλήλαις εἰσίν,
καὶ
προσεκβληθειῶν τῶν ἴσων εὐθειῶν
αἱ ὑπὸ τὴν βάσιν γωνίαι
ἴσαι ἀλλήλαις ἔσονται.

Ἐστω
τρίγωνον ἰσοσκελὲς τὸ $AB\Gamma$
ἴσην ἔχον
τὴν AB πλευρὰν τῇ AG πλευρᾷ,
καὶ προσεκβεβλήσθωσαν
ἐπ' εὐθείας ταῖς AB , AG

İkizkenar üçgenlerde,
tabandaki açılar,
birbirine eşittir,
ve,
eşit doğrular uzatıldığında,
tabanın altında kalan açılar,
birbirine eşit olacaklar.

Verilmiş olsun,
bir $AB\Gamma$ ikizkenar üçgeni;
 AB kenarı eşit olan AG kenarına,
ve varsayalım $B\Delta$ ve ΓE doğrularının
uzatılmış olduğu, AB ve AG
doğrularından.

¹Heath has *coinciding* here, but the verb is just the active form of what, in the passive, is translated as *being applied*.

¹More literally, 'of'.

the STRAIGHTS $B\Delta$ and ΓE .

I say that
angle $AB\Gamma$ to angle $A\Gamma B$
is equal,
and $\Gamma B\Delta$ to $B\Gamma E$.

For, suppose there has been chosen
a random point Z on $B\Delta$,
and there has been taken away
from the greater, AE ,
to the less, AZ ,
an equal, AH ,
and suppose there have been joined
the STRAIGHTS $Z\Gamma$ and HB .

Since then AZ is equal to AH ,
and AB to $A\Gamma$,
so the two AZ and $A\Gamma$
to the two HA , AB ,
will be equal,
either to either;
and they bound a common angle,
[namely] ZAH ;
therefore the base $Z\Gamma$ to the base HB
is equal,
and triangle $AZ\Gamma$ to triangle AHB
will be equal,
and the remaining angles
to the remaining angles
will be equal,
either to either,
those that the equal sides subtend,
 $A\Gamma Z$ to ABH ,
and $AZ\Gamma$ to AHB .
And since AZ as a whole
to AH as a whole
is equal,
of which the [part] AB to $A\Gamma$ is equal,
therefore the remainder BZ
to the remainder ΓH
is equal.
And $Z\Gamma$ was shown equal to HB .
Then the two BZ and $Z\Gamma$
to the two ΓH and HB
are equal,
either to either,
and angle $BZ\Gamma$
to angle ΓHB
[is] equal,
and the common base of them is $B\Gamma$;
and therefore triangle $BZ\Gamma$
to triangle ΓHB
will be equal,
and the remaining angles
to the remaining angles
will be equal,
either to either,
which the equal sides subtend.
Equal therefore is
 $ZB\Gamma$ to $H\Gamma B$,
and $B\Gamma Z$ to $\Gamma B H$.
Since then angle ABH as a whole

$\epsilon\upsilon\theta\epsilon\acute{\iota}\alpha\iota$ αἱ $B\Delta$, ΓE .

λέγω, ὅτι
ἢ μὲν ὑπὸ $AB\Gamma$ γωνία τῆ ὑπὸ $A\Gamma B$
ἴση ἐστίν,
ἢ δὲ ὑπὸ $\Gamma B\Delta$ τῆ ὑπὸ $B\Gamma E$.

Εἰλήφθω γὰρ
ἐπὶ τῆς $B\Delta$ τυχὸν σημεῖον τὸ Z ,
καὶ ἀφῆρήσθω
ἀπὸ τῆς μείζονος τῆς AE
τῆ ἐλάσσονι τῆ AZ
ἴση ἢ AH ,
καὶ ἐπεζεύχθωσαν
αἱ $Z\Gamma$, HB εὐθεῖαι.

Ἐπεὶ οὖν ἴση ἐστίν ἢ μὲν AZ τῆ AH
ἢ δὲ AB τῆ $A\Gamma$,
δύο δὴ αἱ ZA , $A\Gamma$
δυοὶ ταῖς HA , AB
ἴσαι εἰσὶν
ἑκατέρα ἑκατέρᾳ·
καὶ γωνίαν κοινὴν περιέχουσι
τὴν ὑπὸ ZAH ·
βάσεις ἄρα ἢ $Z\Gamma$ βάσει τῆ HB
ἴση ἐστίν,
καὶ τὸ $AZ\Gamma$ τρίγωνον τῷ AHB τριγώνῳ
ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἑκατέρα ἑκατέρᾳ,
ὅφ' ἄς αἱ ἴσαι πλευραὶ ὑποτείνουσιν,
ἢ μὲν ὑπὸ $A\Gamma Z$ τῆ ὑπὸ ABH ,
ἢ δὲ ὑπὸ $AZ\Gamma$ τῆ ὑπὸ AHB .
καὶ ἐπεὶ ὅλη ἢ AZ
ὅλη τῆ AH
ἐστὶν ἴση,
ὣν ἢ AB τῆ $A\Gamma$ ἐστὶν ἴση,
λοιπὴ ἄρα ἢ BZ
λοιπῆ τῆ ΓH
ἐστὶν ἴση.
ἐδείχθη δὲ καὶ ἢ $Z\Gamma$ τῆ HB ἴση·
δύο δὴ αἱ BZ , $Z\Gamma$
δυοὶ ταῖς ΓH , HB
ἴσαι εἰσὶν
ἑκατέρα ἑκατέρᾳ·
καὶ γωνία ἢ ὑπὸ $BZ\Gamma$
γωνία τῆ ὑπὸ ΓHB
ἴση,
καὶ βάσεις αὐτῶν κοινὴ ἢ $B\Gamma$ ·
καὶ τὸ $BZ\Gamma$ ἄρα τρίγωνον
τῷ ΓHB τριγώνῳ
ἴσον ἔσται,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται
ἑκατέρα ἑκατέρᾳ,
ὅφ' ἄς αἱ ἴσαι πλευραὶ ὑποτείνουσιν·
ἴση ἄρα ἐστὶν
ἢ μὲν ὑπὸ $ZB\Gamma$ τῆ ὑπὸ $H\Gamma B$
ἢ δὲ ὑπὸ $B\Gamma Z$ τῆ ὑπὸ $\Gamma B H$.
ἐπεὶ οὖν ὅλη ἢ ὑπὸ ABH γωνία

İddia ediyorum ki
 $AB\Gamma$ açısı, $A\Gamma B$ açısına,
eşittir
ve $\Gamma B\Delta$ açısı eşittir $B\Gamma E$ açısına.

Çünkü, kabul edelim ki, seçilmiş ol-
sun,
rastgele bir Z noktası $B\Delta$ üzerinde,
ve AH , büyük olan AE doğrusundan
küçük olan AZ doğrusunun ke-
silmiş olsun,
ve $Z\Gamma$ ile HB birleştirilmiş olsun.

Çünkü o zaman AZ eşittir AH
doğrusuna,
ve AB doğrusu $A\Gamma$ doğrusuna,
böylece AZ ve $A\Gamma$ ikilisi eşit olacak
 HA ve AB ikilisinin,
her biri birine;
ve sınırlandırılar ortak bir açıyı,
[yani] ZAH açısını;
dolayısıyla $Z\Gamma$ tabanı eşittir HB ta-
banına,
ve $AZ\Gamma$ üçgeni eşit olacak AHB üçge-
nine,
ve geriye kalan açılar eşit olacaklar
geriye kalan açılarının,
her biri birine,
(yani) eşit kenarları görenler,
 $A\Gamma Z$ açısı ABH açısına,
ve $AZ\Gamma$ açısı AHB açısına.
Böylece AZ bütünüünün eşitliği AH
bütünüüne,
ve bunların AB parçasının eşitliği $A\Gamma$
parçasına,
gerekirir BZ kalanının eşit olmasını
 ΓH kalanına.
Ve $Z\Gamma$ doğrusunun gösterilmişti eşit
olduğu HB doğrusuna.
O zaman BZ ve $Z\Gamma$ ikilisi eşittir ΓH ve
 HB ikilisinin,
her biri birine,
ve $BZ\Gamma$ açısı ΓHB açısına,
ve onların ortak tabanı $B\Gamma$
doğrusudur;
ve bu yüzden $BZ\Gamma$ üçgeni eşit olacak
 ΓHB üçgenine,
ve geriye kalan açılar da eşit olacaklar
geriye kalan açılarının,
her biri birine,
aynı kenarları görenler.
Dolayısıyla $ZB\Gamma$ eşittir $H\Gamma B$ açısına,
ve $B\Gamma Z$ açısı $\Gamma B H$ açısına.
Çünkü gösterilmiş oldu ABH açısının
bütünüünün eşit olduğu $A\Gamma Z$
açısının bütünüüne,
ve bunların $\Gamma B H$ parçasının (eşitliği)
 $B\Gamma Z$ parçasına,
dolayısıyla $AB\Gamma$ kalanı eşittir $A\Gamma B$
kalanına;

to angle AGZ as a whole
was shown equal,
of which the [part] GBH to BGZ
is equal,
therefore the remainder ABF
to the remainder AGB
is equal;
and they are at the base
of the triangle ABG .
And was shown also
 ZBF equal to HGB ;
and they are under the base.

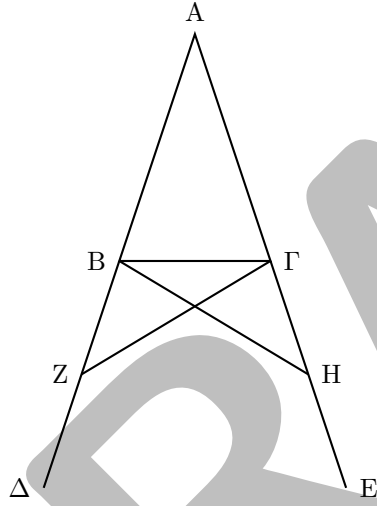
Therefore, in isosceles triangles,
the angles at the base
are equal to one another,
and,
the equal STRAIGHTS being extended,
the angles under the base
will be equal to one another;
—just what it was necessary to show.

ὅλη τῆ ὑπὸ AGZ γωνία
ἐδείχθη ἴση,
ὣν ἡ ὑπὸ GBH τῆ ὑπὸ BGZ
ἴση,
λοιπῆ ἄρα ἡ ὑπὸ ABF
λοιπῆ τῆ ὑπὸ AGB
ἐστὶν ἴση·
καὶ εἰσι πρὸς τῆ βάσει
τοῦ ABG τριγώνου.
ἐδείχθη δὲ καὶ
ἡ ὑπὸ ZBF τῆ ὑπὸ HGB ἴση·
καὶ εἰσιν ὑπὸ τὴν βάσιν.

Τῶν ἰσοσκελῶν τριγώνων
αἱ πρὸς τῆ βάσει γωνίαι
ἴσαι ἀλλήλαις εἰσίν,
καὶ
προσεκβληθειῶν τῶν ἴσων εὐθειῶν
αἱ ὑπὸ τὴν βάσιν γωνίαι
ἴσαι ἀλλήλαις ἔσσονται·
ὅπερ ἔδει δεῖξαι.

ve bunlar ABG üçgeninin tabanıdır.
Ve ZBF açısının eşit olduğu göster-
ilmişti HGB açısına;
ve bunlar tabanın altındadır.

Dolayısıyla bir ikizkenar üçgenin ta-
banındaki açılar birbirine eşit-
tir,
ve, eşit doğrular uzatıldığında,
tabanın altında kalan açılar birbirine
eşit olacaklar.
— gösterilmesi gereken tam buydu.



3.6

If in a triangle
two angles be equal to one another,
also the sides that subtend the equal
angles
will be equal to one another.

Let there be
a triangle, ABG ,
having equal
angle ABG
to angle AGB .

I say that
also side AB to side AG
is equal.

For if unequal is AB to AG ,
one of them is greater.
Suppose AB be greater,
and there has been taken away

Ἐὰν τριγώνου
αἱ δύο γωνίαι ἴσαι ἀλλήλαις ὦσιν,
καὶ αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι
πλευραὶ
ἴσαι ἀλλήλαις ἔσσονται.

Ἐστω
τρίγωνον τὸ ABG
ἴσην ἔχον
τὴν ὑπὸ ABG γωνίαν
τῆ ὑπὸ AGB γωνία·

λέγω, ὅτι
καὶ πλευρὰ ἡ AB πλευρᾶ τῆ AG
ἐστὶν ἴση.

Εἰ γὰρ ἀνίσος ἐστὶν ἡ AB τῆ AG ,
ἡ ἑτέρα αὐτῶν μείζων ἐστίν.
ἔστω μείζων ἡ AB ,
καὶ ἀφηρήσθω

Eğer bir üçgende
birbirine eşit iki açısı varsa,
eşit açılardan gördüğümüz kenarlar da
birbirine eşit olacaklar.

Verilmiş olsun,
bir ABG üçgeni,
 ABG açısı eşit olan
 AGB açısına.

İddia ediyorum ki
 AB kenarı da AG kenarına
eşittir.

Çünkü eğer AB eşit değil ise AG ke-
narına,
biri daha büyüktür.
 AB daha büyük olan olsun,

from the greater, AB,
to the less, AG,
an equal, ΔB,
and there has been joined ΔΓ.

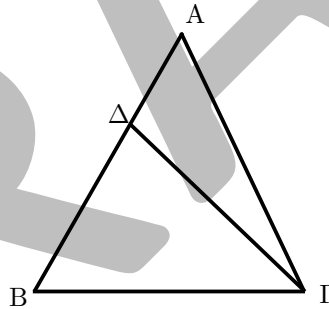
Since then ΔB is equal to AG,
and BΓ is common,
so the two ΔB and BΓ
to the two AG and BΓ
are equal,
either to either,
and angle ΔBΓ
to angle AΓB
is equal;
therefore the base ΔΓ to the base AB
is equal,
and triangle ΔBΓ to triangle AΓB
will be equal,
the less to the greater;
which is absurd.
therefore AB is not unequal to AG;
therefore it is equal.

If therefore in a triangle
two angles be equal to one another,
also the sides that subtend the equal
angles
will be equal to one another;
—just what it was necessary to show.

ἀπὸ τῆς μείζονος τῆς AB
τῆ ἐλάττοني τῆ AG
ἴση ἢ ΔB,
καὶ ἐπεξεύχθω ἢ ΔΓ.

Ἐπεὶ οὖν ἴση ἐστὶν ἢ ΔB τῆ AG
κοινὴ δὲ ἢ BΓ,
δύο δὲ αἱ ΔB, BΓ
δύο ταῖς AG, ΓB
ἴσαι εἰσὶν
ἐκατέρα ἐκατέρα,
καὶ γωνία ἢ ὑπὸ ΔBΓ
γωνία τῆ ὑπὸ AΓB
ἐστὶν ἴση·
βάσις ἄρα ἢ ΔΓ βάσει τῆ AB
ἴση ἐστὶν,
καὶ τὸ ΔBΓ τρίγωνον τῷ AΓB τριγώνῳ
ἴσον ἐστὶν,
τὸ ἐλασσον τῷ μείζονι·
ὅπερ ἄτοπον·
οὐκ ἄρα ἀνίσος ἐστὶν ἢ AB τῆ AG·
ἴση ἄρα.

Ἐὰν τριγώνου
αἱ δύο γωνίαι ἴσαι ἀλλήλαις ᾖσιν,
καὶ αἱ ὑπὸ τὰς ἴσας γωνίας ὑποτείνουσαι
πλευραὶ
ἴσαι ἀλλήλαις ἔσονται·
ὅπερ ἔδει δεῖξαι.



ve diyelim, daha küçük olan AΓ ke-
narına eşit olan, ΔB,
daha büyük olan, AB kenarından ke-
silmiş olsun,
ve ΔΓ birleştirilmiş olsun.

O zaman ΔB eşittir AΓ kenarına,
ve BΓ ortaktır,
böylece ΔB, BΓ ikilisi eşittirler AΓ,
BΓ ikilisinin,
her biri birine,
ve ΔBΓ açısı eşittir AΓB açısına;
dolayısıyla ΔΓ tabanı eşittir AB ta-
banna,
ve ΔBΓ üçgeni eşit olacak AΓB üçge-
nine,
daha küçük daha büyüğe;
ki bu saçmadır.
dolayısıyla AB değildir eşit değil AΓ
kenarına;
dolayısıyla eşittir.

Dolayısıyla eğer bir üçgenin birbirine
eşit iki açısı varsa,
eşit açılardan karşıya gelen kenarlar eşittir;
— gösterilmesi gereken tam buydu.

3·7

On the same STRAIGHT,
to the same two STRAIGHTS,
two other STRAIGHTS,
[which are] equal,
either to either,
will not be constructed
to one and another point,¹
to the same parts,²
having the same extremities
as³ the original lines.

For if possible,
on the same STRAIGHT AB

Ἐπὶ τῆς αὐτῆς εὐθείας
δύο ταῖς αὐταῖς εὐθείαις
ἄλλαι δύο εὐθεῖαι
ἴσαι
ἐκατέρα ἐκατέρα
οὐ συσταθήσονται
πρὸς ἄλλω καὶ ἄλλω σημείῳ
ἐπὶ τὰ αὐτὰ μέρη
τὰ αὐτὰ πέρατα ἔχουσαι
ταῖς ἐξ ἀρχῆς εὐθείαις.

Εἰ γὰρ δυνατόν,
ἐπὶ τῆς αὐτῆς εὐθείας τῆς AB

Aynı doğru üzerinde,
verilmiş iki doğruya,
iki başka doğru,
her biri birine,
inşa edilmeyecek
bir ve başka bir noktaya
aynı tarafta
aynı uçları olan
başlangıçtaki doğrularla.

Çünkü eğer mümkünse,
aynı AB doğrusunda

¹Literally ‘another and another point’; more clearly in English, ‘to different points’.

²In English as apparently in Greek, *parts* can mean ‘region’—in this case, more precisely, ‘side’.

³According to Fowler ([5, as 8, p. 34] and [4, as 9, p. 38]), ‘As

is never to be regarded as a preposition’. This is unfortunate, since it means that the two constructions ‘Equal to X’ and ‘Same as X’ are not grammatically parallel. (We have ‘equal to him’, but ‘same as he’.) The constructions are parallel in Greek: ἴσος + DATIVE and αὐτός + DATIVE.

to two given STRAIGHTS $ΑΓ$, $ΓΒ$,
two other STRAIGHTS $ΑΔ$, $ΔΒ$,
equal
either to either
suppose have been constructed⁴
to one and another point
 $Γ$ and $Δ$,
to the same parts,
having the same extremities,
so that $ΓΑ$ is⁵ equal to $ΔΑ$,
having the same extremity as it, A ,
and $ΓΒ$ to $ΔΒ$,
having the same extremity as it, B ,
and suppose there has been joined
 $ΓΔ$.

Because equal is $ΑΓ$ to $ΑΔ$,
equal is
also angle $ΑΓΔ$ to $ΑΔΓ$;
Greater therefore [is]
 $ΑΔΓ$ than⁶ $ΔΓΒ$;⁷
by much, therefore, [is]
 $ΓΔΒ$ greater than $ΔΓΒ$.
Moreover, since equal is $ΓΒ$ to $ΔΒ$,
equal is also
angle $ΓΔΒ$ to angle $ΔΓΒ$.
But it was also shown than it
much greater;
which is absurd.

Not, therefore,
on the same STRAIGHT,
to the same two STRAIGHTS,
two other STRAIGHTS
[which are] equal,
either to either,
will be constructed
to one and another point
to the same parts
having the same extremities
as the original lines;
—just what it was necessary to show.

δύο ταῖς αὐταῖς εὐθείαις ταῖς $ΑΓ$, $ΓΒ$
ἄλλαι δύο εὐθεῖαι αἱ $ΑΔ$, $ΔΒ$
ἴσαι
ἐκατέρα ἐκατέρα
συνεστάτωσαν
πρὸς ἄλλῃ καὶ ἄλλῃ σημείῳ
τῷ τε $Γ$ καὶ $Δ$
ἐπὶ τὰ αὐτὰ μέρη
τὰ αὐτὰ πέρατα ἔχουσαι,
ὥστε ἴσην εἶναι τὴν μὲν $ΓΑ$ τῇ $ΔΑ$
τὸ αὐτὸ πέρασ ἔχουσαν αὐτῇ τὸ A ,
τὴν δὲ $ΓΒ$ τῇ $ΔΒ$
τὸ αὐτὸ πέρασ ἔχουσαν αὐτῇ τὸ B ,
καὶ ἐπεζεύχθω
ἡ $ΓΔ$.

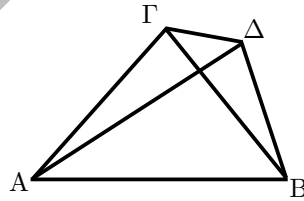
Ἐπεὶ οὖν ἴση ἐστὶν ἡ $ΑΓ$ τῇ $ΑΔ$,
ἴση ἐστὶ
καὶ γωνία ἡ ὑπὸ $ΑΓΔ$ τῇ ὑπὸ $ΑΔΓ$.
μεῖζων ἄρα
ἡ ὑπὸ $ΑΔΓ$ τῆς ὑπὸ $ΔΓΒ$.
πολλῷ ἄρα
ἡ ὑπὸ $ΓΔΒ$ μεῖζων ἐστὶ τῆς ὑπὸ $ΔΓΒ$.
πάλιν ἐπεὶ ἴση ἐστὶν ἡ $ΓΒ$ τῇ $ΔΒ$,
ἴση ἐστὶ καὶ
γωνία ἡ ὑπὸ $ΓΔΒ$ γωνία τῇ ὑπὸ $ΔΓΒ$.
ἐδείχθη δὲ αὐτῆς καὶ
πολλῷ μεῖζων.
ὅπερ ἐστὶν ἀδύνατον.

Οὐκ ἄρα
ἐπὶ τῆς αὐτῆς εὐθείας
δύο ταῖς αὐταῖς εὐθείαις
ἄλλαι δύο εὐθεῖαι
ἴσαι
ἐκατέρα ἐκατέρα
συσταθήσονται
πρὸς ἄλλῃ καὶ ἄλλῃ σημείῳ
ἐπὶ τὰ αὐτὰ μέρη
τὰ αὐτὰ πέρατα ἔχουσαι
ταῖς ἐξ ἀρχῆς εὐθείαις.
ὅπερ ἔδει δεῖξαι.

verilmiş iki $ΑΓ$, $ΓΒ$ doğrusuna
eşit başka iki $ΑΔ$, $ΔΒ$ doğrusu
her biri birine
—diyelim inşa edilmiş olsunlar
bir ve başka bir noktaya
 $Γ$ ve $Δ$,
aynı tarafta,
aynı uçları olan,
şöyle ki $ΓΑ$ eşit olmalı $ΔΑ$ doğrusuna,
aynı A ucuna sahip olan,
ve $ΓΒ$, $ΔΒ$ doğrusuna,
aynı B ucuna sahip olan,
ve $ΓΔ$ birleştirilmiş olsun.

Çünkü $ΑΓ$ eşittir $ΑΔ$ doğrusuna,
böylece $ΑΓΔ$ eşittir $ΑΔΓ$ açısına ;
dolayısıyla $ΑΔΓ$ büyüktür $ΔΓΒ$
açısından;
dolayısıyla $ΓΔΒ$ çok daha büyüktür
 $ΔΓΒ$ açısından.
Üstelik $ΓΒ$ eşit olduğu için $ΔΒ$
doğrusuna,
 $ΓΔΒ$ açısı eşittir $ΔΓΒ$ açısına.
Ama ondan çok daha büyük olduğu
gösterilmiştir;
ki bu saçmadır.

Şöyle olmaz, dolayısıyla; aynı doğru
üzerinde,
verilmiş iki doğruya,
iki başka doğru, eşit,
her biri birine,
inşa edilecek
başka bir noktaya
aynı tarafta
aynı uçları olan
başlangıçtaki doğrularla.
— gösterilmesi gereken tam buydu.



⁴The Perseus Project Word Study Tool does not recognize $\sigma\upsilon\nu\epsilon\sigma\tau\acute{\alpha}\tau\omega\sigma\alpha\upsilon$ here, but it should be just the plural form of $\sigma\upsilon\nu\epsilon\sigma\tau\acute{\alpha}\tau\omega$, which is used for example in Proposition I.2 and which Perseus declares to be a passive perfect imperative. The active third-person imperative ending $-\tau\omega\sigma\alpha\upsilon$ (instead of the older $-\tau\omega\upsilon$) is said by Smyth [16, 466] to appear in prose after Thucydides. This describes Euclid. However, I cannot explain from Smyth the use of an active perfect (as opposed to aorist) form with passive meaning. Presumably the verb is used 'impersonally'. The LJS lexicon [10] cites the

present proposition under $\sigma\upsilon\nu\iota\sigma\tau\eta\mu$. See also the note at I.21.

⁵The Greek verb is an infinitive. An infinitive clause may follow ὥστε [16, ¶2260, p. 507]. Compare the enunciation of Proposition 1.

⁶Fowler ([5, **than** 6, p. 629] and [4, textbfthan 6, p. 619]) does grant the possibility of construing 'than' as a preposition, though he disapproves. Then English cannot exactly mirror the Greek μεῖζων + GENITIVE. Turkish does mirror it with *-den büyük*. See note 3 above.

⁷Here one must refer to the diagram.

3.8

If two triangles
two sides
to two sides
have equal,
either to either,
and have also base equal to base,
also angle to angle
they will have equal,
[namely] that by the equal STRAIGHTS
subtended.

Let there be
two triangles, $AB\Gamma$ and ΔEZ ,
the two sides AB and $A\Gamma$
to the two sides ΔE and ΔZ
having equal,
either to either,
 AB to ΔE ,
and $A\Gamma$ to ΔZ ;
and let them have
base $B\Gamma$ equal to base EZ .

I say that
also angle BAG
to angle $E\Delta Z$
is equal.

For, there being applied
triangle $AB\Gamma$
to triangle ΔEZ ,
and there being placed
the point B on the point E ,
and the STRAIGHT $B\Gamma$ on EZ ,
also the point Γ will apply to Z ,
by the equality of $B\Gamma$ to EZ .
Then, $B\Gamma$ applying to EZ ,
also will apply
 BA and ΓA to $E\Delta$ and ΔZ .
For if base $B\Gamma$ to the base EZ
apply,
and sides BA , $A\Gamma$ to $E\Delta$, ΔZ
do not apply,
but deviate,
as EH , HZ ,
there will be constructed
on the same STRAIGHT,
to two given STRAIGHTS,
two other STRAIGHTS equal,
either to either,
to one and another point
to the same parts
having the same extremities.
But they are not constructed;
therefore it is not [the case] that,
there being applied
the base $B\Gamma$ to the base EZ ,
there do not apply
sides BA , $A\Gamma$ to $E\Delta$, ΔZ .
Therefore they apply.
So angle BAG

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δύο πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκατέρα,
ἔχη δὲ καὶ τὴν βάσιν τῆς βάσει ἴσην,
καὶ τὴν γωνίαν τῆς γωνία
ἴσην ἔξει
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην.

Ἔστω
δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ
τὰς δύο πλευρὰς τὰς AB , $A\Gamma$
ταῖς δύο πλευραῖς ταῖς ΔE , ΔZ
ἴσας ἔχοντα
ἐκατέραν ἐκατέρα,
τὴν μὲν AB τῆς ΔE
τὴν δὲ $A\Gamma$ τῆς ΔZ .
ἔχέτω δὲ
καὶ βάσιν τὴν $B\Gamma$ βάσει τῆς EZ ἴσην.

λέγω, ὅτι
καὶ γωνία ἡ ὑπὸ BAG
γωνία τῆς ὑπὸ $E\Delta Z$
ἴσος ἐστίν.

Ἐφαρμοζομένου γὰρ
τοῦ $AB\Gamma$ τριγώνου
ἐπὶ τὸ ΔEZ τρίγωνον
καὶ τιθεμένου
τοῦ μὲν B σημείου ἐπὶ τὸ E σημεῖον
τῆς δὲ $B\Gamma$ εὐθείας ἐπὶ τὴν EZ
ἐφαρμόσει καὶ τὸ Γ σημεῖον ἐπὶ τὸ Z
διὰ τὸ ἴσην εἶναι τὴν $B\Gamma$ τῆς EZ .
ἐφαρμοσάσης δὲ τῆς $B\Gamma$ ἐπὶ τὴν EZ
ἐφαρμόσουσι καὶ
αἱ BA , ΓA ἐπὶ τὰς $E\Delta$, ΔZ .
εἰ γὰρ βάσις μὲν ἡ $B\Gamma$ ἐπὶ βάσιν τὴν EZ
ἐφαρμόσει,
αἱ δὲ BA , $A\Gamma$ πλευραὶ ἐπὶ τὰς $E\Delta$, ΔZ
οὐκ ἐφαρμόσουσιν
ἀλλὰ παραλλάξουσιν
ὡς αἱ EH , HZ ,
συσταθήσονται
ἐπὶ τῆς αὐτῆς εὐθείας
δύο ταῖς αὐταῖς εὐθείαις
ἄλλαι δύο εὐθεῖαι ἴσαι
ἐκατέρα ἐκατέρα
πρὸς ἄλλω καὶ ἄλλω σημείω
ἐπὶ τὰ αὐτὰ μέρη
τὰ αὐτὰ πέρατα ἔχουσαι.
οὐ συνίστανται δὲ
οὐκ ἄρα
ἐφαρμοζομένης
τῆς $B\Gamma$ βάσεως ἐπὶ τὴν EZ βάσιν
οὐκ ἐφαρμόσουσι
καὶ αἱ BA , $A\Gamma$ πλευραὶ ἐπὶ τὰς $E\Delta$, ΔZ .
ἐφαρμόσουσιν ἄρα
ὥστε καὶ γωνία ἡ ὑπὸ BAG

Eğer iki üçgenin, varsa iki kenarı eşit
olan iki kenara,
her bir (kenar) birine,
ve varsa tabana eşit tabanı,
ayrıca olacak açığa eşit açıları,
(yani) eşit kenarları göröneler.

Verilmiş olsun
iki üçgen, $AB\Gamma$ ve ΔEZ ,
iki kenarı AB , $A\Gamma$ eşit olan ΔE , ΔZ
iki kenarının
her biri birine,
 AB , ΔE kenarına,
ve $A\Gamma$, ΔZ kenarına;
ve onların
 $B\Gamma$ tabanı eşit olsun EZ tabanına.

İddia ediyorum ki
 BAG açısı da
eşittir $E\Delta Z$ açısına.

Çünkü, üstüne koyulursa
 $AB\Gamma$ üçgeni ΔEZ üçgeninin,
ve yerleştirilirse
 B noktası E noktasına,
ve $B\Gamma$ on EZ doğrusuna,
 Γ noktası da yerleşecek Z noktasına,
sayesinde eşitliğinin $B\Gamma$ doğrusunun
 EZ doğrusuna.
O zaman, $B\Gamma$ yerleştirilince EZ
doğrusuna,
 BA ve ΓA doğruları da yerleşecekler
 $E\Delta$ ve ΔZ doğrularına.
Çünkü eğer $B\Gamma$ yerleşirse EZ ta-
banına,
ve BA , $A\Gamma$ kenarları yerleşmezse $E\Delta$,
 ΔZ kenarlarına,
ama kayarsa,
 EH ve HZ olarak
inşa edilmiş olacak
aynı doğru üzerinde,
verilmiş iki doğruya,
iki başka doğru eşit,
her biri birine,
başka bir noktaya
aynı tarafta
aynı uçları olan.
Ama inşa edilmediler;
dolayısıyla (durum) şöyle değil,
 $B\Gamma$ tabanı yerleştirilince EZ tabanına,
 BA , $A\Gamma$ kenarları yerleşmez $E\Delta$, ΔZ
kenarlarına.
Dolayısıyla yerleşirler.
Böylece BAG açısı yerleşecek $E\Delta Z$

to angle $E\Delta Z$
will apply
and will be equal to it.

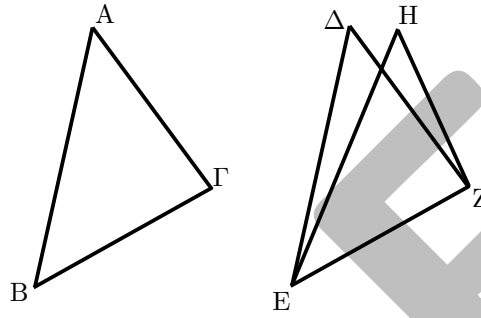
If, therefore, two triangles
two sides
to two sides
have equal,
either to either,
and have also base equal to base,
also angle to angle
they will have equal,
[namely] that by the equal STRAIGHTS
subtended;
—just what it was necessary to show.

ἐπὶ γωνίαν τὴν ὑπὸ $E\Delta Z$
ἐφαρμόσει
καὶ ἴση αὐτῇ ἔσται.

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δύο πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκατέρα,
ἔχη δὲ καὶ τὴν βάσιν τῆς βάσει ἴσην,
καὶ τὴν γωνίαν τῆς γωνία
ἴσην ἔξει
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην·
ὅπερ ἔδει δεῖξαι.

açısına
ve ona eşit olacak.

Eğer, dolayısıyla, iki üçgenin,
varsa iki kenarı
eşit olan
iki kenara,
her bir (kenar) birine,
ve varsa tabana eşit tabanı,
ayrıca olacak açıya eşit açıları,
(yani) eşit kenarları görenler;
— gösterilmesi gereken tam buydu.



3.9

The¹ given rectilinear angle
to cut in two.²

Let be
the given rectilinear angle
 $B\Delta\Gamma$.

Then it is necessary
to cut it in two.

Suppose there has been chosen
on AB at random a point Δ ,
and there has been taken from $A\Gamma$
 AE , equal to $A\Delta$,
and ΔE has been joined,
and there has been constructed on ΔE
an equilateral triangle, ΔEZ ,
and AZ has been joined.

I say that
angle $B\Delta\Gamma$ has been cut in two
by the STRAIGHT AZ .
For, because $A\Delta$ is equal to AE ,
and AZ is common,
then the two, ΔA and AZ
to the two, EA and AZ ,
are equal,

Τὴν δοθεῖσαν γωνίαν εὐθύγραμμον
δίχα τεμεῖν.

Ἐστω
ἡ δοθεῖσα γωνία εὐθύγραμμος
ἡ ὑπὸ $B\Delta\Gamma$.

δεῖ δὲ
αὐτὴν δίχα τεμεῖν.

Εἰλήφθω
ἐπὶ τῆς AB τυχὸν σημείον τὸ Δ ,
καὶ ἀφῆρήσθω ἀπὸ τῆς $A\Gamma$
τῆς $A\Delta$ ἴση ἡ AE ,
καὶ ἐπεζεύχθω ἡ ΔE ,
καὶ συνεστάτω ἐπὶ τῆς ΔE
τρίγωνον ἰσόπλευρον τὸ ΔEZ ,
καὶ ἐπεζεύχθω ἡ AZ .

λέγω, ὅτι
ἡ ὑπὸ $B\Delta\Gamma$ γωνία δίχα τέμνεται
ὑπὸ τῆς AZ εὐθείας.
Ἐπεὶ γὰρ ἴση ἔστιν ἡ $A\Delta$ τῆς AE ,
κοινὴ δὲ ἡ AZ ,
δύο δὲ αἱ ΔA , AZ
δυσὶ ταῖς EA , AZ
ἴσαι εἰσὶν

Verilen düzkenar açıyı
ikiye kesmek.

Verilmiş olsun
düzkenar bir açı, $B\Delta\Gamma$.

Şimdi gereklidir
onun ikiye kesilmesi.

Diyelim seçilmiş olsun
 AB üzerinde rastgele bir nokta, Δ ,
ve kesilmiş olsun $A\Gamma$ doğrusundan
 AE , eşit olan $A\Delta$ doğrusuna,
ve ΔE birleştirilmiş olsun,
ve inşa edilmiş olsun ΔE üzerinde
bir eşkenar üçgen, ΔEZ ,
ve AZ birleştirilmiş olsun.

İddia ediyorum ki
 $B\Delta\Gamma$ açısı ikiye kesilmiş oldu
 AZ doğrusu tarafından.
Çünkü, olduğundan, $A\Delta$ eşit AE ke-
narına,
ve AZ ortak,
 ΔA , AZ ikilisi eşittirler EA , AZ ikil-
isinin

¹Here the generic article (see note 1 to Proposition 1 above) is particularly appropriate. Suppose we take a straight line with a point A on it and draw a circle with center A cutting the line at B and C . Then the straight line BC has been bisected at A . In particular, a line has been bisected. But this does not mean we have solved the problem of the present proposition. In modern mathemat-

ical English, the proposition could indeed be 'To bisect a rectilinear angle'; but then 'a' must be understood as 'an arbitrary' or 'a given'. Of course, Euclid does supply this qualification in any case.

²For 'cut in two' we could say 'bisect'; but in at least one place, in Proposition 12, $\delta\acute{\iota}\chi\alpha$ $\tau\epsilon\mu\epsilon\acute{\iota}\nu$ will be separated.

either to either,
and the base ΔZ to the base EZ
is equal;
therefore angle ΔAZ
to angle EAZ
is equal.

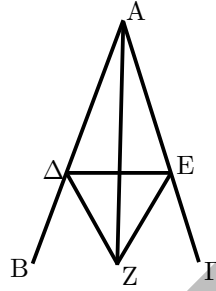
Therefore the given rectilinear angle
 BAG
has been cut in two
by the STRAIGHT AZ ;
—just what it was necessary to do.

ἐκατέρα ἐκατέρα.
καὶ βάσις ἡ ΔZ βάσει τῆ EZ
ἴση ἐστίν.
γωνία ἄρα ἡ ὑπὸ ΔAZ
γωνία τῆ ὑπὸ EAZ
ἴση ἐστίν.

Ἦ ἄρα δοθεῖσα γωνία εὐθύγραμμος
ἡ ὑπὸ BAG
δίχα τέτμηται
ὑπὸ τῆς AZ εὐθείας.
ὅπερ ἔδει ποιῆσαι.

her biri birine ,
ve ΔZ tabanı eşittir EZ tabanına;
dolayısıyla ΔAZ açısı EAZ eşittir.

Dolayısıyla verilen düzkenar açı BAG
kesilmiş oldu ikiye
 AZ doğrusunca;
— yapılması gereken tam buydu.



3.10

The given bounded STRAIGHT
to cut in two.

Let be
the given bounded straight line AB .

It is necessary then
the bounded straight line AB to cut
in two.

Suppose there has been constructed
on it
an equilateral triangle, ABG ,
and suppose has been cut in two
the angle APB by the STRAIGHT $\Gamma\Delta$.

I say that
the STRAIGHT AB has been cut in two
at the point Δ .
For, because AG is equal to AB ,
and $\Gamma\Delta$ is common,
the two, AG and $\Gamma\Delta$,
to the two, BG , $B\Delta$,
are equal,
either to either,
and angle $AG\Delta$
to angle $BG\Delta$
is equal;
therefore the base $A\Delta$ to the base $B\Delta$
is equal.

Therefore the given bounded
STRAIGHT,
 AB ,
has been cut in two at Δ ;

Τὴν δοθεῖσαν εὐθεῖαν πεπερασμένην
δίχα τεμεῖν.

Ἐστω
ἡ δοθεῖσα εὐθεῖα πεπερασμένη ἡ AB .

δεῖ δὲ
τὴν AB εὐθεῖαν πεπερασμένην δίχα
τεμεῖν.

Συνεστάτω ἐπ' αὐτῆς
τρίγωνον ἰσόπλευρον τὸ ABG ,
καὶ τετμήσθω
ἡ ὑπὸ AGB γωνία δίχα τῆ $\Gamma\Delta$ εὐθείας.

λέγω, ὅτι
ἡ AB εὐθεῖα δίχα τέτμηται
κατὰ τὸ Δ σημείον.
Ἐπεὶ γὰρ ἴση ἐστίν ἡ AG τῆ BG ,
κοινὴ δὲ ἡ $\Gamma\Delta$,
δύο δὲ αἱ AG , $\Gamma\Delta$
δύο ταῖς BG , $\Gamma\Delta$
ἴσαι εἰσὶν
ἐκατέρα ἐκατέρα.
καὶ γωνία ἡ ὑπὸ $AG\Delta$
γωνία τῆ ὑπὸ $BG\Delta$
ἴση ἐστίν.
βάσις ἄρα ἡ $A\Delta$ βάσει τῆ $B\Delta$
ἴση ἐστίν.

Ἦ ἄρα δοθεῖσα εὐθεῖα πεπερασμένη
ἡ AB
δίχα τέτμηται κατὰ τὸ Δ .
ὅπερ ἔδει ποιῆσαι.

Verilen sınırlı doğruyu
ikiye kesmek.

Verilmiş olsun
bir sınırlı doğru, AB .

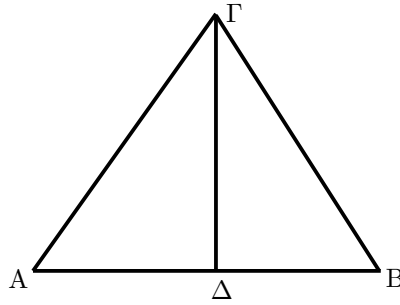
Gereklidir
kesmek,
verilmiş AB sınırlı doğrusunu,
ikiye.

Kabul edelim ki üzerinde inşa edilmiş
olsun
bir eşkenar üçgen, ABG ,
ve AGB açısı kesilmiş olsun ikiye
 $\Gamma\Delta$ doğrusunca.

İddia ediyorum ki
 AB doğrusu ikiye kesilmiş oldu
 Δ noktasında. Çünkü, AG eşit
olduğundan AB kenarına,
ve $\Gamma\Delta$ ortak,
 AG ve $\Gamma\Delta$ ikilisi, eşittirler BG , $B\Delta$ ik-
ilisinin,
her biri birine,
ve $AG\Delta$ açısı eşittir $BG\Delta$ açısına;
dolayısıyla $A\Delta$ tabanı, $B\Delta$ tabanına,
eşittir.

Dolayısıyla
verilmiş sınırlı AB doğrusu Δ
noktasında ikiye kesilmiş oldu;
— yapılması gereken tam buydu.

—just what it was necessary to do.



3.11

To the given STRAIGHT
from the given point on it
at right angles
to draw¹ a straight line.²

Let be
the given STRAIGHT AB,
and the given point on it, Γ.

It is necessary then
from the point Γ
to the STRAIGHT AB
at right angles
to draw a straight line.

Suppose there has been chosen
on AΓ at random a point Δ,
and there has been laid down
an equal to ΓΔ, [namely] ΓΕ,
and there has been constructed
on ΔΕ
an equilateral triangle, ΖΔΕ,
and there has been joined ΖΓ.

I say that
to the given straight line AB
from the given point on it,
Γ,
at right angles
has been drawn a straight line, ΖΓ.
For, since ΔΓ is equal to ΓΕ,
and ΓΖ is common,
the two, ΔΓ and ΓΖ,
to the two, ΕΓ and ΓΖ,
are equal,
either to either;
and the base ΔΖ to the base ΖΕ
is equal;
therefore angle ΔΓΖ
to angle ΕΓΖ
is equal;
and they are adjacent.
Whenever a STRAIGHT,
standing on a STRAIGHT,
the adjacent angles

Τῇ δοθείσῃ εὐθείᾳ
ἀπὸ τοῦ πρὸς αὐτῇ δοθέντος σημείου
πρὸς ὀρθὰς γωνίας
εὐθεῖαν γραμμὴν ἀγαγεῖν.

Ἐστω
ἡ μὲν δοθεῖσα εὐθεῖα ἡ ΑΒ
τὸ δὲ δοθὲν σημεῖον ἐπ' αὐτῆς τὸ Γ.

δεῖ δὴ
ἀπὸ τοῦ Γ σημείου
τῇ ΑΒ εὐθείᾳ
πρὸς ὀρθὰς γωνίας
εὐθεῖαν γραμμὴν ἀγαγεῖν.

Εἰλήφθω
ἐπὶ τῆς ΑΓ τυχὸν σημεῖον τὸ Δ,
καὶ κείσθω
τῇ ΓΔ ἴση ἡ ΓΕ,
καὶ συνεστάτω
ἐπὶ τῆς ΔΕ τρίγωνον ἰσόπλευρον
τὸ ΖΔΕ,
καὶ ἐπεζεύχθω ἡ ΖΓ.

λέγω, ὅτι
τῇ δοθείσῃ εὐθείᾳ τῇ ΑΒ
ἀπὸ τοῦ πρὸς αὐτῇ δοθέντος σημείου
τοῦ Γ
πρὸς ὀρθὰς γωνίας
εὐθεῖα γραμμὴ ἤχται ἡ ΖΓ.
Ἐπεὶ γὰρ ἴση ἐστὶν ἡ ΔΓ τῇ ΓΕ,
κοινὴ δὲ ἡ ΓΖ,
δύο δὴ αἱ ΔΓ, ΓΖ
δυσὶ ταῖς ΕΓ, ΓΖ
ἴσαι εἰσὶν
ἐκατέρα ἐκατέρα·
καὶ βάσις ἡ ΔΖ βάσει τῇ ΖΕ
ἴση ἐστὶν·
γωνία ἄρα ἡ ὑπὸ ΔΓΖ
γωνία τῇ ὑπὸ ΕΓΖ
ἴση ἐστὶν·
καὶ εἰσὶν ἐφεξῆς.
ὅταν δὲ εὐθεῖα
ἐπ' εὐθεῖαν σταθεῖσα
τὰς ἐφεξῆς γωνίας

Verilen bir doğruya
üzerinde verilen bir noktada
dik açılarda
bir doğru çizmek.

Verilmiş olsun
bir doğru, ΑΒ,
ve üzerinde bir nokta, Γ.

Gereklidir
Γ noktasında
ΑΒ doğrusuna
dik açılarda
bir doğru.

Kabul edelim ki seçilmiş olsun
ΑΓ doğrusunda rastgele bir nokta, Δ,
ve yerleştirilmiş olsun
ΓΕ eşit olarak ΓΔ doğrusuna,
ve inşa edilmiş olsun
ΔΕ üzerinde bir eşkenar üçgen, ΖΔΕ,
ve ΖΓ birleştirilmiş olsun.

İddia ediyorum ki
verilen ΑΒ doğrusuna
üzerindeki Γ noktasında
dik açılarda
bir ΖΓ doğrusu çizilmiş oldu.
Çünkü, ΔΓ eşit olduğundan ΓΕ
doğrusuna,
ve ΓΖ ortak olduğundan,
ΔΓ ve ΓΖ ikilisi,
eşittirler ΕΓ ve ΓΖ ikilisininin,
her biri birine;
ve ΔΖ tabanı eşittir ΖΕ tabanına;
dolayısıyla ΔΓΖ açısı eşittir ΕΓΖ
açısına;
ve bitişiktirler.
Ne zaman bir doğru,
bir doğru üzerinde dikilen,
bitişik açıları birbirine eşit yaparsa,
bu açılardan her biri dik olur.
Dolayısıyla ΔΓΖ, ΖΓΕ açılarının her
ikisi de diktir.

¹This is the first time among the propositions that Euclid writes out *straight line* (εὐθεῖα γραμμὴ) and not just *straight* (εὐθεῖα).

²Literally 'lead, conduct'.

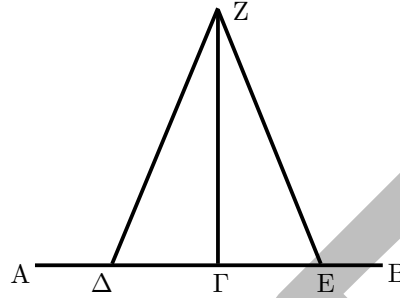
equal to one another
make,
either of the equal angles is right.
Right therefore is either of the angles
 $\Delta\Gamma Z$ and $Z\Gamma E$.

Therefore, to the given STRAIGHT AB,
from the given point on it,
 Γ ,
at right angles,
has been drawn the straight line ΓZ ;
—just what it was necessary to do.

ἴσας ἀλλήλαις
ποιῆ,
ὀρθὴ ἑκατέρα τῶν ἴσων γωνιῶν ἐστίν·
ὀρθὴ ἄρα ἐστίν ἑκατέρα τῶν
ὑπὸ $\Delta\Gamma Z$, $Z\Gamma E$.

Τῆ ἄρα δοθείσῃ εὐθείᾳ τῆ AB
ἀπὸ τοῦ πρὸς αὐτῇ δοθέντος σημείου
τοῦ Γ
πρὸς ὀρθὰς γωνίας
εὐθεῖα γραμμὴ ἤχται ἡ ΓZ ·
ὅπερ ἔδει ποιῆσαι.

Dolayısıyla, verilen AB doğrusuna,
üzerinde verilmiş Γ noktasında,
dik açılarda,
bir ΓZ doğrusu çizilmiş oldu;
— yapılması gereken tam buydu.



3.12

To the given unbounded STRAIGHT,
from the given point,
which is not on it,
to draw a perpendicular straight line.

Let be
the given unbounded STRAIGHT AB,
and the given point,
which is not on it,
 Γ .

It is necessary then
to the given unbounded STRAIGHT,
AB
from the given point Γ ,
which is not on it,
to draw a perpendicular straight line.

For suppose there has been chosen
on the other parts
of the STRAIGHT AB
at random a point Δ ,
and to the center Γ ,
at the distance $\Gamma\Delta$,
a circle has been drawn, EZH,
and has been cut
the STRAIGHT EH
in two at Θ ,
and there have been joined
the STRAIGHTS ΓH , $\Gamma\Theta$, and ΓE .

I say that
to the given unbounded STRAIGHT
AB,
from the given point Γ ,
which is not on it,

Ἐπὶ τὴν δοθείσαν εὐθεῖαν ἄπειρον
ἀπὸ τοῦ δοθέντος σημείου,
ὃ μὴ ἐστὶν ἐπ' αὐτῆς,
κάθετον εὐθεῖαν γραμμὴν ἀγαγεῖν.

Ἐστω
ἡ μὲν δοθείσα εὐθεῖα ἄπειρος ἡ AB
τὸ δὲ δοθὲν σημεῖον,
ὃ μὴ ἐστὶν ἐπ' αὐτῆς,
τὸ Γ .

δεῖ δὴ
ἐπὶ τὴν δοθείσαν εὐθεῖαν ἄπειρον
τὴν AB
ἀπὸ τοῦ δοθέντος σημείου τοῦ Γ ,
ὃ μὴ ἐστὶν ἐπ' αὐτῆς,
κάθετον εὐθεῖαν γραμμὴν ἀγαγεῖν.

Εἰλήφθω γὰρ
ἐπὶ τὰ ἕτερα μέρη
τῆς AB εὐθείας
τυχὸν σημεῖον τὸ Δ ,
καὶ κέντρῳ μὲν τῷ Γ
διαστήματι δὲ τῷ $\Gamma\Delta$
κύκλος γεγράφθω ὁ EZH,
καὶ τετμήσθω
ἡ EH εὐθεῖα
δίχα κατὰ τὸ Θ ,
καὶ ἐπεζεύχθωσαν
αἱ ΓH , $\Gamma\Theta$, ΓE εὐθεῖαι·

λέγω, ὅτι
ἐπὶ τὴν δοθείσαν εὐθεῖαν ἄπειρον
τὴν AB
ἀπὸ τοῦ δοθέντος σημείου τοῦ Γ ,
ὃ μὴ ἐστὶν ἐπ' αὐτῆς,

Verilen sınırlanmamış doğruya,
verilen bir noktadan,
üzerinde olmayan,
bir dik doğru çizmek.

Verilmiş olsun
bir sınırlanmamış doğru, AB,
ve bir nokta,
üzerinde olmayan, Γ .

Gereklidir
verilmiş AB sınırlanmamış doğruya
verilmiş Γ noktasından,
üzerinde olmayan,
bir dik doğru çizmek.

Çünkü kabul edelim ki seçilmiş olsun
AB doğrusunun diğer tarafında
rastgele bir Δ noktası,
ve Γ merkezinde,
 $\Gamma\Delta$ uzaklığında,
bir çember çizilmiş olsun, EZH,
ve EH doğrusu Θ noktasında ikiye ke-
silmiş olsun,
ve birleştirilmiş olsun
 ΓH , $\Gamma\Theta$, ve ΓE doğruları.

İddia ediyorum ki
verilen sınırlanmamış AB doğruya,
verilen Γ noktasından,
üzerinde olmayan,
çizilmiş oldu dik $\Gamma\Theta$ doğrusu.

has been drawn a perpendicular, $\Gamma\Theta$.
 For, because $H\Theta$ is equal to ΘE ,
 and $\Theta\Gamma$ is common,
 the two, $H\Theta$ and $\Theta\Gamma$,
 to the two, $E\Theta$ and $\Theta\Gamma$, are equal,
 either to either;
 and the base ΓH to the base ΓE
 is equal;
 therefore angle $\Gamma\Theta H$
 to angle $E\Theta\Gamma$
 is equal;
 and they are adjacent.
 Whenever a STRAIGHT,
 standing on a STRAIGHT,
 the adjacent angles
 equal to one another make,
 right
 either of the equal angles is,
 and
 the STRAIGHT that has been stood
 is called perpendicular
 to that on which it has been stood.

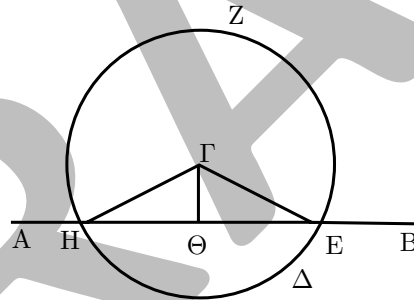
Therefore, to the given unbounded
 STRAIGHT AB ,
 from the given point Γ ,
 which is not on it,
 a perpendicular $\Gamma\Theta$ has been drawn;
 —just what it was necessary to do.

κάθετος ἤχται ἡ $\Gamma\Theta$.
 Ἐπεὶ γὰρ ἴση ἐστὶν ἡ $H\Theta$ τῇ ΘE ,
 κοινὴ δὲ ἡ $\Theta\Gamma$,
 δύο δὲ αἱ $H\Theta$, $\Theta\Gamma$
 δύο ταῖς $E\Theta$, $\Theta\Gamma$ ἴσαι εἰσὶν
 ἑκατέρωθεν ἑκατέρωθεν·
 καὶ βάσις ἡ ΓH βάσει τῇ ΓE
 ἐστὶν ἴση·
 γωνία ἄρα ἡ ὑπὸ $\Gamma\Theta H$
 γωνία τῇ ὑπὸ $E\Theta\Gamma$
 ἐστὶν ἴση.
 καὶ εἰσὶν ἐφ' ἑξῆς.
 ὅταν δὲ εὐθεῖα
 ἐπ' εὐθεῖαν σταθεῖσα
 τὰς ἐφ' ἑξῆς γωνίας
 ἴσας ἀλλήλαις ποιῇ,
 ὀρθὴ
 ἑκατέρωθεν τῶν ἴσων γωνιῶν ἐστὶν,
 καὶ
 ἡ ἐφ' ἑστηκυῖα εὐθεῖα
 κάθετος καλεῖται
 ἐφ' ἣν ἐφέστηκεν.

Ἐπὶ τὴν δοθεῖσαν ἄρα εὐθεῖαν ἄπειρον
 τὴν AB
 ἀπὸ τοῦ δοθέντος σημείου τοῦ Γ ,
 ὃ μὴ ἐστὶν ἐπ' αὐτῆς,
 κάθετος ἤχται ἡ $\Gamma\Theta$.
 ὅπερ ἔδει ποιῆσαι.

Çünkü, $H\Theta$ eşit olduğundan ΘE
 doğrusuna,
 ve $\Theta\Gamma$ ortak,
 $H\Theta$ ve $\Theta\Gamma$ ikilisi,
 eşittirler $E\Theta$ ve $\Theta\Gamma$ ikilisinin,
 her biri birine;
 ve ΓH tabanı eşittir ΓE tabanına;
 dolayısıyla $\Gamma\Theta H$ açısı eşittir $E\Theta\Gamma$
 açısına.
 Ve onlar bitişiktirler.
 Ne zaman bir doğru,
 bir doğru üzerinde dikildiğinde,
 bitişik açılardan birbirine eşit yaparsa,
 açılardan her biri eşittir,
 ve diktilen doğru
 üzerinde
 dikildiği doğruya diktir denir.

Dolayısıyla, verilen AB sınırlandırıl-
 mamış doğruya,
 verilen Γ noktasından,
 üzerinde olmayan,
 bir dik, $\Gamma\Theta$, çizilmiş oldu;
 — yapılması gereken tam buydu.



3.13

If a STRAIGHT,
 stood on a STRAIGHT,
 make angles,
 either two RIGHTS
 or equal to two RIGHTS
 it will make [them].

For, some STRAIGHT, AB ,
 stood on the STRAIGHT $\Gamma\Delta$,
 —suppose it makes¹ angles
 $\Gamma B A$ and $A B \Delta$.

I say that
 the angles $\Gamma B A$ and $A B \Delta$
 either are two RIGHTS
 or [are] equal to two RIGHTS.

If equal is

Ἐὰν εὐθεῖα
 ἐπ' εὐθεῖαν σταθεῖσα
 γωνίας ποιῇ,
 ἤτοι δύο ὀρθὰς
 ἢ δυοῖν ὀρθαῖς ἴσας
 ποιήσει.

Εὐθεῖα γὰρ τις ἡ AB
 ἐπ' εὐθεῖαν τὴν $\Gamma\Delta$ σταθεῖσα
 γωνίας ποιεῖτω
 τὰς ὑπὸ $\Gamma B A$, $A B \Delta$.

λέγω, ὅτι
 αἱ ὑπὸ $\Gamma B A$, $A B \Delta$ γωνίαι
 ἤτοι δύο ὀρθαὶ εἰσὶν
 ἢ δυοῖν ὀρθαῖς ἴσαι.

Εἰ μὲν οὖν ἴση ἐστὶν

Eğer bir doğru,
 dikiltirirse bir doğrunun üzerine,
 yaptığı açılar,
 ya iki dik
 ya da iki dik açuya eşit
 olacak.

Çünkü, bir AB doğrusuda,
 dikiltirilsin $\Gamma\Delta$ doğrusu,
 —kabul edelim ki $\Gamma B A$ ve $A B \Delta$
 açılarını oluştursun.

İddia ediyorum ki
 $\Gamma B A$ ve $A B \Delta$ açılardan
 ya iki dik açıdır
 ya da iki dik açuya eşittir(ler).

Eğer $\Gamma B A$ eşitse $A B \Delta$ açısına,

¹Euclid uses a *present, active* imperative here.

ΓBA to $AB\Delta$,
they are two RIGHTS.

If not,
suppose there has been drawn,
from the point B,
to the [STRAIGHT] $\Gamma\Delta$,
at right angles,
BE.

Therefore ΓBE and $EB\Delta$
are two RIGHTS;
and since ΓBE
to the two, ΓBA and ABE , is equal
let there be added in common $EB\Delta$.
Therefore ΓBE and $EB\Delta$
to the three, ΓBA , ABE , and $EB\Delta$,
are equal.

Moreover,
since ΔBA
to the two, ΔBE and EBA , is equal
let there be added in common $AB\Gamma$;
therefore ΔBA and $AB\Gamma$
to the three, ΔBE , EBA , and $AB\Gamma$,
are equal.

And ΓBE and $EB\Delta$ were shown
equal to the same three.
And equals to the same
are also equal to one another;
also, therefore, ΓBE and $EB\Delta$
to ΔBA and $AB\Gamma$ are equal;
but ΓBE and $EB\Delta$
are two RIGHTS;
and therefore ΔBA and $AB\Gamma$
are equal to two RIGHTS.

If, therefore, a STRAIGHT,
stood on a STRAIGHT,
make angles,
either two RIGHTS
or equal to two RIGHTS
it will make;
—just what it was necessary to show.

ἢ ὑπὸ ΓBA τῆ ὑπὸ $AB\Delta$,
δύο ὀρθαί εἰσιν.

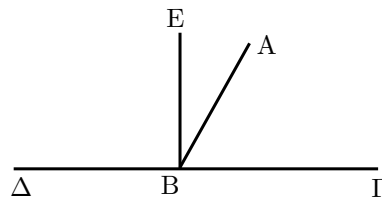
εἰ δὲ οὐ,
ἦχθω
ἀπὸ τοῦ B σημείου
τῆ $\Gamma\Delta$ [εὐθεία]
πρὸς ὀρθὰς
ἢ BE.

αἰ ἄρα ὑπὸ ΓBE , $EB\Delta$
δύο ὀρθαί εἰσιν·
καὶ ἐπεὶ ἢ ὑπὸ ΓBE
δυσὶ ταῖς ὑπὸ ΓBA , ABE ἴση ἐστίν,
κοινῇ προσκεῖσθω ἢ ὑπὸ $EB\Delta$.
αἰ ἄρα ὑπὸ ΓBE , $EB\Delta$
τρισὶ ταῖς ὑπὸ ΓBA , ABE , $EB\Delta$
ἴσαι εἰσίν.

πάλιν,
ἐπεὶ ἢ ὑπὸ ΔBA
δυσὶ ταῖς ὑπὸ ΔBE , EBA ἴση ἐστίν,
κοινῇ προσκεῖσθω ἢ ὑπὸ $AB\Gamma$.
αἰ ἄρα ὑπὸ ΔBA , $AB\Gamma$
τρισὶ ταῖς ὑπὸ ΔBE , EBA , $AB\Gamma$
ἴσαι εἰσίν.

ἐδείχθησαν δὲ καὶ αἰ ὑπὸ ΓBE , $EB\Delta$
τρισὶ ταῖς αὐταῖς ἴσαι·
τὰ δὲ τῶ αὐτῶ ἴσα
καὶ ἀλλήλοις ἐστὶν ἴσα·
καὶ αἰ ὑπὸ ΓBE , $EB\Delta$ ἄρα
ταῖς ὑπὸ ΔBA , $AB\Gamma$ ἴσαι εἰσίν·
ἀλλὰ αἰ ὑπὸ ΓBE , $EB\Delta$
δύο ὀρθαί εἰσιν·
καὶ αἰ ὑπὸ ΔBA , $AB\Gamma$ ἄρα
δυσὶν ὀρθαῖς ἴσαι εἰσίν.

Ἐὰν ἄρα εὐθεῖα
ἐπ' εὐθεῖαν σταθεῖσα
γωνίας ποιῆ,
ἦτοι δύο ὀρθὰς
ἢ δυσὶν ὀρθαῖς ἴσας
ποιήσῃ [τήρημ].
ὅπερ ἔδει δεῖξαι.



iki dik açıdırlar.

Eğer değilse,
kabul edelim ki çizilmiş olsun,
B noktasından,
 $\Gamma\Delta$ doğrusuna,
dik açılarda,
BE.

Dolayısıyla ΓBE ve $EB\Delta$ iki diktir;
ve olduğundan ΓBE
eşit ΓBA ve ABE ikilisine,
 $EB\Delta$ her birine eklenmiş olsun.
Dolayısıyla ΓBE ve $EB\Delta$
eşittirler,
 ΓBA , ABE ve $EB\Delta$ üçlüsüne.

Dahası,
olduğundan ΔBA
eşit, ΔBE ve EBA ikilisine,
 $AB\Gamma$ her birine eklenmiş olsun;
dolayısıyla ΔBA ve $AB\Gamma$
eşittirler,
 ΔBE , EBA ve $AB\Gamma$ üçlüsüne.

Ve ΓBE ve $EB\Delta$ açılarının göster-
ilmişti
eşitliği aynı üçlüye.
Ve aynı şeye eşit olanlar birbirine eşit-
tir;
ve, dolayısıyla, ΓBE ve $EB\Delta$
eşittirle ΔBA ve $AB\Gamma$ açılarna;
ama ΓBE ve $EB\Delta$ iki diktir;
ve dolayısıyla ΔBA ve $AB\Gamma$
iki dike eşittirler.

Eğer, dolayısıyla, bir doğru,
dikiltilirse bir doğrunun üzerine,
yaptığı açılar,
ya iki dik
ya da iki dik açıya eşit
olacak.
— gösterilmesi gereken tam buydu.

3.14

If to some STRAIGHT,
and at the same point,
two STRAIGHTS,
not lying to the same parts,
the adjacent angles
to two RIGHTS
make equal,

Ἐὰν πρὸς τινι εὐθείᾳ
καὶ τῶ πρὸς αὐτῇ σημείῳ
δύο εὐθεῖαι
μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
δυσὶν ὀρθαῖς ἴσας
ποιῶσιν,

Eğer bir doğruya,
ve aynı noktasında,
iki doğru,
aynı tarafında kalmayan,
yaparsa
iki dik açıya eşit
bitişik açılar,

on a STRAIGHT
will be with one another
the STRAIGHTS.

For, to some STRAIGHT, AB,
and at the same point, B,
two STRAIGHTS BG and BD,
not lying to the same parts,
the adjacent angles
ABG and ABD
equal to two RIGHTS
—suppose they make.

I say that
on a STRAIGHT
with GB is BΔ.

For, if it is not
with BG on a STRAIGHT,
[namely] BΔ,
let there be,
with BG in a STRAIGHT,
BE.

For, since the STRAIGHT AB
has stood¹ to the STRAIGHT GBE,
therefore angles ABG and ABE
are equal to two RIGHTS.
Also ABG and ABD
are equal to two RIGHTS.
Therefore GBA and ABE
are equal to GBA and ABD.
In common
suppose there has been taken away
GBA;
therefore the remainder ABE
to the remainder ABD is equal,
the less to the greater;
which is impossible.
Therefore it is not [the case that]
BE is on a STRAIGHT with GB.
Similarly we² shall show that
no other [is so], except BΔ.
Therefore on a STRAIGHT
is GB with BΔ.

If, therefore, to some STRAIGHT,
and at the same point,
two STRAIGHTS,
not lying in the same parts,
adjacent angles
two right angles
make,
on a STRAIGHT
will be with one another
the STRAIGHTS;
—just what it was necessary to show.

ἐπ' εὐθείας
ἔσσονται ἀλλήλαις
αἱ εὐθεῖαι.

Πρὸς γὰρ τινὶ εὐθείᾳ τῇ AB
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ B
δύο εὐθεῖαι αἱ BG, BD
μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
τὰς ὑπὸ ABG, ABD
δύο ὀρθαῖς ἴσας
ποιείωσαν·

λέγω, ὅτι
ἐπ' εὐθείας
ἔστι τῇ GB ἢ BΔ.

Εἰ γὰρ μὴ ἔστι
τῇ BG ἐπ' εὐθείας
ἢ BΔ,
ἔστω
τῇ GB ἐπ' εὐθείας
ἢ BE.

Ἐπεὶ οὖν εὐθεῖα ἢ AB
ἐπ' εὐθεῖαν τὴν GBE ἐφέστηκεν,
αἱ ἄρα ὑπὸ ABG, ABE γωνία
δύο ὀρθαῖς ἴσαι εἰσίν·
εἰσὶ δὲ καὶ αἱ ὑπὸ ABG, ABD
δύο ὀρθαῖς ἴσαι·
αἱ ἄρα ὑπὸ GBA, ABE
ταῖς ὑπὸ GBA, ABD ἴσαι εἰσίν.
κοινὴ
ἀφηρήσθω
ἢ ὑπὸ GBA·
λοιπὴ ἄρα ἢ ὑπὸ ABE
λοιπῇ τῇ ὑπὸ ABD ἔστιν ἴση,
ἢ ἐλάσσων τῇ μείζονι·
ὅπερ ἔστιν ἀδύνατον.
οὐκ ἄρα
ἐπ' εὐθείας ἔστιν ἢ BE τῇ GB.
ὁμοίως δὲ δεῖξομεν, ὅτι
οὐδὲ ἄλλη τις πλὴν τῆς BΔ·
ἐπ' εὐθείας ἄρα
ἔστιν ἢ GB τῇ BΔ.

Ἐὰν ἄρα πρὸς τινὶ εὐθείᾳ
καὶ τῷ πρὸς αὐτῇ σημείῳ
δύο εὐθεῖαι
μὴ ἐπὶ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
δυοῖν ὀρθαῖς ἴσας
ποιῶσιν,
ἐπ' εὐθείας
ἔσσονται ἀλλήλαις
αἱ εὐθεῖαι·
ὅπερ ἔδει δεῖξαι.

bir doğruda
kalacaklar ikisi birlikte,
doğruların.

Bir AB doğrusuna,
ve bir B noktasında,
aynı tarafında kalmayan,
iki BG ve BD doğrularının,
ABG ve ABD
bitişik açılarının iki dik açı
—olduğu kabul edilsin.

İddia ediyorum ki
BΔ ile GB bir doğrudadır.

Çünkü, eğer değilse
bir doğruda BG ile,
BΔ,
olsun,
bir doğruda BG ile,
BE.

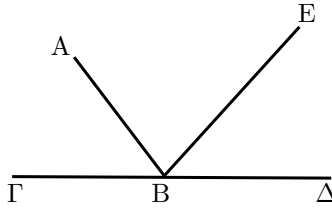
Çünkü, AB doğrusu
dikiltilmiş olur GBE doğrusuna,
dolayısıyla ABG ve ABE açıları
eşittirler iki dik açıya.
Ayrıca ABG ve ABD
eşittirler iki dik açıya.
Dolayısıyla GBA ve ABE
eşittirler GBA ve ABD açılarna.
Ortak GBA açısının çıkartıldığı kabul
edilsin.

Dolayısıyla ABE kalanı
eşittir ABD kalanına,
küçük olan büyüğe;
ki bu imkansızdır.
Dolayısıyla değildir [durum] şöyle;
BE bir doğrudadır GB doğrusuyla.
Benzer şekilde göstereceğiz ki
hiçbiri [öyledir], BΔ dışında.
Dolayısıyla GB bir doğrudadır BΔ ile.

Eğer, dolayısıyla, bir doğruya,
ve aynı noktasında,
iki doğru,
aynı tarafında kalmayan,
yaparsa
iki dik açıya eşit
bitişik açılar,
bir doğruda
kalacaklar ikisi birlikte,
doğruların.
— gösterilmesi gereken tam buydu.

¹The English perfect sounds strange here, but the point may be that the standing has already come to be and will continue.

²This seems to be the first use of the first person *plural*.



3.15

If two STRAIGHTS cut one another, the vertical¹ angles they make equal to one another.

For, let the STRAIGHTS AB and ΓΔ cut one another at the point E.

I say that equal are angle AEG to ΔEB, and ΓEB to AED.

For, since the STRAIGHT AE has stood to the STRAIGHT ΓΔ, making angles ΓEA and AED, therefore angles ΓEA and AED are equal to two RIGHTS.

Moreover, since the STRAIGHT ΔE has stood to the STRAIGHT AB, making angles AED and ΔEB, therefore angles AED and ΔEB are equal to two RIGHTS.

And ΓEA and AED were shown equal to two RIGHTS; therefore ΓEA and AED are equal to AED and ΔEB.

In common suppose there has been taken away AED;

therefore the remainder ΓEA is equal to the remainder ΔEB; similarly it will be shown that also ΓEB and ΔEA are equal.²

If, therefore, two STRAIGHTS cut one another, the vertical angles they make equal to one another; —just what it was necessary to show.

Ἐὰν δύο εὐθεῖαι τέμνωσιν ἀλλήλας, τὰς κατὰ κορυφὴν γωνίας ἴσας ἀλλήλαις ποιούσιν.

Δύο γὰρ εὐθεῖαι αἱ AB, ΓΔ τεμνέτωσαν ἀλλήλας κατὰ τὸ E σημείον·

λέγω, ὅτι ἴση ἐστὶν ἡ μὲν ὑπὸ AEG γωνία τῇ ὑπὸ ΔEB, ἡ δὲ ὑπὸ ΓEB τῇ ὑπὸ AED.

Ἐπεὶ γὰρ εὐθεῖα ἡ AE ἐπ' εὐθεῖαν τὴν ΓΔ ἐφέστηκε γωνίας ποιούσα τὰς ὑπὸ ΓEA, AED, αἱ ἄρα ὑπὸ ΓEA, AED γωνία δυσὶν ὀρθαῖς ἴσαι εἰσὶν. πάλιν, ἐπεὶ εὐθεῖα ἡ ΔE ἐπ' εὐθεῖαν τὴν AB ἐφέστηκε γωνίας ποιούσα τὰς ὑπὸ AED, ΔEB, αἱ ἄρα ὑπὸ AED, ΔEB γωνία δυσὶν ὀρθαῖς ἴσαι εἰσὶν. ἐδείχθησαν δὲ καὶ αἱ ὑπὸ ΓEA, AED δυσὶν ὀρθαῖς ἴσαι· αἱ ἄρα ὑπὸ ΓEA, AED ταῖς ὑπὸ AED, ΔEB ἴσαι εἰσὶν. κοινὴ ἀφηρήσθω ἡ ὑπὸ AED· λοιπὴ ἄρα ἡ ὑπὸ ΓEA λοιπῇ τῇ ὑπὸ ΔEB ἴση ἐστὶν· ὁμοίως δὲ δεῖχθήσεται, ὅτι καὶ αἱ ὑπὸ ΓEB, ΔEA ἴσαι εἰσὶν.

Ἐὰν ἄρα δύο εὐθεῖαι τέμνωσιν ἀλλήλας, τὰς κατὰ κορυφὴν γωνίας ἴσας ἀλλήλαις ποιούσιν· ὅπερ ἔδει δεῖξαι.

Eğer iki doğru keserse birbirini, dikey açılar oluşturlar eşit bir birine.

Çünkü, AB ve ΓΔ doğruları kessinler bir birlerini E noktasında.

İddia ediyorum ki eşittirler AEG açısı ΔEB açısına, ve ΓEB açısı AED açısına.

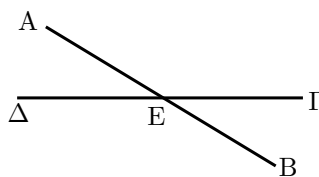
Çünkü, AE doğrusu dikiltildiği ΓΔ doğrusuna, oluşturarak ΓEA ve AED açılarını, dolayısıyla ΓEA ve AED açıları eşittirler iki dik açıya.

Dahası, ΔE doğrusu dikiltildiği AB doğrusuna, oluşturarak AED ve ΔEB açılarını, dolayısıyla AED ve ΔEB açıları eşittirler iki dik açıya.

Ve ΓEA ve AED açılarının gösterilmişti eşitliği iki dik açıya, dolayısıyla ΓEA ve AED eşittirler AED ve ΔEB açılara. Ortak AED açısının çıkartılmış olduğu kabul edilsin;

dolayısıyla ΓEA kalanı eşittir ΔEB kalanına; benzer şekilde gösterilecek ki ΓEB açısı da eşittir ΔEA açısına.

Eğer, dolayısıyla, iki doğru keserse bir birini, dikey açılar oluşturlar eşit birbirine — gösterilmesi gereken tam buydu.



¹The Greek is κατὰ κορυφὴν, which might be translated as 'at a head', just as, in the conclusion of I.10, AB has been cut in two 'at Δ', κατὰ τὸ Δ. But κορυφή and the Latin *vertex* can both mean *crown of the head*, and in anatomical use, the English *vertical* refers

to this crown. Apollonius uses κορυφή for the vertex of a cone [17, pp. 286–7].

²This is a rare moment when two things are said to be equal *simply*, and not equal to one another.

3.16

One of the sides of any triangle being extended, the exterior angle than either of the interior and opposite angles is greater.

Let there be a triangle, $AB\Gamma$, and let there have been extended its side $B\Gamma$, to Δ .

I say that the exterior angle $A\Gamma\Delta$ is greater than either of the two interior and opposite angles, $\Gamma B A$ and $B A \Gamma$.

Suppose $A\Gamma$ has been cut in two at E , and BE , being joined, —suppose it has been extended on a STRAIGHT to Z , and there has been laid down, equal to BE , EZ , and there has been joined $Z\Gamma$, and there has been drawn through $A\Gamma$ to H .

Since equal are AE to $E\Gamma$, and BE to EZ , the two, AE and EB to the two, ΓE and EZ , are equal, either to either; and angle AEB is equal to angle $ZE\Gamma$; for they are vertical; therefore the base AB is equal to the base $Z\Gamma$, and triangle ABE is equal to triangle $ZE\Gamma$, and the remaining angles are equal to the remaining angles, either to either, which the equal sides subtend. Therefore equal are $E\Gamma\Delta$ and $E\Gamma Z$. but greater is $E\Gamma\Delta$ than $E\Gamma Z$; therefore greater [is] $A\Gamma\Delta$ than $B A E$. Similarly $B\Gamma$ having been cut in two, it will be shown that $B\Gamma H$, which is $A\Gamma\Delta$,

Παντὸς τριγώνου μιᾶς τῶν πλευρῶν προσεκβληθείσης ἢ ἐκτὸς γωνία ἐκατέρας τῶν ἐντὸς καὶ ἀπεναντίον γωνιῶν μείζων ἐστίν.

Ἐστω τρίγωνον τὸ $AB\Gamma$, καὶ προσεκβεβλήσθω αὐτοῦ μία πλευρὰ ἢ $B\Gamma$ ἐπὶ τὸ Δ .

λέγω, ὅτι ἢ ἐκτὸς γωνία ἢ ὑπὸ $A\Gamma\Delta$ μείζων ἐστὶν ἐκατέρας τῶν ἐντὸς καὶ ἀπεναντίον τῶν ὑπὸ $\Gamma B A$, $B A \Gamma$ γωνιῶν.

Τετμησθῶ ἢ $A\Gamma$ δίχα κατὰ τὸ E , καὶ ἐπιζευχθεῖσα ἢ BE ἐκβεβλήσθω ἐπ' εὐθείας ἐπὶ τὸ Z , καὶ κείσθω τῇ BE ἴση ἢ EZ , καὶ ἐπεζεύχθω ἢ $Z\Gamma$, καὶ διήχθω ἢ $A\Gamma$ ἐπὶ τὸ H .

Ἐπεὶ οὖν ἴση ἐστὶν ἢ μὲν AE τῇ $E\Gamma$, ἢ δὲ BE τῇ EZ , δύο δὲ αἱ AE , EB δυοὶ ταῖς ΓE , EZ ἴσαι εἰσὶν ἐκατέρα ἐκατέρᾳ· καὶ γωνία ἢ ὑπὸ AEB γωνία τῇ ὑπὸ $ZE\Gamma$ ἴση ἐστίν· κατὰ κορυφὴν γάρ· βάσις ἄρα ἢ AB βάσει τῇ $Z\Gamma$ ἴση ἐστίν, καὶ τὸ ABE τρίγωνον τῷ $ZE\Gamma$ τριγώνῳ ἐστὶν ἴσον, καὶ αἱ λοιπαὶ γωνίαι ταῖς λοιπαῖς γωνίαις ἴσαι εἰσὶν ἐκατέρα ἐκατέρᾳ, ὅφ' ἂς αἱ ἴσαι πλευραὶ ὑποτείνουσιν· ἴση ἄρα ἐστὶν ἢ ὑπὸ BAE τῇ ὑπὸ $E\Gamma Z$. μείζων δὲ ἐστὶν ἢ ὑπὸ $E\Gamma\Delta$ τῆς ὑπὸ $E\Gamma Z$. μείζων ἄρα ἢ ὑπὸ $A\Gamma\Delta$ τῆς ὑπὸ BAE . Ὅμοίως δὲ τῆς $B\Gamma$ τετμημένης δίχα δειχθήσεται καὶ ἢ ὑπὸ $B\Gamma H$, τουτέστιν ἢ ὑπὸ $A\Gamma\Delta$,

Herhangi bir üçgenin kenarlarından biri uzatıldığında, dış açı her bir iç ve karşıt açıdan büyüktür.

Verilmiş olsun, bir $AB\Gamma$ üçgeni ve uzatılmış olsun onun $B\Gamma$ kenarı Δ noktasına.

İddia ediyorum $A\Gamma\Delta$ dış açısı büyüktür her iki $\Gamma B A$ ve $B A \Gamma$ iç ve karşıt açılarından.

$A\Gamma$ kenarı, E noktasından ikiye kesilmiş olsun, ve birleştirilen BE , —uzatılmış olsun Z noktasına bir doğruya ve yerleştirilmiş olsun, BE doğrusuna eşit olan EZ , ve birleştirilmiş olsun $Z\Gamma$, ve çizilmiş olsun $A\Gamma$ doğrusu H noktasına kadar.

Eşit olduğundan AE , $E\Gamma$ doğrusuna, ve BE , EZ doğrusuna, AE ve EB ikilisi, eşittirler ΓE ve EZ ikilisinin, her biri birine; ve AEB açısı eşittir $ZE\Gamma$ açısına; dikey olduklarından; dolayısıyla AB tabanı eşittir $Z\Gamma$ tabanına, ve ABE üçgeni eşittir $ZE\Gamma$ üçgenine, ve kalan açılar eşittirler kalan açılarım, her biri birine, (yani) eşit kenarları görenler. Dolayısıyla eşittirler $E\Gamma\Delta$ ve $E\Gamma Z$. Ama büyüktür $E\Gamma\Delta$, $E\Gamma Z$ açısından; dolayısıyla büyüktür $A\Gamma\Delta$, BAE açısından. Benzer şekilde ikiye kesilmiş olduğundan $B\Gamma$, gösterilecek ki $B\Gamma H$, $A\Gamma\Delta$ açısına eşit olan, büyüktür $AB\Gamma$ açısından.

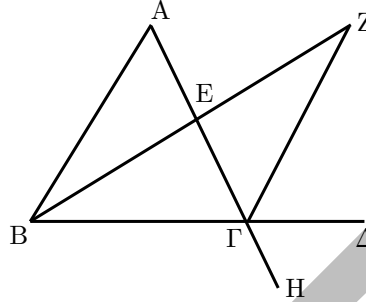
[is] greater than $AB\Gamma$.

Therefore, of any triangle,
one of the sides
being extended,
the exterior angle
than either
of the interior and opposite angles
is greater;
—just what it was necessary to show.

μείζων καὶ τῆς ὑπὸ $AB\Gamma$.

Παντὸς ἄρα τριγώνου
μῆς τῶν πλευρῶν
προσεκβληθείσης
ἡ ἔκτος γωνία
εκατέρας
τῶν ἐντὸς καὶ ἀπεναντίον γωνιῶν
μείζων ἐστίν·
ὅπερ ἔδει δεῖξαι.

Dolayısıyla, herhangi bir üçgenin,
kenarlarından biri
uzatıldığında,
dış açı
her bir
iç ve karşıt açıdan
büyüktür;
— gösterilmesi gereken tam buydu.



3.17

Two angles of any triangle
are greater than two RIGHTS
—taken anyhow.

Let there be
a triangle, $AB\Gamma$.

I say that
two angles of triangle $AB\Gamma$
are greater than two RIGHTS
—taken anyhow.

For, suppose there has been extended
 $B\Gamma$ to Δ .

And since, of triangle $AB\Gamma$,
 $A\Gamma\Delta$ is an exterior angle,
it is greater
than the interior and opposite $AB\Gamma$.
Let $A\Gamma B$ be added in common;
therefore $A\Gamma\Delta$ and $A\Gamma B$
are greater than $AB\Gamma$ and $B\Gamma A$.
But $A\Gamma\Delta$ and $A\Gamma B$
are equal to two RIGHTS;
therefore $AB\Gamma$ and $B\Gamma A$
are less than two RIGHTS.
Similarly we shall show that
also $B\Gamma A$ and $A\Gamma B$
are less than two RIGHTS,
and yet [so are] $\Gamma A B$ and $AB\Gamma$.

Therefore two angles of any triangle
are greater than two RIGHTS
—taken anyhow;
—just what it was necessary to show.

Παντὸς τριγώνου αἱ δύο γωνίαι
δύο ὀρθῶν ἐλάσσονές εἰσι
πάντῃ μεταλαμβάνομεναι.

Ἐστω
τρίγωνον τὸ $AB\Gamma$.

λέγω, ὅτι
τοῦ $AB\Gamma$ τριγώνου αἱ δύο γωνίαι
δύο ὀρθῶν ἐλάττονές εἰσι
πάντῃ μεταλαμβάνομεναι.

Ἐκβεβλήσθω γὰρ
ἡ $B\Gamma$ ἐπὶ τὸ Δ .

Καὶ ἐπεὶ τριγώνου τοῦ $AB\Gamma$
ἔκτος ἐστὶ γωνία ἡ ὑπὸ $A\Gamma\Delta$,
μείζων ἐστὶ
τῆς ἐντὸς καὶ ἀπεναντίον τῆς ὑπὸ $AB\Gamma$.
κοινῇ προσκεῖσθω ἡ ὑπὸ $A\Gamma B$ ·
αἱ ἄρα ὑπὸ $A\Gamma\Delta$, $A\Gamma B$
τῶν ὑπὸ $AB\Gamma$, $B\Gamma A$ μείζονές εἰσιν.
ἀλλ' αἱ ὑπὸ $A\Gamma\Delta$, $A\Gamma B$
δύο ὀρθαῖς ἴσαι εἰσιν·
αἱ ἄρα ὑπὸ $AB\Gamma$, $B\Gamma A$
δύο ὀρθῶν ἐλάσσονές εἰσιν.
ὁμοίως δὲ δεῖξομεν, ὅτι
καὶ αἱ ὑπὸ $B\Gamma A$, $A\Gamma B$
δύο ὀρθῶν ἐλάσσονές εἰσι
καὶ ἔτι αἱ ὑπὸ $\Gamma A B$, $AB\Gamma$.

Παντὸς ἄρα τριγώνου αἱ δύο γωνίαι
δύο ὀρθῶν ἐλάσσονές εἰσι
πάντῃ μεταλαμβάνομεναι·
ὅπερ ἔδει δεῖξαι.

Herhangi bir üçgenin iki açısı
küçüktür iki dik açıdan
—nasıl alırsa alınsın.

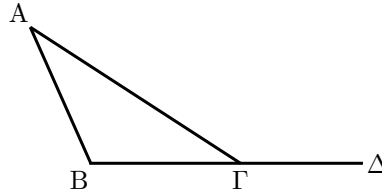
Verilmiş olsun
bir $AB\Gamma$ üçgeni.

İddia ediyorum ki
 $AB\Gamma$ üçgeninin iki açısı
küçüktür iki dik açıdan
—nasıl alırsa alınsın.

Çünkü, uzatılmış olsun,
 $B\Gamma$, Δ noktasına.

Ve $AB\Gamma$ üçgeninin,
bir dış açısı olduğundan $A\Gamma\Delta$,
büyüktür
iç ve karşıt $AB\Gamma$ açısından.
 $A\Gamma B$ ortak açısı eklenmiş olsun;
dolayısıyla $A\Gamma\Delta$ ve $A\Gamma B$
büyüktürler $AB\Gamma$ ve $B\Gamma A$ açılarından.
Ama $A\Gamma\Delta$ ve $A\Gamma B$
eşittirler iki dik açıya;
dolayısıyla $AB\Gamma$ ve $B\Gamma A$
küçüktürler iki dik açıdan.
Benzer şekilde göstereceğiz ki
 $B\Gamma A$ ve $A\Gamma B$ de
küçüktürler iki dik açıdan,
ve sonra [öyledirler] $\Gamma A B$ ve $AB\Gamma$.

Dolayısıyla herhangi bir üçgenin iki
açısı
küçüktür iki dik açıdan
—nasıl alırsa alınsın;
— gösterilmesi gereken tam buydu.



3.18

Of any triangle,
the greater side
subtends the greater angle.¹

For, let there be
a triangle, ABG,
having side AG greater than AB.

I say that
also angle ABG
is greater than BGA.

For, since AG is greater than AB,
suppose there has been laid down,
equal to AB,
AD,
and let BΔ be joined.

Since also, of triangle BGD,
angle ADB is exterior,
it is greater
than the interior and opposite ΔGB;
and ADB is equal to ABΔ,
since side AB is equal to AD;
greater therefore
is ABΔ than AGB;
by much, therefore,
ABG is greater
than AGB.

Therefore, of any triangle,
the greater side
subtends the greater angle;
—just what it was necessary to show.

Παντὸς τριγώνου
ἡ μείζων πλευρὰ
τὴν μείζονα γωνίαν ὑποτείνει.

Ἐστω γὰρ
τρίγωνον τὸ ABΓ
μείζονα ἔχον τὴν ΑΓ πλευρὰν τῆς ΑΒ·

λέγω, ὅτι
καὶ γωνία ἡ ὑπὸ ABΓ
μείζων ἐστὶ τῆς ὑπὸ BΓΑ·

Ἐπεὶ γὰρ μείζων ἐστὶν ἡ ΑΓ τῆς ΑΒ,
κείσθω
τῆ ΑΒ ἴση
ἡ ΑΔ,
καὶ ἐπέξεύχθω ἡ ΒΔ.

Καὶ ἐπεὶ τριγώνου τοῦ BΓΔ
ἔκτος ἐστὶ γωνία ἡ ὑπὸ ΑΔΒ,
μείζων ἐστὶ
τῆς ἐντὸς καὶ ἀπεναντίον τῆς ὑπὸ ΔΓΒ·
ἴση δὲ ἡ ὑπὸ ΑΔΒ τῆ ὑπὸ ΑΒΔ,
ἐπεὶ καὶ πλευρὰ ἡ ΑΒ τῆ ΑΔ ἐστὶν ἴση·
μείζων ἄρα
καὶ ἡ ὑπὸ ΑΒΔ τῆς ὑπὸ ΑΓΒ·
πολλῶ ἄρα
ἡ ὑπὸ ΑΒΓ μείζων ἐστὶ
τῆς ὑπὸ ΑΓΒ.

Παντὸς ἄρα τριγώνου
ἡ μείζων πλευρὰ
τὴν μείζονα γωνίαν ὑποτείνει·
ὅπερ ἔδει δεῖξαι.

Herhangi bir üçgende
daha büyük bir kenar,
daha büyük bir açıyı karşılar.

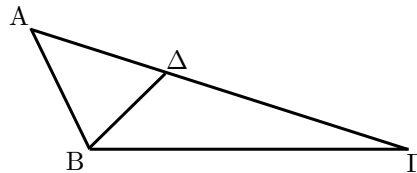
Çünkü, verilmiş olsun
bir ABG üçgeni,
AG kenarı daha büyük olan, AB ke-
narından.

İddia ediyorum ki
ABG açısı da
daha büyüktür, BGA açısından.

Çünkü AG, AB kenarından daha
büyük olduğundan,
yerleştirilmiş olsun,
eşit olan AB kenarına,
AD,
ve BΔ birleştirilmiş olsun.

Ayrıca, BGD üçgeninin,
ADB açısı dış açı olduğundan,
büyüktür
iç ve karşıt ΔGB açısından;
ve ADB eşittir ABΔ açısına,
AB kenarı eşit olduğundan AD ke-
narına;
büyüktür dolayısıyla
ABΔ, AGB açısından;
dolayısıyla, çok daha
büyüktür ABG,
AGB açısından.

Dolayısıyla, herhangi bir üçgende
daha büyük bir kenar,
daha büyük bir açıyı karşılar;
— gösterilmesi gereken tam buydu.



3.19

Of any triangle,

Παντὸς τριγώνου

Herhangi bir üçgende,

¹This enunciation has almost the same words as that of the next proposition. The object of the verb ὑποτείνει is preceded by the preposition ὑπό in the next enunciation, and not here. But the more

important difference would seem to be word order: SUBJECT-OBJECT-VERB here, and OBJECT-SUBJECT-VERB in I.19. This difference in order ensures that I.19 is the converse of I.18.

under the greater angle
the greater side subtends.¹

For, let there be
a triangle, $AB\Gamma$,
having angle $AB\Gamma$ greater
than $B\Gamma A$.

I say that
also side $A\Gamma$
is greater than side AB .

For if not,
either $A\Gamma$ is equal to AB
or less;
[but] $A\Gamma$ is not equal to AB ;
for [if it were],
also $AB\Gamma$ would be² equal to $A\Gamma B$;
but it is not;
therefore $A\Gamma$ is not equal to AB .
Nor is $A\Gamma$ less than AB ;
for [if it were],
also angle $AB\Gamma$ would be [less]
than $A\Gamma B$;
but it is not;
therefore $A\Gamma$ is not less than AB .
And it was shown that
it is not equal.
Therefore $A\Gamma$ is greater than AB .

Therefore, of any triangle,
under the greater angle
the greater side subtends;
—just what it was necessary to show.

ὕπὸ τὴν μείζονα
γωνίαν ἢ μείζων πλευρὰ ὑποτείνει.

Ἐστω
τρίγωνον τὸ $AB\Gamma$
μείζονα ἔχον τὴν ὑπὸ $AB\Gamma$ γωνίαν
τῆς ὑπὸ $B\Gamma A$.

λέγω, ὅτι
καὶ πλευρὰ ἢ $A\Gamma$
πλευρᾶς τῆς AB μείζων ἐστίν.

Εἰ γὰρ μή,
ἢτοι ἴση ἐστὶν ἢ $A\Gamma$ τῇ AB
ἢ ἐλάσσων.
ἴση μὲν οὖν οὐκ ἔστιν ἢ $A\Gamma$ τῇ AB .
ἴση γὰρ ἂν
ἦν καὶ γωνία ἢ ὑπὸ $AB\Gamma$ τῇ ὑπὸ $A\Gamma B$.
οὐκ ἔστι δέ.
οὐκ ἄρα ἴση ἐστὶν ἢ $A\Gamma$ τῇ AB .
οὐδὲ μὴν ἐλάσσων ἐστὶν ἢ $A\Gamma$ τῆς AB .
ἐλάσσων γὰρ
ἂν ἦν καὶ γωνία ἢ ὑπὸ $AB\Gamma$
τῆς ὑπὸ $A\Gamma B$.
οὐκ ἔστι δέ.
οὐκ ἄρα ἐλάσσων ἐστὶν ἢ $A\Gamma$ τῆς AB .
ἐδείχθη δέ, ὅτι
οὐδὲ ἴση ἐστίν.
μείζων ἄρα ἐστὶν ἢ $A\Gamma$ τῆς AB .

Παντὸς ἄρα τριγώνου
ὕπὸ τὴν μείζονα γωνίαν
ἢ μείζων πλευρὰ ὑποτείνει.
ὅπερ ἔδει δεῖξαι.

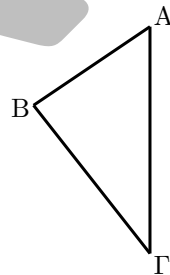
daha büyük bir açı,
daha büyük bir kenarca karşılanır.

Çünkü, verilmiş olsun
bir $AB\Gamma$ üçgeni,
 $AB\Gamma$ açısı daha büyük olan,
 $B\Gamma A$ açısından.

İddia ediyorum ki
 $A\Gamma$ kenarı da
daha büyüktür AB kenarından.

Çünkü değil ise,
ya $A\Gamma$ eşittir AB kenarına
ya da daha küçüktür;
(ama) $A\Gamma$ eşit değildir AB kenarına;
çünkü (eğer olsaydı),
 $AB\Gamma$ da eşit olurdu $A\Gamma B$ açısına;
ama değildir;
dolayısıyla $A\Gamma$ eşit değildir AB ke-
narına.
 $A\Gamma$ küçük de değildir AB kenarından;
çünkü (eğer olsaydı),
 $AB\Gamma$ açısı da olurdu (küçük)
 $A\Gamma B$ açısından;
ama değildir;
dolayısıyla $A\Gamma$ küçük değildir AB ke-
narından.
Ve gösterilmişti ki
eşit değildir.
Dolayısıyla $A\Gamma$ daha büyüktür AB ke-
narından.

Dolayısıyla, herhangi bir üçgende,
daha büyük bir açı,
daha büyük bir kenarca karşılanır;
— gösterilmesi gereken tam buydu.



3.20

Two sides of any triangle
are greater than the remaining one
—taken anyhow.

For, let there be
a triangle, $AB\Gamma$.

I say that
two sides of triangle $AB\Gamma$

Παντὸς τριγώνου αἱ δύο πλευραὶ
τῆς λοιπῆς μείζονός εἰσι
πάντη μεταλαμβάνομεναί.

Ἐστω γὰρ
τρίγωνον τὸ $AB\Gamma$.

λέγω, ὅτι
τοῦ $AB\Gamma$ τριγώνου αἱ δύο πλευραὶ

Herhangi bir üçgenin iki kenarı
daha büyüktür geriye kalandan
—nasıl seçilirse seçilsin.

Çünkü verilmiş olsun
bir $AB\Gamma$ üçgeni.

İddia ediyorum ki
 $AB\Gamma$ üçgeninin iki kenarı

¹Heath here uses the expedient of the passive: 'The greater angle is subtended by the greater side.'

²Literally 'was'; but this conditional use of *was* is archaic in English.

are greater than the remaining one,
—taken anyhow,
BA and AG, than BG,
AB and BG, than AG,
BG and GA, than AB.

For, suppose has been drawn through
BA to a point Δ ,
and there has been laid down
AD equal to GA,
and there has been joined
 $\Delta\Gamma$.

Since ΔA is equal to AG,
equal also is
angle $A\Delta\Gamma$ to $AG\Delta$.
Therefore $B\Gamma\Delta$ is greater than $A\Delta\Gamma$;
also, since there is a triangle, $\Delta\Gamma B$,¹
having angle $\Gamma B\Delta$ greater
than $\Delta B\Gamma$,
and under the greater angle
the greater side subtends,
therefore ΔB is greater than BG.
But ΔA is equal to AG;
therefore BA and AG are greater
than BG;
similarly we shall show that
AB and BG than GA
are greater,
and BG and GA than AB.

Therefore two sides of any triangle
are greater than the remaining one
—taken anyhow;
—just what it was necessary to show.

τῆς λοιπῆς μείζονές εἰσι
πάντη μεταλαμβάνομεναι,
αἱ μὲν BA, AG τῆς BG,
αἱ δὲ AB, BG τῆς AG,
αἱ δὲ BG, GA τῆς AB.

Διήχθω γὰρ
ἢ BA ἐπὶ τὸ Δ σημείον,
καὶ κείσθω
τῆ GA ἴση ἢ AD,
καὶ ἐπεζεύχθω
ἢ $\Delta\Gamma$.

Ἐπεὶ οὖν ἴση ἐστὶν ἢ ΔA τῆ AG,
ἴση ἐστὶ καὶ
γωνία ἢ ὑπὸ $A\Delta\Gamma$ τῆ ὑπὸ $AG\Delta$.
μείζων ἄρα ἢ ὑπὸ $B\Gamma\Delta$ τῆς ὑπὸ $A\Delta\Gamma$.
καὶ ἐπεὶ τρίγωνόν ἐστι τὸ $\Delta\Gamma B$
μείζονα ἔχον τὴν ὑπὸ $B\Gamma\Delta$ γωνίαν
τῆς ὑπὸ $\Delta B\Gamma$,
ὑπὸ δὲ τὴν μείζονα γωνίαν
ἢ μείζων πλευρὰ ὑποτείνει,
ἢ ΔB ἄρα τῆς BG ἐστὶ μείζων.
ἴση δὲ ἢ ΔA τῆ AG.
μείζονες ἄρα αἱ BA, AG
τῆς BG.
ὁμοίως δὲ δεῖξομεν, ὅτι
καὶ αἱ μὲν AB, BG τῆς GA
μείζονές εἰσιν,
αἱ δὲ BG, GA τῆς AB.

Πάντος ἄρα τριγώνου αἱ δύο πλευραὶ
τῆς λοιπῆς μείζονές εἰσι
πάντη μεταλαμβάνομεναι.
ὅπερ ἔδει δεῖξαι.

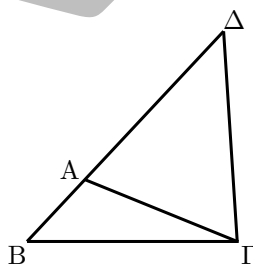
daha büyüktür geriye kalandan
—nasıl seçilirse seçilsin,
BA ve AG, BG kenarından,
AB ve BG, AG kenarından,
BG ve GA, AB kenarından.

Çünkü, çizilmiş olsun
BA kenarı geçerek bir Δ noktasından,
ve yerleştirilmiş olsun
AD, GA kenarına eşit olan,
ve birleştirilmiş olsun
 $\Delta\Gamma$.

ΔA eşit olduğundan AG kenarına,
eşittir ayrıca
 $A\Delta\Gamma$, $AG\Delta$ açısına.
Dolayısıyla $B\Gamma\Delta$ büyüktür, $A\Delta\Gamma$
açısından;
yine, $\Delta\Gamma B$, bir üçgen olduğundan,
 $\Gamma B\Delta$ daha büyük olan
 $\Delta B\Gamma$ açısından,
daha büyük açı
daha büyük kenarca karşılandığından,
dolayısıyla ΔB büyüktür BG kenarın-
dan.

Ama ΔA eşittir AG kenarına;
dolayısıyla BA ve AG büyüktürler
BG kenarından;
benzer şekilde göstereceğiz ki
AB ve BG, GA kenarından
büyüktürler,
ve BG ve GA, AB kenarından.

Dolayısıyla, herhangi bir üçgenin iki
kenarı
daha büyüktür geriye kalandan
—nasıl seçilirse seçilsin;
— gösterilmesi gereken tam buydu.



3.21

If, of a triangle,
on one of the sides,
from its extremities,
two STRAIGHTS
be constructed within,¹
the constructed [STRAIGHTS],
than the remaining two sides of the
triangle
will be less,

Ἐὰν τριγώνου
ἐπὶ μιᾶς τῶν πλευρῶν
ἀπὸ τῶν περάτων
δύο εὐθεῖαι
ἐντὸς συσταθῶσιν,
αἱ συσταθεῖσαι
τῶν λοιπῶν τοῦ τριγώνου δύο πλευρῶν
ἐλάττωες μὲν ἔσονται,
μείζονα δὲ γωνίαν περιέξουσιν.

Eğer bir üçgende,
kenarlardan birinin
uçlarından,
iki doğru
içeride inşa edilirse,
inşa edilen doğrular,
üçgenin geriye kalan iki kenarından
daha küçük olacak,
ama daha büyük bir açıyı içerecekler.

¹Heath's version is, 'Since DCB [$\Delta\Gamma B$] is a triangle. . .'

but will contain the a greater angle.

For, of a triangle, $AB\Gamma$,
on one of the sides, $B\Gamma$,
from its extremities, B and Γ ,
suppose two STRAIGHTS have been
constructed within,
 $B\Delta$ and $\Delta\Gamma$.

I say that
 $B\Delta$ and $\Delta\Gamma$
than the remaining two sides of the
triangle,
 BA and $A\Gamma$,
are less,
but contain a greater angle,
 $B\Delta\Gamma$, than BAG .

For, let $B\Delta$ be drawn through to E .

And since, of any triangle,
two sides than the remaining one
are greater,
of the triangle ABE ,
the two sides AB and AE
are greater than BE ;
suppose has been added in common
 EG ;
therefore BA and $A\Gamma$ than BE and EG
are greater.
Moreover,
since, of the triangle GED ,
the two sides GE and ED
are greater than GD ,
suppose has been added in common
 ΔB ;
therefore GE and EB than GD and ΔB
are greater.
But than BE and EG
 BA and $A\Gamma$ were shown greater;
therefore by much
 BA and $A\Gamma$ than $B\Delta$ and $\Delta\Gamma$
are greater.

Again,
since of any triangle
the external angle
than the interior and opposite angle
is greater,
therefore, of the triangle $G\Delta E$
the exterior angle $B\Delta\Gamma$
is greater than $G\Delta E$.
For the same [reason] again,
of the triangle ABE ,
the exterior angle GEB
is greater than BAG .
But than GEB

Τριγώνου γὰρ τοῦ $AB\Gamma$
ἐπὶ μιᾶς τῶν πλευρῶν τῆς $B\Gamma$
ἀπὸ τῶν περάτων τῶν B , Γ
δύο εὐθεΐαι ἐντὸς συνεστᾶτωςαν
αἱ $B\Delta$, $\Delta\Gamma$.

λέγω, ὅτι
αἱ $B\Delta$, $\Delta\Gamma$
τῶν λοιπῶν τοῦ τριγώνου δύο πλευρῶν
τῶν BA , $A\Gamma$
ἐλάσσονες μὲν εἰσιν,
μεῖζονα δὲ γωνίαν περιέχουσι
τὴν ὑπὸ $B\Delta\Gamma$ τῆς ὑπὸ BAG .

Διήχθω γὰρ ἡ $B\Delta$ ἐπὶ τὸ E .

καὶ ἐπεὶ παντὸς τριγώνου
αἱ δύο πλευραὶ τῆς λοιπῆς
μεῖζονές εἰσιν,
τοῦ ABE ἄρα τριγώνου
αἱ δύο πλευραὶ αἱ AB , AE
τῆς BE μεῖζονές εἰσιν·
κοινὴ προσκεῖσθω
ἡ EG .
αἱ ἄρα BA , $A\Gamma$ τῶν BE , EG
μεῖζονές εἰσιν.
πάλιν,
ἐπεὶ τοῦ GED τριγώνου
αἱ δύο πλευραὶ αἱ GE , ED
τῆς GD μεῖζονές εἰσιν,
κοινὴ προσκεῖσθω
ἡ ΔB .
αἱ GE , EB ἄρα τῶν GD , ΔB
μεῖζονές εἰσιν.
ἀλλὰ τῶν BE , EG
μεῖζονες ἐδείχθησαν αἱ BA , $A\Gamma$.
πολλῶ ἄρα
αἱ BA , $A\Gamma$ τῶν $B\Delta$, $\Delta\Gamma$
μεῖζονές εἰσιν.

Πάλιν,
ἐπεὶ παντὸς τριγώνου
ἡ ἔκτὸς γωνία
τῆς ἐντὸς καὶ ἀπεναντίον
μεῖζων ἔστιν,
τοῦ $G\Delta E$ ἄρα τριγώνου
ἡ ἔκτὸς γωνία ἡ ὑπὸ $B\Delta\Gamma$
μεῖζων ἔστι τῆς ὑπὸ $G\Delta E$.
διὰ ταῦτὰ τοίνυν
καὶ τοῦ ABE τριγώνου
ἡ ἔκτὸς γωνία ἡ ὑπὸ GEB
μεῖζων ἔστι τῆς ὑπὸ BAG .
ἀλλὰ τῆς ὑπὸ GEB

Çünkü, $AB\Gamma$ üçgeninin,
bir $B\Gamma$ kenarının
 B ve Γ uçlarından,
içeride iki doğru inşa edilmiş olsun;
 $B\Delta$ ve $\Delta\Gamma$.

İddia ediyorum ki
 $B\Delta$ ve $\Delta\Gamma$
üçgenin geriye kalan iki
 BA ve $A\Gamma$ kenarından,
daha küçütürler,
ama içerirler,
 BAG açısından daha büyük $B\Delta\Gamma$
açısını.

Çünkü, $B\Delta$ çizilmiş olsun E noktasına
doğru.

Ve herhangi bir üçgenin
iki kenarı, geriye kalandan
büyük olduğundan,
 ABE üçgeninin,
iki kenarı, AB ve AE
büyüktür BE kenarından;
ortak olarak eklenmiş olsun
 EG ;
dolayısıyla BA ve $A\Gamma$, BE ve EG ke-
narlarından
büyüktürler.
Dahası,
 GED üçgeninin,
iki kenarları, GE ve ED
büyüktür GD kenarından,
ortak olarak eklenmiş olsun
 ΔB ;
dolayısıyla GE ve EB , GD ve ΔB ke-
narlarından
büyüktürler.
Ama BE ve EG kenarlarından
 BA ve $A\Gamma$ kenarlarının gösterilmişti
büyüklüğü;
dolayısıyla çok daha büyüktür
 BA ve $A\Gamma$, $B\Delta$ ve $\Delta\Gamma$ kenarlarından.

Tekrar,
herhangi bir üçgenin
dış açısı
iç ve karşıt açısından
daha büyüktür,
dolayısıyla, $G\Delta E$ üçgeninin
dış açısı $B\Delta\Gamma$
büyüktür $G\Delta E$ açısından.
Aynı [sebep] tekrar,
 ABE üçgeninin,
dış açısı GEB
büyüktür BAG açısından.
Ama GEB açısından,

¹Here the Greek verb, συνίστημι, is the same one used in I.1 for the construction of a triangle on a given straight line. Is it supposed to be obvious to the reader, even without a diagram, that now the

two constructed straight lines are supposed to meet at a point? See also I.2 and note.

$B\Delta\Gamma$ was shown greater;
therefore by much
 $B\Delta\Gamma$ is greater than BAG .

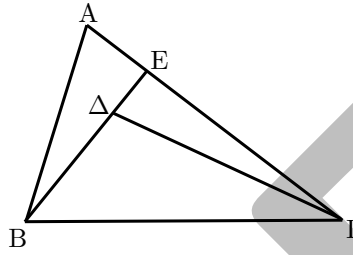
If, therefore, of a triangle,
on one of the sides,
from its extremities,
two STRAIGHTS
be constructed within,
the constructed [STRAIGHTS],
than the remaining two sides of the
triangle
will be less,
but will contain the a greater angle;
—just what it was necessary to show.

μείζων ἐδείχθη ἢ ὑπὸ $B\Delta\Gamma$.
πολλῷ ἄρα
ἢ ὑπὸ $B\Delta\Gamma$ μείζων ἐστὶ τῆς ὑπὸ BAG .

Ἐὰν ἄρα τριγώνου
ἐπὶ μιᾶς τῶν πλευρῶν
ἀπὸ τῶν περάτων
δύο εὐθεῖαι
ἐντὸς συσταθῶσιν,
αἱ συσταθεῖσαι
τῶν λοιπῶν τοῦ τριγώνου δύο πλευρῶν
ἐλάττονες μὲν εἰσιν,
μείζονα δὲ γωνίαν περιέχουσιν.
ὅπερ εἶδει δεῖξαι.

$B\Delta\Gamma$ açısının büyüklüğü gösterilmişti;
dolayısıyla çok daha
büyüktür $B\Delta\Gamma$, BAG açısından.

Eğer, dolayısıyla, bir üçgenin,
kenarlardan birinin
uçlarından,
iki doğru
içeride inşa edilirse,
inşa edilen doğrular,
üçgenin geriye kalan iki kenarından
daha küçük olacak,
ama daha büyük bir açıyı içerecekler;
— gösterilmesi gereken tam buydu.



3.22

From three STRAIGHTS,
which are equal
to three given [STRAIGHTS],
a triangle to be constructed;
and it is necessary
for two than the remaining one
to be greater
[because of any triangle,
two sides
are¹ greater than the remaining one
taken anyhow].

Let be
the given three STRAIGHTS
 A , B , and Γ ,
of which two than the remaining one
are greater,
taken anyhow,
 A and B than Γ ,
 A and Γ than B ,
and B and Γ than A .

Is is necessary
from equals to A , B , and Γ
for a triangle to be constructed.

Suppose there is laid out
some straight line, ΔE ,
bounded at Δ ,
but unbounded at E ,

Ἐκ τριῶν εὐθειῶν,
αἱ εἰσιν ἴσαι
τρισὶ ταῖς δοθείσαις [εὐθείαις],
τρίγωνον συστήσασθαι.
δεῖ δὲ²
τὰς δύο τῆς λοιπῆς
μείζονας εἶναι
πάντη μεταλαμβανομένας
[διὰ τὸ καὶ παντὸς τριγώνου
τὰς δύο πλευρὰς
τῆς λοιπῆς μείζονας εἶναι
πάντη μεταλαμβανομένας].

Ἐστωσαν
αἱ δοθεῖσαι τρεῖς εὐθεῖαι
αἱ A , B , Γ ,
ᾧν αἱ δύο τῆς λοιπῆς
μείζονες ἔστωσαν
πάντη μεταλαμβανόμεναι,
αἱ μὲν A , B τῆς Γ ,
αἱ δὲ A , Γ τῆς B ,
καὶ ἔτι αἱ B , Γ τῆς A .

δεῖ δὲ
ἐκ τῶν ἴσων ταῖς A , B , Γ
τρίγωνον συστήσασθαι.

Ἐκκεῖσθω
τις εὐθεῖα ἡ ΔE
πεπερασμένη μὲν κατὰ τὸ Δ
ἄπειρος δὲ κατὰ τὸ E ,

Üç doğrudan,
eşit olan
verilmiş üç doğruya,
bir üçgen oluşturulması;
ve gereklidir
ikisinin, kalandan
daha büyük olması
(çünkü herhangi bir üçgenin,
iki kenarı
büyüktür geriye kalandan
nasıl seçilirse seçilsin).

Verilmiş olsun
üç doğru
 A , B , ve Γ ,
ikisi, kalandan
büyük olan,
nasıl seçilirse seçilsin,
 A ile B , Γ kenarından,
 A ile Γ , B kenarından,
ve B ile Γ , A kenarından.

Gereklidir
 A , B ve Γ doğrularına eşit olanlardan
bir üçgenin inşa edilmesi.

Yerleştirilmiş olsun
bir ΔE doğrusu,
 Δ noktasında sınırlanmış,
ama E noktasında sınırlandırılmamış,

¹In the Greek this is the infinitive εἶναι 'to be', as in the previous clause.

²According to Heiberg, the manuscripts have δεῖ δὲ here, as at

the beginnings of specifications (see §1.3); but Proclus and Eutocius have δεῖ δὲ in their commentaries.

and there is laid down
 ΔZ equal to A,
 ZH equal to B,
and $H\Theta$ equal to Γ ;
and to center Z
at distance $Z\Delta$
a circle has been drawn, $\Delta\text{ΚΛ}$;
moreover,
to center H,
at distance $H\Theta$,
circle $\text{ΚΛ}\Theta$ has been drawn,
and KZ and KH have been joined.

I say that
from three STRAIGHTS
equal to A, B, and Γ ,
a triangle has been constructed, ΚΖΗ .

For, since the point Z
is the center of circle $\Delta\text{ΚΛ}$,
 $Z\Delta$ is equal to $Z\text{Κ}$;
but $Z\Delta$ is equal to A.
And KZ is therefore equal to A.
Moreover,
since the point H
is the center of circle $\text{ΚΛ}\Theta$,
 $H\Theta$ is equal to $H\text{Κ}$;
but $H\Theta$ is equal to Γ ;
and KH is therefore equal to Γ .
and ZH is equal to B;
therefore the three STRAIGHTS,
 KZ , ZH , and HK
are equal to the three, A, B, and Γ .

Therefore, from the three STRAIGHTS
 KZ , ZH , and HK ,
which are equal
to the three given STRAIGHTS
A, B, and Γ ,
a triangle has been constructed, ΚΖΗ ;
—just what it was necessary to show.

Dolayısıyla, üç doğrudan;
 KZ , ZH ve HK ,
eşit olan
verilmiş üç doğruya
A, B ve Γ ,
bir ΚΖΗ üçgeni inşa edilmiştir;
— gösterilmesi gereken tam buydu.

καὶ κείσθω
τῆ μὲν A ἴση ἡ ΔZ ,
τῆ δὲ B ἴση ἡ ZH ,
τῆ δὲ Γ ἴση ἡ $H\Theta$.
καὶ κέντρῳ μὲν τῷ Z,
διαστήματι δὲ τῷ $Z\Delta$
κύκλος γεγράφθω ὁ $\Delta\text{ΚΛ}$.
πάλιν
κέντρῳ μὲν τῷ H,
διαστήματι δὲ τῷ $H\Theta$
κύκλος γεγράφθω ὁ $\text{ΚΛ}\Theta$,
καὶ ἐπεζεύχθωσαν αἱ KZ , KH .

λέγω, ὅτι
ἐκ τριῶν εὐθειῶν
τῶν ἴσων ταῖς A, B, Γ
τρίγωνον συνέσταται τὸ ΚΖΗ .

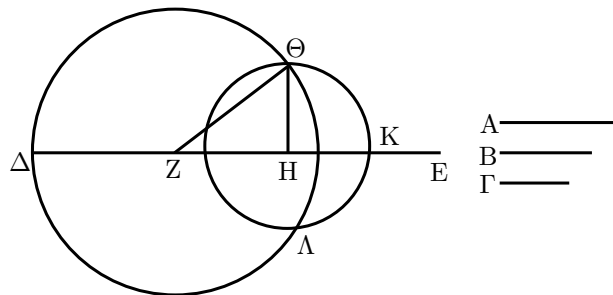
Ἐπεὶ γὰρ τὸ Z σημεῖον
κέντρον ἐστὶ τοῦ $\Delta\text{ΚΛ}$ κύκλου,
ἴση ἐστὶν ἡ $Z\Delta$ τῆ $Z\text{Κ}$.
ἀλλὰ ἡ $Z\Delta$ τῆ A ἐστὶν ἴση.
καὶ ἡ KZ ἄρα τῆ A ἐστὶν ἴση.
πάλιν,
ἐπεὶ τὸ H σημεῖον
κέντρον ἐστὶ τοῦ $\text{ΚΛ}\Theta$ κύκλου,
ἴση ἐστὶν ἡ $H\Theta$ τῆ $H\text{Κ}$.
ἀλλὰ ἡ $H\Theta$ τῆ Γ ἐστὶν ἴση.
καὶ ἡ KH ἄρα τῆ Γ ἐστὶν ἴση.
ἐστὶ δὲ καὶ ἡ ZH τῆ B ἴση.
αἱ τρεῖς ἄρα εὐθεῖαι
αἱ KZ , ZH , HK
τριῶν ταῖς A, B, Γ ἴσαι εἰσίν.

Ἐκ τριῶν ἄρα εὐθειῶν
τῶν KZ , ZH , HK ,
αἱ εἰσὶν ἴσαι
τριῶν ταῖς δοθείσασιν εὐθείαις
ταῖς A, B, Γ ,
τρίγωνον συνέσταται τὸ ΚΖΗ .
ὅπερ ἔδει ποιῆσαι.

yerleştirilmiş olsun
A doğrusuna eşit ΔZ ,
B doğrusuna eşit ZH ,
ve Γ doğrusuna eşit $H\Theta$;
ve Z merkezine
 $Z\Delta$ uzaklığında
bir $\Delta\text{ΚΛ}$ çemberi çizilmiş olsun;
dahası,
H merkezine,
 $H\Theta$ uzaklığında,
 $\text{ΚΛ}\Theta$ çemberi çizilmiş olsun,
ve KZ ile KH birleştirilmiş olsun.

İddia ediyorum ki
üç doğrudan
A, B ve Γ doğrularına eşit olan
bir ΚΖΗ üçgeni inşa edilmiştir.

Çünkü merkezi olduğundan Z noktası,
 $\Delta\text{ΚΛ}$ çemberinin,
 $Z\Delta$ eşittir $Z\text{Κ}$ doğrusuna;
ama $Z\Delta$ eşittir A doğrusuna.
Ve KZ dolayısıyla A doğrusuna eşittir.
Dahası,
merkezi olduğundan H noktası
 $\text{ΚΛ}\Theta$ çemberinin,
 $H\Theta$ eşittir $H\text{Κ}$ doğrusuna;
ama $H\Theta$ eşittir Γ doğrusuna;
ve KH dolayısıyla Γ doğrusuna eşittir.
ve ZH eşittir B doğrusuna;
dolayısıyla üç doğru,
 KZ , ZH ve HK
eşittirler A, B ve Γ üçlüsüne.



3.23

At the given STRAIGHT,

Πρὸς τῆ δοθείσῃ εὐθείᾳ

Verilmiş bir doğruya,

and at the given point on it,
equal to the given rectilinear angle,
a rectilinear angle to be constructed.

Let be
the given STRAIGHT AB,
the point on it, A,
the given rectilinear angle,
 $\Delta\Gamma E$.

It is necessary then,
on the given STRAIGHT, AB,
and at the point A on it,
to the given rectilinear angle
 $\Delta\Gamma E$
equal,
for a rectilinear angle
to be constructed.

Suppose there have been chosen
on either of $\Gamma\Delta$ and ΓE
random points Δ and E,
and ΔE has been joined,
and from three STRAIGHTS,
which are equal to the three,
 $\Gamma\Delta$, ΔE , and ΓE ,
triangle AZH has been constructed,
so that equal are
 $\Gamma\Delta$ to AZ,
 ΓE to AH,
and ΔE to ZH.

Since then the two, $\Delta\Gamma$ and ΓE ,
are equal to the two, ZA and AH,
either to either,
and the base ΔE to the base ZH
is equal,
therefore the angle $\Delta\Gamma E$
is equal to ZAH.

Therefore, on the given STRAIGHT,
AB,
and at the point A on it,
equal to the given rectilinear angle,
 $\Delta\Gamma E$,
the rectilinear angle $\Delta\Gamma E$ has been
constructed;
—just what it was necessary to do.

καὶ τῷ πρὸς αὐτῇ σημείῳ
τῇ δοθείσῃ γωνίᾳ εὐθύγραμμῳ ἴσην
γωνίαν εὐθύγραμμον συστήσασθαι.

Ἐστω
ἡ μὲν δοθεῖσα εὐθεῖα ἡ AB,
τὸ δὲ πρὸς αὐτῇ σημείον τὸ A,
ἡ δὲ δοθεῖσα γωνία εὐθύγραμμος
ἡ ὑπὸ $\Delta\Gamma E$.

δεῖ δὴ
πρὸς τῇ δοθείσῃ εὐθείᾳ τῇ AB
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ A
τῇ δοθείσῃ γωνίᾳ εὐθύγραμμῳ
τῇ ὑπὸ $\Delta\Gamma E$
ἴσην
γωνίαν εὐθύγραμμον
σύστησασθαι.

Εἰλήφθω
ἐφ' ἑκατέρας τῶν $\Gamma\Delta$, ΓE
τυχόντα σημεία τὰ Δ , E,
καὶ ἐπεζεύχθω ἡ ΔE .
καὶ ἐκ τριῶν εὐθειῶν,
αἱ εἰσὶν ἴσαι τρισὶ
ταῖς $\Gamma\Delta$, ΔE , ΓE ,
τρίγωνον συνεστάτω τὸ AZH,
ὥστε ἴσην εἶναι
τὴν μὲν $\Gamma\Delta$ τῇ AZ,
τὴν δὲ ΓE τῇ AH,
καὶ ἔτι τὴν ΔE τῇ ZH.

Ἐπεὶ οὖν δύο αἱ $\Delta\Gamma$, ΓE
δύο ταῖς ZA, AH ἴσαι εἰσὶν
ἑκατέρα ἑκατέρᾳ,
καὶ βᾶσις ἡ ΔE βᾶσει τῇ ZH
ἴση,
γωνία ἄρα ἡ ὑπὸ $\Delta\Gamma E$ γωνία
τῇ ὑπὸ ZAH ἴστιν ἴση.

Πρὸς ἄρα τῇ δοθείσῃ εὐθείᾳ
τῇ AB
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ A τῇ
δοθείσῃ γωνίᾳ εὐθύγραμμῳ τῇ ὑπὸ
 $\Delta\Gamma E$ ἴση
γωνία εὐθύγραμμος συνέσεται ἡ ὑπὸ
ZAH.
ὅπερ ἔδει ποιῆσαι.

ve üzerinde verilmiş noktada,
verilmiş düzkenar açığa eşit olan,
bir düzkenar açı inşa edilmesi.

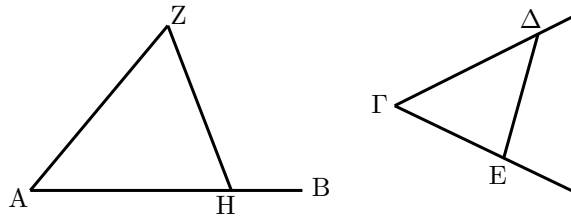
Verilmiş olsun
AB doğrusu,
üzerindeki A noktası,
verilmiş olsun düzkenar açı,
 $\Delta\Gamma E$.

Gereklidir şimdi,
verilmiş AB doğrusunda,
ve üzerindeki A noktasında,
verilmiş düzkenar
 $\Delta\Gamma E$ açısına
eşit,
bir düzkenar açının
inşa edilmesi.

Seçilmiş olsun
 $\Gamma\Delta$ ve ΓE doğrularının her birinden
rastgele Δ ve E noktaları,
ve ΔE birleştirilmiş olsun,
ve üç doğrudan,
eşit olan verilmiş üç,
 $\Gamma\Delta$, ΔE ve ΓE doğrularına,
bir AZH üçgen inşa edilmiş olsun,
öyle ki, eşit olsun
 $\Gamma\Delta$, AZ doğrusuna,
 ΓE , AH doğrusuna, ve ΔE , ZH
doğrusuna.

O zaman $\Delta\Gamma$ ve ΓE ikilisi,
eşit olduğundan ZA ve AH ikilisininin,
her biri birine,
ve ΔE tabanı, ZH tabanına
eşit,
dolayısıyla $\Delta\Gamma E$ açısı
eşittir ZAH açısına.

Dolayısıyla,
AB doğrusunda,
ve üzerindeki A noktasında,
verilen düzkenar $\Delta\Gamma E$ açısına eşit,
 $\Delta\Gamma E$ düzkenar açısı inşa edilmiştir;
— yapılması gereken tam buydu.



3.24

If two triangles
two sides
to two sides

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
[ταῖς] δύο πλευραῖς

Eğer iki üçgenin
(birinin) iki kenarı
(diğerinin) iki kenarına

have equal
either to either,
but angle
than angle
have greater,
[namely] that by the equal sides
contained,
also base
than base
they will have greater.

Let there be
two triangles, $AB\Gamma$ and ΔEZ ,
—two sides, AB and AG ,
to two sides, ΔE and ΔZ ,
having equal,
either to either,
 AB to ΔE ,
and AG to ΔZ ,
—and the angle at A ,
than the angle at Δ ,
let it be greater.

I say that
also the base $B\Gamma$
than the base EZ
is greater.

For since [it] is greater,
[namely] angle BAG
than angle $E\Delta Z$,
suppose has been constructed
on the STRAIGHT, ΔE ,
and at the point Δ on it,
equal to angle BAG ,
 $E\Delta H$,
and suppose is laid down,
to either of AG and ΔZ equal,
 ΔH ,
and suppose have been joined
 EH and ZH .

Since [it] is equal,
 AB to ΔE ,
and AG to ΔH ,
the two, BA and AG ,
to the two, $E\Delta$ and ΔH ,
are equal,
either to either;
and angle BAG
to angle $E\Delta H$ is equal;
therefore the base $B\Gamma$
to the base EH is equal.
Moreover,
since [it] is equal,
[namely] ΔZ to ΔH ,
[it] too is equal,
[namely] angle ΔHZ to ΔZH ;
therefore [it] is greater,
[namely] ΔZH than EZH ;
therefore [it] is much greater,
[namely] EZH than EZH .
And since there is a triangle, EZH ,

ἴσας ἔχῃ
ἑκατέραν ἑκατέρα,
τὴν δὲ γωνίαν
τῆς γωνίας
μεῖζονα ἔχῃ
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην,
καὶ τὴν βάσιν
τῆς βάσεως
μεῖζονα ἔξει.

Ἐστω
δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ
τὰς δύο πλευρὰς τὰς AB , AG
ταῖς δύο πλευραῖς ταῖς ΔE , ΔZ
ἴσας ἔχοντα
ἑκατέραν ἑκατέρα,
τὴν μὲν AB τῆ ΔE
τὴν δὲ AG τῆ ΔZ ,
ἢ δὲ πρὸς τῷ A γωνία
τῆς πρὸς τῷ Δ γωνίας
μεῖζων ἔστω.

λέγω, ὅτι
καὶ βάσις ἢ $B\Gamma$
βάσεως τῆς EZ
μεῖζων ἔστί.

Ἐπεὶ γὰρ μεῖζων
ἢ ὑπὸ BAG γωνία
τῆς ὑπὸ $E\Delta Z$ γωνίας,
συνεστάτω
πρὸς τῆ ΔE εὐθείᾳ
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ Δ
τῆ ὑπὸ BAG γωνία ἴση
ἢ ὑπὸ $E\Delta H$,
καὶ κείσθω
ὁποτέρᾳ τῶν AG , ΔZ ἴση
ἢ ΔH ,
καὶ ἐπεζεύχθωσαν
αἱ EH , ZH .

Ἐπεὶ οὖν ἴση ἔστί
ἢ μὲν AB τῆ ΔE ,
ἢ δὲ AG τῆ ΔH ,
δύο δὲ αἱ BA , AG
δυσὶ ταῖς $E\Delta$, ΔH
ἴσαι εἰσὶν
ἑκατέρα ἑκατέρα·
καὶ γωνία ἢ ὑπὸ BAG
γωνία τῆ ὑπὸ $E\Delta H$ ἴση·
βάσις ἄρα ἢ $B\Gamma$
βάσει τῆ EH ἔστί ἴση.
πάλιν,
ἐπεὶ ἴση ἔστί
ἢ ΔZ τῆ ΔH ,
ἴση ἔστί καὶ
ἢ ὑπὸ ΔHZ γωνία τῆ ὑπὸ ΔZH ·
μεῖζων ἄρα
ἢ ὑπὸ ΔZH τῆς ὑπὸ EZH ·
πολλῶ ἄρα μεῖζων ἔστί
ἢ ὑπὸ EZH τῆς ὑπὸ EZH .
καὶ ἐπεὶ τρίγωνόν ἐστι τὸ EZH

eşitse,
her biri birine,
ama açısı
açısından
büyükse,
[yani] eşit kenarlarca
içerilen(ler),
tabanı da
tabanından
büyük olacak.

Verilmiş olsun
iki $AB\Gamma$ ve ΔEZ üçgeni,
— iki AB ve AG kenarı,
iki ΔE ve ΔZ kenarına,
eşit olan,
her biri birine,
 AB , ΔE kenarına,
ve AG , ΔZ kenarına,
—ve A noktasındaki açısı,
 Δ doktasındakinden,
büyük olsun.

İddia ediyorum ki
 $B\Gamma$ tabanı da
 EZ tabanından
büyüktür.

Çünkü büyük olduğundan,
 BAG açısı
 $E\Delta Z$ açısından,
inşa edilmiş olsun
 ΔE doğrusunda,
ve üzerindeki Δ noktasında,
 BAG açısına eşit,
 $E\Delta H$,
ve yerleştirilmiş olsun
 AG ve ΔZ kenarlarının ikisine de eşit,
 ΔH ,
ve birleştirilmiş olsun
 EH ve ZH .

Eşit olduğundan,
 AB , ΔE kenarına,
ve AG , ΔH kenarına,
 BA ve AG ikilisi,
 $E\Delta$ ve ΔH ikilisine,
eşittirler,
her biri birine;
ve BAG açısı
 $E\Delta H$ açısına eşittir;
dolayısıyla $B\Gamma$ tabanı
 EH tabanına eşittir.
Dahası,
eşit olduğundan,
 ΔZ , ΔH kenarına,
yine eşittir,
 ΔHZ açısı, ΔZH açısına;
dolayısıyla büyüktür
 ΔZH , EZH açısından;
dolayısıyla çok daha büyüktür
 EZH , EZH açısından.
Ve EZH bir üçgen olduğundan,

having greater
angle EZH than EHZ,
and the greater angle,
—the greater side subtends it;
greater therefore also is
side EH than EZ.
And [it] is equal, EH to BG;
greater therefore is BG than EZ.

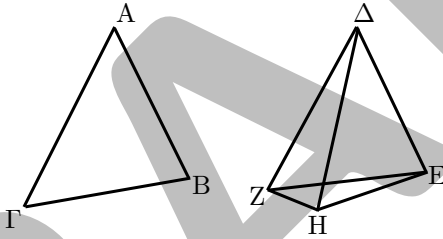
If, therefore, two triangles
two sides
to two sides
have equal,
either to either,
but angle
than angle
have greater,
[namely] that by the equal sides
contained,
also base
than base
they will have greater;
—just what it was necessary to show.

μείζονα ἔχον
τὴν ὑπὸ EZH γωνίαν τῆς ὑπὸ EHZ,
ὑπὸ δὲ τὴν μείζονα γωνίαν
ἢ μείζων πλευρὰ ὑποτείνει,
μείζων ἄρα καὶ
πλευρὰ ἢ EH τῆς EZ.
ἴση δὲ ἢ EH τῆς BG·
μείζων ἄρα καὶ ἢ BG τῆς EZ.

Ἐὰν ἄρα δύο τρίγωνα
τὰς δύο πλευρὰς
δυσὶ πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκατέρα,
τὴν δὲ γωνίαν
τῆς γωνίας
μείζονα ἔχη
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην,
καὶ τὴν βάσιν
τῆς βάσεως
μείζονα ἔξει·
ὅπερ ἔδει δεῖξαι.

büyük olan
EZH açısı EHZ açısından,
ve daha büyük açı,
—daha büyük açı tarafından karşı-
landığından;
büyüktür dolayısıyla
EH kenarı da EZ kenarından.
Ve eşittir, EH , BG kenarına;
büyüktür dolayısıyla BG, EZ kenarın-
dan.

Eğer, dolayısıyla, iki üçgenin
(birinin) iki kenarı
(diğerinin) iki kenarına
eşitse
her biri birine,
ama açısı
açısından
büyükse,
[yani] eşit kenarlarca
içerilen(ler),
tabanı da
tabanından
büyük olacak;
— gösterilmesi gereken tam buydu.



3.25

If two triangles
two sides
to two sides
have equal,
either to either,
but base
than base
have greater,
also angle
than angle
they will have greater
—that by the equal STRAIGHTS
contained.

Let there be
two triangles, ABG and DEZ,
two sides, AB and AG,
to two sides, DE and DZ,
having equal,
either to either,
AB to DE
and AG to DZ;
and the base BG
than the base EZ
—let it be greater.

Ἐὰν δύο τρίγωνα
τὰς δύο πλευρὰς
δυσὶ πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκατέρα,
τὴν δὲ βάσιν
τῆς βάσεως
μείζονα ἔχη,
καὶ τὴν γωνίαν
τῆς γωνίας
μείζονα ἔξει
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην.

Ἐστω
δύο τρίγωνα τὰ ABG, ΔEZ
τὰς δύο πλευρὰς τὰς AB, AG
ταῖς δύο πλευραῖς ταῖς DE, ΔZ
ἴσας ἔχοντα
ἐκατέραν ἐκατέρα,
τὴν μὲν AB τῆς DE,
τὴν δὲ AG τῆς ΔZ·
βάσις δὲ ἢ BG
βάσεως τῆς EZ
μείζων ἔστω·

Eğer iki üçgenin
(birinin) iki kenarı
(diğerinin) iki kenarına
eşitse
her biri birine,
ama tabanı
tabanından
büyükse,
açısı da
açısından
büyük olacak
—(yani) eşit doğrularca
içerilenler.

Verilmiş olsun
ABG ve ΔEZ üçgenleri,
iki AB ve AG kenarı,
iki DE ve ΔZ kenarına,
eşit olan,
her biri birine,
AB, DE kenarına
ve AG, ΔZ kenarına;
ve BG tabanı
EZ tabanından
—büyük olsun.

I say that
also the angle BAF
than the angle EΔZ
is greater.

For if not,
[it] is either equal to it, or less;
but it is not equal
—BAF to EΔZ;
for if it is equal,
also the base BF to EZ;
but it is not.
Therefore it is not equal,
angle BAF to EΔZ;
neither is it less,
BAF than EΔZ;
for if it is less,
also base BF than EZ;
but it is not;
therefore it is not less,
BAF than angle EΔZ.
And it was shown that
it is not equal;
therefore it is greater,
BAF than EΔZ.

If, therefore, two triangles
two sides
to two sides
have equal,
either to either,
but base
than base
have greater,
also angle
than angle
they will have greater
—that by the equal STRAIGHTS
contained
—just what it was necessary to show.

λέγω, ὅτι
καὶ γωνία ἢ ὑπὸ BAF
γωνίας τῆς ὑπὸ EΔZ
μείζων ἐστίν.

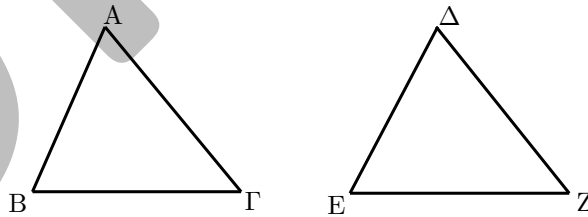
Εἰ γὰρ μή,
ἤτοι ἴση ἐστὶν αὐτῇ ἢ ἐλάσσων·
ἴση μὲν οὖν οὐκ ἔστιν
ἢ ὑπὸ BAF τῇ ὑπὸ EΔZ·
ἴση γὰρ ἂν ᾦν
καὶ βάσις ἢ BF βάσει τῇ EZ·
οὐκ ἔστι δέ.
οὐκ ἄρα ἴση ἐστὶ
γωνία ἢ ὑπὸ BAF τῇ ὑπὸ EΔZ·
οὐδὲ μὴν ἐλάσσων ἐστὶν
ἢ ὑπὸ BAF τῆς ὑπὸ EΔZ·
ἐλάσσων γὰρ ἂν ᾦν
καὶ βάσις ἢ BF βάσει τῆς EZ·
οὐκ ἔστι δέ·
οὐκ ἄρα ἐλάσσων ἐστὶν
ἢ ὑπὸ BAF γωνία τῆς ὑπὸ EΔZ.
ἐδείχθη δέ, ὅτι
οὐδὲ ἴση·
μείζων ἄρα ἐστὶν
ἢ ὑπὸ BAF τῆς ὑπὸ EΔZ.

Ἐὰν ἄρα δύο τρίγωνα
τὰς δύο πλευρὰς
δυσὶ πλευραῖς
ἴσας ἔχη
ἐκατέραν ἐκάτερα,
τὴν δὲ βασὶν
τῆς βάσεως
μείζονα ἔχη,
καὶ τὴν γωνίαν
τῆς γωνίας
μείζονα ἔξει
τὴν ὑπὸ τῶν ἴσων εὐθειῶν
περιεχομένην·
ὅπερ ἔδει δεῖξαι.

İddia ediyorum ki
BAF açısı da
EΔZ açısından
büyüktür.

Çünkü eğer değilse,
ya ona eşittir, ya da ondan küçük;
ama eşit değildir
—BAF, EΔZ açısına;
çünkü eğer eşit ise,
BF tabanı da EZ tabanına (eşittir);
ama değil.
Dolayısıyla eşit değildir,
BAF, EΔZ açısına;
küçük de değildir,
BAF, EΔZ açısından;
çünkü eğer küçük ise,
BF tabanı da EZ tabanından (küçük-
tür);
ama değil;
dolayısıyla küçük değildir,
BAF, EΔZ açısından.
Ama gösterilmişti ki
eşit değildir;
dolayısıyla büyüktür,
BAF, EΔZ açısından.

Eğer, dolayısıyla, iki üçgenin
(birinin) iki kenarı
(diğerin) iki kenarına
eşitse
her biri birine,
ama tabanı
tabanından
büyükse,
açısı da
açısından
büyük olacak
—(yani) eşit doğrularca
içerilenler;
— gösterilmesi gereken tam buydu.



3.26

If two triangles
two angles
to two angles
have equal,
either to either,
and one side
to one side
equal,
either that near the equal sides
or that subtending

Ἐὰν δύο τρίγωνα
τὰς δύο γωνίας
δυσὶ γωνίαις
ἴσας ἔχη
ἐκατέραν ἐκάτερα
καὶ μίαν πλευρὰν
μῖα πλευρᾷ
ἴσην
ἤτοι τὴν πρὸς ταῖς ἴσαις γωνίαις
ἢ τὴν ὑποτείνουσαν

Eğer iki üçgenin
iki açısı
iki açısına
eşitse,
her biri birine,
ve bir kenar
bir kenara
eşitse,
ya eşit açılımlar arasında olan
ya da karşılayan

one of the equal sides,
also the remaining sides
to the remaining sides
they will have equal,
also the remaining angle
to the remaining angle.

Let there be
two triangles, $AB\Gamma$ and ΔEZ
the two angles $AB\Gamma$ and $B\Gamma A$
to the two angles ΔEZ and $EZ\Delta$
having equal,
either to either,
 $AB\Gamma$ to ΔEZ ,
and $B\Gamma A$ to $EZ\Delta$;
and let them also have
one side
to one side
equal,
first that near the equal angles,
 $B\Gamma$ to EZ ;

I say that
the remaining sides
to the remaining sides
they will have equal,
either to either,
 AB to ΔE
and $A\Gamma$ to ΔZ ,
also the remaining angle
to the remaining angle,
 $BA\Gamma$ to $E\Delta Z$.

For, if it is unequal,
 AB to ΔE ,
one of them is greater.
Let be greater
 AB ,
and let there be cut
to ΔE equal
 BH ,
and suppose there has been joined
 $H\Gamma$.

Because then it is equal,
 BH to ΔE ,
and $B\Gamma$ to EZ ,
the two, BH^1 and $B\Gamma$
to the two ΔE and EZ
are equal,
either to either,
and the angle $H\text{B}\Gamma$
to the angle ΔEZ
is equal;
therefore the base $H\Gamma$
to the base ΔZ
is equal,
and the triangle $H\text{B}\Gamma$
to the triangle ΔEZ
is equal,
and the remaining angles
to the remaining angles
will be equal,

ὕπο μίαν τῶν ἴσων γωνιῶν,
καὶ τὰς λοιπὰς πλευρὰς
ταῖς λοιπαῖς πλευραῖς
ἴσας ἔξει
καὶ τὴν λοιπὴν γωνίαν
τῇ λοιπῇ γωνίᾳ.

Ἐστω
δύο τρίγωνα τὰ $AB\Gamma$, ΔEZ
τὰς δύο γωνίας τὰς ὑπὸ $AB\Gamma$, $B\Gamma A$
δυσὶ ταῖς ὑπὸ ΔEZ , $EZ\Delta$
ἴσας ἔχοντα
ἑκατέραν ἑκατέρᾳ,
τὴν μὲν ὑπὸ $AB\Gamma$ τῇ ὑπὸ ΔEZ ,
τὴν δὲ ὑπὸ $B\Gamma A$ τῇ ὑπὸ $EZ\Delta$.
ἔχέτω δὲ
καὶ μίαν πλευρὰν
μιᾷ πλευρᾷ
ἴσην,
πρότερον τὴν πρὸς ταῖς ἴσαις γωνίαις
τὴν $B\Gamma$ τῇ EZ .

λέγω, ὅτι
καὶ τὰς λοιπὰς πλευρὰς
ταῖς λοιπαῖς πλευραῖς
ἴσας ἔξει
ἑκατέραν ἑκατέρᾳ,
τὴν μὲν AB τῇ ΔE
τὴν δὲ $A\Gamma$ τῇ ΔZ ,
καὶ τὴν λοιπὴν γωνίαν
τῇ λοιπῇ γωνίᾳ,
τὴν ὑπὸ $BA\Gamma$ τῇ ὑπὸ $E\Delta Z$.

Εἰ γὰρ ἄνισός ἐστιν
ἢ AB τῇ ΔE ,
μία αὐτῶν μείζων ἐστίν.
ἔστω μείζων
ἢ AB ,
καὶ κείσθω
τῇ ΔE ἴση
ἢ BH ,
καὶ ἐπεζεύχθω
ἢ $H\Gamma$.

Ἐπεὶ οὖν ἴση ἐστίν
ἢ μὲν BH τῇ ΔE ,
ἢ δὲ $B\Gamma$ τῇ EZ ,
δύο δὴ αἱ BH , $B\Gamma$
δυσὶ ταῖς ΔE , EZ
ἴσαι εἰσὶν
ἑκατέρα ἑκατέρᾳ.
καὶ γωνία ἢ ὑπὸ $H\text{B}\Gamma$
γωνία τῇ ὑπὸ ΔEZ
ἴση ἐστίν.
βάσις ἄρα ἢ $H\Gamma$
βάσει τῇ ΔZ
ἴση ἐστίν,
καὶ τὸ $H\text{B}\Gamma$ τρίγωνον
τῷ ΔEZ τριγώνῳ
ἴσον ἐστίν,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσονται,

eşit açılardan birini,
kalan kenarları da
kalan kenarlarına
eşit olacak,
kalan açıları da
kalan açılara.

Verilmiş olsun
iki $AB\Gamma$ ve ΔEZ üçgeni
iki $AB\Gamma$ ve $B\Gamma A$ açıları
iki ΔEZ ve $EZ\Delta$ açılara
eşit olan,
her biri birine,
 $AB\Gamma$, ΔEZ açısına
ve $B\Gamma A$, $EZ\Delta$ açısına;
ayrıca olsun
bir kenarı
bir kenarına
eşit,
önce eşit açılardan yanındaki olan,
 $B\Gamma$, EZ kenarına;

İddia ediyorum ki
kalan kenarlar
kalan kenarlara
eşit olacaklar,
her biri birine,
 AB , ΔE kenarına
ve $A\Gamma$, ΔZ kenarına,
ayrıca kalan açı
kalan açılara,
 $BA\Gamma$, $E\Delta Z$ açısına.

Çünkü, eğer eşit değilse,
 AB , ΔE kenarına,
biri daha büyüktür.
Büyük olan
 AB olsun,
ve kesilmiş olsun
 ΔE kenarına eşit
 BH ,
ve birleştirilmiş olsun
 $H\Gamma$.

Çünkü o zaman eşittir,
 BH , ΔE kenarına
ve $B\Gamma$, EZ kenarına,
 BH ve $B\Gamma$ ikilisi
 ΔE ve EZ ikilisine
eşittirler,
her biri birine,
ve $H\text{B}\Gamma$ açısı
 ΔEZ açısına
eşittir;
dolayısıyla $H\Gamma$ tabanı
 ΔZ tabanına
eşittir,
ve $H\text{B}\Gamma$ üçgeni
 ΔEZ üçgenine
eşittir,
ve kalan açılar
kalan açılara
eşit olacaklar,

those that the equal sides subtend.
 Equal therefore is angle BGA
 to ΔZE .
 But ΔZE
 to BGA
 is supposed equal;
 therefore also BGA
 to BGA
 is equal,
 the lesser to the greater,
 which is impossible.
 Therefore it is not unequal,
 AB to ΔE .
 Therefore it is equal.
 It is also the case that
 BG to EZ is equal;
 then the two AB and BG
 to the two ΔE and EZ
 are equal,
 either to either;
 also the angle ABG
 to the angle ΔEZ
 is equal;
 therefore the base AG
 to the base ΔZ
 is equal,
 and the remaining angle BAG
 to the remaining angle ΔZ
 is equal.

But then again let them be
 —[those angles] equal sides
 subtending—
 equal,
 as AB to ΔE ;
 I say again that
 also the remaining sides
 to the remaining sides
 will be equal,
 AG to ΔZ ,
 and BG to EZ,
 and also the remaining angle BAG
 to the remaining angle ΔZ
 is equal.

For, if it is unequal,
 BG to EZ,
 one of them is greater.
 Let be greater,
 if possible,
 BG,
 and let there be cut
 to EZ equal
 B Θ ,
 and suppose there has been joined
 A Θ .
 Because also it is equal
 —B Θ to EZ

ὄφ' ἄς αἰ ἴσαι πλευραὶ ὑποτείνουσιν·
 ἴση ἄρα ἡ ὑπὸ ΗΓΒ γωνία
 τῇ ὑπὸ ΔZE .
 ἀλλὰ ἡ ὑπὸ ΔZE
 τῇ ὑπὸ BGA
 ὑπόκειται ἴση·
 καὶ ἡ ὑπὸ BGA ἄρα
 τῇ ὑπὸ BGA
 ἴση ἐστίν,
 ἡ ἐλάσσων τῇ μείζονι·
 ὅπερ ἀδύνατον.
 οὐκ ἄρα ἀνισός ἐστιν
 ἡ AB τῇ ΔE .
 ἴση ἄρα.
 ἔστι δὲ καὶ
 ἡ BG τῇ EZ ἴση·
 δύο δὲ αἰ AB, BG
 δυοὶ ταῖς ΔE , EZ
 ἴσαι εἰσὶν
 ἑκατέρα ἑκατέρᾳ·
 καὶ γωνία ἡ ὑπὸ ABG
 γωνία τῇ ὑπὸ ΔEZ
 ἐστὶν ἴση·
 βάσις ἄρα ἡ AG
 βάσει τῇ ΔZ
 ἴση ἐστίν,
 καὶ λοιπὴ γωνία ἡ ὑπὸ BAG
 τῇ λοιπῇ γωνίᾳ τῇ ὑπὸ ΔZ
 ἴση ἐστίν.

Ἄλλὰ δὲ πάλιν ἔστωσαν
 αἰ ὑπὸ τὰς ἴσας γωνίας πλευραὶ
 ὑποτείνουσαι
 ἴσαι,
 ὡς ἡ AB τῇ ΔE .
 λέγω πάλιν, ὅτι
 καὶ αἰ λοιπαὶ πλευραὶ
 ταῖς λοιπαῖς πλευραῖς
 ἴσαι ἔσονται,
 ἡ μὲν AG τῇ ΔZ ,
 ἡ δὲ BG τῇ EZ
 καὶ ἔτι ἡ λοιπὴ γωνία ἡ ὑπὸ BAG
 τῇ λοιπῇ γωνίᾳ τῇ ὑπὸ ΔZ
 ἴση ἐστίν.

Εἰ γὰρ ἀνισός ἐστιν
 ἡ BG τῇ EZ,
 μία αὐτῶν μείζων ἐστίν.
 ἔστω μείζων,
 εἰ δυνατόν,
 ἡ BG,
 καὶ κείσθω
 τῇ EZ ἴση
 ἡ B Θ ,
 καὶ ἐπεζεύχθω
 ἡ A Θ .
 καὶ ἐπεὶ ἴση ἐστὶν
 ἡ μὲν B Θ τῇ EZ

eşit kenarların karşıladıkları.
 Eşittir dolayısıyla BGA açısı
 ΔZE açısına.
 Ama ΔZE ,
 BGA açısına
 eşit kabul edilmişti
 dolayısıyla BGA ve
 BGA açısına
 eşittir,
 daha küçük olan daha büyük olana,
 ki bu imkansızdır.
 Dolayısıyla değildir eşit değil,
 AB, ΔE kenarına.
 Dolayısıyla eşittir.
 Ayrıca durum şöyledir;
 BG, EZ kenarına eşittir;
 o zaman AB ve BG ikilisi
 ΔE ve EZ ikilisine
 eşittirler,
 her biri birine;
 ABG açısı da
 ΔEZ açısına
 eşittir;
 dolayısıyla AG tabanı
 ΔZ tabanına
 eşittir,
 ve kalan BAG açısı
 kalan ΔZ açısına
 eşittir.

Ama o zaman, yine olsunlar
 — kenarlar eşit [açıları]
 karşılayan—
 eşit,
 AB, ΔE kenarına gibi;
 Yine iddia ediyorum ki
 kalan kenarlar da
 kalan kenarlara
 eşit olacaklar,
 AG, ΔZ kenarına
 ve BG, EZ kenarına
 ve kalan BAG açısı da
 kalan ΔZ açısına
 eşittir.

Çünkü, eğer eşit değil ise,
 BG, EZ kenarına,
 biri daha büyüktür.
 Daha büyük olsun,
 eğer mümkünse,
 BG,
 ve kesilmiş olsun
 EZ kenarına eşit
 B Θ ,
 ve kabul edilsin birleştirilmiş olduğu
 A Θ kenarının.
 Ayrıca eşit olduğundan
 —B Θ , EZ kenarına

¹Fitzpatrick considers this way of denoting the line to be a 'mistake'; apparently he thinks Euclid should (and perhaps did originally) write HB, for parallelism with ΔE . But HB and BH are the same line, and for all we know, Euclid preferred to write BH because it was in alphabetical order. Netz [12, Ch. 2] studies the general

Greek mathematical practice of using the letters in different order for the same mathematical object. He concludes that changes in order are made on purpose, though he does not address examples like the present one.

and AB to ΔE ,
then the two AB and B Θ
to the two ΔE and EZ
are equal,
either to either;
and they contain equal angles;
therefore the base A Θ
to the base ΔZ
is equal,
and the triangle AB Θ
to the triangle ΔEZ
is equal,
and the remaining angles
to the remaining angles
are equal,
which the equal sides
subtend.

Therefore equal is
angle B Θ A
to EZ Δ .
But EZ Δ
to B Γ A
is equal;
then of triangle A Θ Γ
the exterior angle B Θ A
is equal
to the interior and opposite
B Γ A;
which is impossible.

Therefore it is not unequal,
B Γ to EZ;
therefore it is equal.
And it is also,
AB,
to ΔE ,
equal.

Then the two AB and B Γ
to the two ΔE and EZ
are equal,
either to either;
and equal angles
they contain;
therefore the base A Γ
to the base ΔZ
is equal,
and triangle AB Γ
to triangle ΔEZ
is equal,
and the remaining angle BA Γ
to the remaining angle E ΔZ
is equal.

If therefore two triangles
two angles
to two angles
have equal,
either to either,
and one side
to one side
equal,
either that near the equal sides
or that subtending
one of the equal sides,

ἡ δὲ AB τῆ ΔE ,
δύο δὲ αἱ AB, B Θ
δυοσι ταῖς ΔE , EZ
ἴσαι εἰσὶν
ἑκατέρω ἑκατέρω·
καὶ γωνίας ἴσας περιέχουσιν·
βάσις ἄρα ἡ A Θ
βάσει τῆ ΔZ
ἴση ἐστίν,
καὶ τὸ AB Θ τρίγωνον
τῷ ΔEZ τριγώνῳ
ἴσον ἐστίν,
καὶ αἱ λοιπαὶ γωνίαι
ταῖς λοιπαῖς γωνίαις
ἴσαι ἔσσονται,
ὅφ' ἃς αἱ ἴσας πλευραὶ
ὑποτείνουσιν·
ἴση ἄρα ἐστίν
ἡ ὑπὸ B Θ A γωνία
τῆ ὑπὸ EZ Δ .
ἀλλὰ ἡ ὑπὸ EZ Δ
τῆ ὑπὸ B Γ A
ἐστὶν ἴση·
τριγώνου δὲ τοῦ A Θ Γ
ἡ ἐκτὸς γωνία ἡ ὑπὸ B Θ A
ἴση ἐστὶ
τῆ ἐντὸς καὶ ἀπεναντίον
τῆ ὑπὸ B Γ A·
ὅπερ ἀδύνατον.
οὐκ ἄρα ἀνίσος ἐστὶν
ἡ B Γ τῆ EZ·
ἴση ἄρα.
ἐστὶ δὲ καὶ
ἡ AB
τῆ ΔE
ἴση.
δύο δὲ αἱ AB, B Γ
δύο ταῖς ΔE , EZ
ἴσαι εἰσὶν
ἑκατέρω ἑκατέρω·
καὶ γωνίας ἴσας
περιέχουσιν·
βάσις ἄρα ἡ A Γ
βάσει τῆ ΔZ
ἴση ἐστίν,
καὶ τὸ AB Γ τρίγωνον
τῷ ΔEZ τριγώνῳ
ἴσον
καὶ λοιπὴ γωνία ἡ ὑπὸ BA Γ
τῆ λοιπῆ γωνίᾳ τῆ ὑπὸ E ΔZ
ἴση.

Ἐὰν ἄρα δύο τρίγωνα
τὰς δύο γωνίας
δυοσι γωνίαις
ἴσας ἔχη
ἑκατέραν ἑκατέρω
καὶ μίαν πλευρὰν
μὴ πλευρᾶ
ἴσην
ἦτοι τὴν πρὸς ταῖς ἴσαις γωνίαις,
ἢ τὴν ὑποτείνουσαν
ὑπὸ μίαν τῶν ἴσων γωνιῶν,

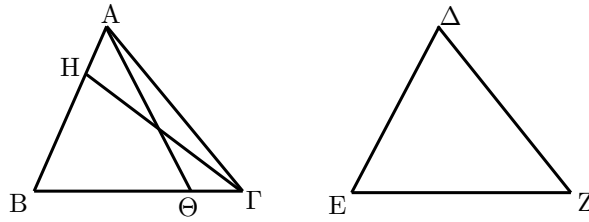
ve AB, ΔE kenarına
AB ve B Θ ikilisi
 ΔE ve EZ ikilisine
eşittirler,
her biri birine;
ama içerirler eşit açılırlı;
dolayısıyla A Θ tabanı
 ΔZ tabanına
eşittir,
ve AB Θ üçgeni
 ΔEZ üçgenine
eşittir,
ve kalan açılar
kalan açılara
eşittirler,
eşit kenarların
karşılıkları.
Dolayısıyla eşittir
B Θ A,
EZ Δ açısına.
Ama EZ Δ ,
B Γ A açısına
eşittir;
o zaman A Θ Γ üçgeninin
B Θ A dış açısı
eşittir
iç ve karşıt
B Γ A açısına;
ki bu imkansızdır.
Dolayısıyla eşit değil değildir,
B Γ , EZ kenarına;
dolayısıyla eşittir.
Ve yine
AB,
 ΔE kenarına,
eşittir.
O zaman AB ve B Γ ikilisi
 ΔE ve EZ ikilisine
eşittirler,
her biri birine;
eşit açılar
içerirler;
dolayısıyla A Γ tabanı
 ΔZ tabanına
eşittir,
ve AB Γ üçgeni
 ΔEZ üçgenine
eşittir,
ve kalan BA Γ açısı
kalan E ΔZ açısına
eşittir.

Eğer, dolayısıyla, iki üçgenin
iki açısı
iki açısına
eşitse,
her biri birine,
ve bir kenar
bir kenara
eşitse,
ya eşit açılırlar arasında olan
ya da karşılayan
eşit açılardan birini;

also the remaining sides
to the remaining sides
they will have equal,
also the remaining angle
to the remaining angle;
—just what it was necessary to show.

καὶ τὰς λοιπὰς πλευρὰς
ταῖς λοιπαῖς πλευραῖς
ἴσας ἔξει
καὶ τὴν λοιπὴν γωνίαν
τῇ λοιπῇ γωνίᾳ·
ὅπερ ἔδει δεῖξαι.

kalan kenarları da
kalan kenarlarına
eşit olacak,
kalan açıları da
kalan açılarna;
— gösterilmesi gereken tam buydu.



3.27

If on two STRAIGHTS
a STRAIGHT falling
the alternate angles
equal to one another
make,
parallel will be to one another
the STRAIGHTS.

For, on the two STRAIGHTS
AB and ΓΔ
[suppose] the STRAIGHT falling,
[namely] EZ,
the alternate angles
AEZ and EZΔ
equal to one another
make.

I say that
parallel is AB to ΓΔ.

For if not,
extended,
AB and ΓΔ will meet,
either in the B-Δ parts,
or in the A-Γ.
Suppose they have been extended,
and let them meet
in the B-Δ parts
at H.
Of the triangle HEZ
the exterior angle AEZ
is equal
to the interior and opposite
EZH;
which is impossible.
Therefore it is not [the case] that
AB and ΓΔ,
extended,
meet in the B-Δ parts.
Similarly it will be shown that
neither on the A-Γ.
Those that in neither parts
meet
are parallel;
therefore, parallel is AB to ΓΔ.

Ἐὰν εἰς δύο εὐθείας
εὐθεῖα ἐμπίπτουσα
τὰς ἐναλλάξ γωνίας
ἴσας ἀλλήλαις
ποιῆ,
παράλληλοι ἔσονται ἀλλήλαις
αἱ εὐθεῖαι.

Εἰς γὰρ δύο εὐθείας
τὰς AB, ΓΔ
εὐθεῖα ἐμπίπτουσα
ἢ EZ
τὰς ἐναλλάξ γωνίας
τὰς ὑπὸ AEZ, EZΔ
ἴσας ἀλλήλαις
ποιεῖται·

λέγω, ὅτι
παράλληλός ἐστιν ἢ AB τῇ ΓΔ.

Εἰ γὰρ μή,
ἐκβαλλόμενα
αἱ AB, ΓΔ συμπεσοῦνται
ἢτοι ἐπὶ τὰ B, Δ μέρη
ἢ ἐπὶ τὰ A, Γ.
ἐκβεβλήσθωσαν
καὶ συμπίπτωσαν
ἐπὶ τὰ B, Δ μέρη
κατὰ τὸ H.
τριγώνου δὲ τοῦ HEZ
ἢ ἐκτὸς γωνία ἢ ὑπὸ AEZ
ἴση ἐστὶ
τῇ ἐντὸς καὶ ἀπεναντίον
τῇ ὑπὸ EZH·
ὅπερ ἐστὶν ἀδύνατον·
οὐκ ἄρα
αἱ AB, ΓΔ
ἐκβαλλόμενα
συμπεσοῦνται ἐπὶ τὰ B, Δ μέρη.
ὅμοιος δὲ δειχθήσεται, ὅτι
οὐδὲ ἐπὶ τὰ A, Γ·
αἱ δὲ ἐπὶ μηδέτερα τὰ μέρη
συμπίπτουσαι
παράλληλοι εἰσιν·
παράλληλος ἄρα ἐστὶν ἢ AB τῇ ΓΔ.

Eğer iki doğru üzerine
düşen bir doğrunun
yaptığı ters açılar
birbirine eşitse
birbirine paralel olacak
doğrular.

Çünkü, iki doğru üzerine,
AB ve ΓΔ,
[kabul edilsin] düşen,
EZ doğrusunun,
ters
AEZ ve EZΔ açılarını
birbirine eşit
oluşturduğunu.

İddia ediyorum ki
paraleldir AB, ΓΔ doğrusuna.

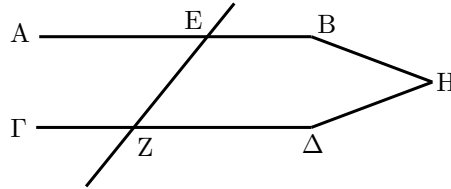
Çünkü eğer değilse,
uzatılmış,
AB ve ΓΔ buluşacaklar,
ya B-Δ parçalarında,
ya da A-Γ parçalarında.
Uzatılmış oldukları kabul edilsin,
ve buluşsunlar
B-Δ parçalarında,
H noktasında.
HEZ üçgeninin
AEZ dış açısı
eşittir
iç ve karşıt
EZH açısına;
ki bu imkansızdır.
Dolayısıyla şöyle değildir (durum)
AB ve ΓΔ,
uzatılmış,
buluşurlar B-Δ parçalarında.
Benzer şekilde gösterilecek ki
A-Γ parçalarında da.
Hiçbir parçada
buluşmayanlar
paraleldir;
dolayısıyla,

If therefore on two STRAIGHTS
a STRAIGHT falling
the alternate angles
equal to one another
make,
parallel will be to one another
the STRAIGHTS;
—just what it was necessary to show.

Ἐάν ἄρα εἰς δύο εὐθείας
εὐθεῖα ἐπίπτουσα
τὰς ἐναλλάξ γωνίας
ἴσας ἀλλήλαις
ποιῆ,
παράλληλοι ἔσονται
αἱ εὐθεῖαι·
ὅπερ ἔδει δεῖξαι.

paraleldir AB, ΓΔ doğrusuna.

Eğer, dolayısıyla, iki doğru üzerine
düşen bir doğrunun
yaptığı ters açılar
birbirine eşitse
birbirine paralel olacak
doğrular;
— gösterilmesi gereken tam buydu.



3.28

If on two STRAIGHTS
a STRAIGHT falling¹
the exterior angle
to the interior and opposite
and in the same parts
make equal,
or the interior and in the same parts
to two RIGHTS
equal,
parallel will be to one another
the STRAIGHTS.

Ἐάν εἰς δύο εὐθείας
εὐθεῖα ἐπίπτουσα
τὴν ἐκτὸς γωνίαν
τῆ ἐντὸς καὶ ἀπεναντίον
καὶ ἐπὶ τὰ αὐτὰ μέρη
ἴσην ποιῆ
ἢ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη
δυσὶν ὀρθαῖς
ἴσας,
παράλληλοι ἔσονται ἀλλήλαις
αἱ εὐθεῖαι.

Eğer iki doğru üzerine
düşen bir doğru,
dış açıyı,
iç ve karşıt
ve aynı tarafta kalan açıya,
eşit yaparsa,
veya iç ve aynı tarafta kalanları,
iki dik açıya
eşit,
birbirine paralel olacak
doğrular.

For, on the two STRAIGHTS AB and
ΓΔ,
the STRAIGHT falling—EZ—
the exterior angle EHB
to the interior and opposite angle
HΘΔ
equal
—suppose it makes,
or the interior and in the same parts,
BΗΘ and HΘΔ,
to two RIGHTS
equal.

Εἰς γὰρ δύο εὐθείας τὰς AB, ΓΔ
εὐθεῖα ἐπίπτουσα ἡ EZ
τὴν ἐκτὸς γωνίαν τὴν ὑπὸ EHB
τῆ ἐντὸς καὶ ἀπεναντίον γωνία
τῆ ὑπὸ HΘΔ
ἴσην
ποιεῖτω
ἢ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη
τὰς ὑπὸ BΗΘ, HΘΔ
δυσὶν ὀρθαῖς
ἴσας·

Çünkü, AB ve ΓΔ doğruları üzerine
düşen EZ doğrusu
EHB dış açısını
iç ve karşıt
HΘΔ açısına
eşit
—yaptığı varsayılımsın,
veya iç ve aynı tarafta kalan,
BΗΘ ve HΘΔ açılarının,
iki dik açıya
equal.

I say that
parallel is
AB to ΓΔ.

λέγω, ὅτι
παράλληλός ἐστιν
ἡ AB τῆ ΓΔ.

İddia ediyorum ki
paraleldir
AB, ΓΔ doğrusuna.

For, since equal is
EHB to HΘΔ,
while EHB to AHΘ
is equal,
therefore also AHΘ to HΘΔ
is equal;
and they are alternate;
parallel therefore is AB to ΓΔ.

Ἐπεὶ γὰρ ἴση ἐστὶν
ἡ ὑπὸ EHB τῆ ὑπὸ HΘΔ,
ἀλλὰ ἡ ὑπὸ EHB τῆ ὑπὸ AHΘ
ἐστὶν ἴση,
καὶ ἡ ὑπὸ AHΘ ἄρα τῆ ὑπὸ HΘΔ
ἐστὶν ἴση·
καὶ εἰσὶν ἐναλλάξ·
παράλληλος ἄρα ἐστὶν ἡ AB τῆ ΓΔ.

Çünkü, eşit olduğundan
EHB, HΘΔ açısına,
aynı zamanda EHB, AHΘ açısına
eşitken,
dolayısıyla AHΘ de HΘΔ açısına
eşittir;
ve terstirler;
paraleldirler dolayısıyla AB ve ΓΔ.

¹It is perhaps impossible to maintain the Greek word order comprehensibly in English. The normal English order would be, 'If a straight line, falling on two straight lines'. But the proposition is

ultimately about the *two* straight lines; perhaps that is why Euclid mentions them before the one straight line that falls on them.

Alternatively, since $BH\theta$ and $H\theta\Delta$ to two RIGHTS are equal, and also are $AH\theta$ and $BH\theta$ to two RIGHTS equal, therefore $AH\theta$ and $BH\theta$ to $BH\theta$ and $H\theta\Delta$ are equal; suppose the common has been taken away — $BH\theta$; therefore the remaining $AH\theta$ to the remaining $H\theta\Delta$ is equal; also they are alternate; parallel therefore are AB and $\Gamma\Delta$.

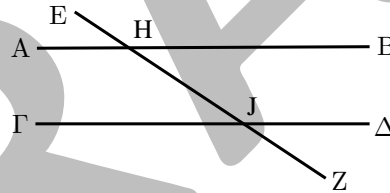
If therefore on two STRAIGHTS a STRAIGHT falling the exterior angle to the interior and opposite and in the same parts make equal, or the interior and in the same parts to two RIGHTS equal, parallel will be to one another the STRAIGHTS; —just what it was necessary to show.

Πάλιν, ἐπεὶ αἱ ὑπὸ $BH\theta$, $H\theta\Delta$ δύο ὀρθαῖς ἴσαι εἰσὶν, εἰσὶ δὲ καὶ αἱ ὑπὸ $AH\theta$, $BH\theta$ δυσὶν ὀρθαῖς ἴσαι, αἱ ἄρα ὑπὸ $AH\theta$, $BH\theta$ ταῖς ὑπὸ $BH\theta$, $H\theta\Delta$ ἴσαι εἰσὶν· κοινὴ ἀφῆρησθῶ ἡ ὑπὸ $BH\theta$ · λοιπὴ ἄρα ἡ ὑπὸ $AH\theta$ λοιπῇ τῇ ὑπὸ $H\theta\Delta$ ἐστὶν ἴση· καὶ εἰσὶν ἐναλλάξ· παράλληλος ἄρα ἐστὶν ἡ AB τῇ $\Gamma\Delta$.

Ἐὰν ἄρα εἰς δύο εὐθείας εὐθεῖα ἐμπίπτουσα τὴν ἔκτος γωνίαν τῇ ἐντὸς καὶ ἀπεναντίον καὶ ἐπὶ τὰ αὐτὰ μέρη ἴσην ποιῇ ἢ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη δυσὶν ὀρθαῖς ἴσας, παράλληλοι ἔσονται αἱ εὐθεῖαι· ὅπερ εἶδει δεῖξαι.

Ya da $BH\theta$ ve $H\theta\Delta$, iki dik açığıya eşittir, ve $AH\theta$ ve $BH\theta$ de iki dik açığıya eşittir, dolayısıyla $AH\theta$ ve $BH\theta$, $BH\theta$ ve $H\theta\Delta$ açıklarına eşittirle; varsayalım çıkartılmış olduğu ortak olan $BH\theta$ açısının; dolayısıyla $AH\theta$ kalanı $H\theta\Delta$ kalanına eşittir ve bunlar terstirler; paraleldir dolayısıyla AB ve $\Gamma\Delta$.

Eğer dolayısıyla iki doğru üzerine düşen bir doğru, dış açığı, iç ve karşıt ve aynı tarafta kalan açığıya, eşit yaparsa, veya iç ve aynı tarafta kalanları, iki dik açığıya eşit, birbirine paralel olacak doğrular; — gösterilmesi gereken tam buydu.



3.29

The STRAIGHT falling on parallel STRAIGHTS the alternate angles makes equal to one another, and the exterior to the interior and opposite equal, and the interior and in the same parts to two RIGHTS equal.

For, on the parallel STRAIGHTS AB and $\Gamma\Delta$ let the STRAIGHT EZ fall.

I say that the alternate angles $AH\theta$ and $H\theta\Delta$ equal it makes, and the exterior angle EHT

Ἡ εἰς τὰς παραλλήλους εὐθείας εὐθεῖα ἐμπίπτουσα τὰς τε ἐναλλάξ γωνίας ἴσας ἀλλήλαις ποιεῖ καὶ τὴν ἔκτος τῇ ἐντὸς καὶ ἀπεναντίον ἴσην καὶ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη δυσὶν ὀρθαῖς ἴσας.

Εἰς γὰρ παραλλήλους εὐθείας τὰς AB , $\Gamma\Delta$ εὐθεῖα ἐμπίπτέτω ἡ EZ .

λέγω, ὅτι τὰς ἐναλλάξ γωνίας τὰς ὑπὸ $AH\theta$, $H\theta\Delta$ ἴσας ποιεῖ καὶ τὴν ἔκτος γωνίαν τὴν ὑπὸ EHT

Paralel doğrular üzerine düşen bir doğru ters açıları birbirine eşit yapar, ve dış açığı iç ve karşıt açığıya eşit, ve iç ve aynı tarafta kalanları iki dik açığıya eşit.

Çünkü, paralel AB ve $\Gamma\Delta$ doğruları üzerine EZ doğrusu düşsün.

İddia ediyorum ki ters $AH\theta$ ve $H\theta\Delta$ açılarını eşit yapar, ve EHT dış açısını

to the interior and opposite $H\Theta\Delta$ equal,
and the interior and in the same parts $BH\Theta$ and $H\Theta\Delta$ to two RIGHTS equal.

For, if it is unequal,
 $AH\Theta$ to $H\Theta\Delta$,
one of them is greater.
Let the greater be $AH\Theta$;
let be added in common $BH\Theta$;
therefore $AH\Theta$ and $BH\Theta$ than $BH\Theta$ and $H\Theta\Delta$ are greater.
However, $AH\Theta$ and $BH\Theta$ to two RIGHTS equal are.
Therefore [also] $BH\Theta$ and $H\Theta\Delta$ than two RIGHTS less are.
And [STRAIGHTS] from [angles] that are less than two RIGHTS, extended to the infinite, fall together.
Therefore AB and $\Gamma\Delta$, extended to the infinite, will fall together.
But they do not fall together, by their being assumed parallel.
Therefore is not unequal $AH\Theta$ to $H\Theta\Delta$.
Therefore it is equal.
However, $AH\Theta$ to EHB is equal;
therefore also EHB to $H\Theta\Delta$ is equal;
let $BH\Theta$ be added in common;
therefore EHB and $BH\Theta$ to $BH\Theta$ and $H\Theta\Delta$ is equal.
But EHB and $BH\Theta$ to two RIGHTS are equal.
Therefore also $BH\Theta$ and $H\Theta\Delta$ to two RIGHTS are equal.
Therefore the on-parallel-STRAIGHTS STRAIGHT falling the alternate angles makes equal to one another, and the exterior to the interior and opposite equal,
and the interior and in the same parts to two RIGHTS equal;
—just what it was necessary to show.

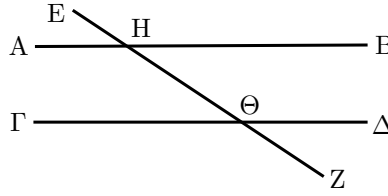
τῆ ἐντὸς καὶ ἀπεναντίον τῆ ὑπὸ $H\Theta\Delta$ ἴσην
καὶ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη τὰς ὑπὸ $BH\Theta$, $H\Theta\Delta$ δυσὶν ὀρθαῖς ἴσας.

Εἰ γὰρ ἀνίσος ἐστὶν ἢ ὑπὸ $AH\Theta$ τῆ ὑπὸ $H\Theta\Delta$, μία αὐτῶν μείζων ἐστίν. ἔστω μείζων ἢ ὑπὸ $AH\Theta$ κοινῇ προσκείσθω ἢ ὑπὸ $BH\Theta$ · αἱ ἄρα ὑπὸ $AH\Theta$, $BH\Theta$ τῶν ὑπὸ $BH\Theta$, $H\Theta\Delta$ μείζονές εἰσιν. ἀλλὰ αἱ ὑπὸ $AH\Theta$, $BH\Theta$ δυσὶν ὀρθαῖς ἴσαι εἰσίν.
[καὶ] αἱ ἄρα ὑπὸ $BH\Theta$, $H\Theta\Delta$ δύο ὀρθῶν ἐλάσσονές εἰσιν.
αἱ δὲ ἀπ' ἐλασσόνων ἢ δύο ὀρθῶν ἐκβαλλόμεναι εἰς ἄπειρον συμπίπτουσιν· αἱ ἄρα AB , $\Gamma\Delta$ ἐκβαλλόμεναι εἰς ἄπειρον συμπεσοῦνται· οὐ συμπίπτουσι δὲ διὰ τὸ παραλλήλους αὐτὰς ὑποκείσθαι· οὐκ ἄρα ἀνίσος ἐστὶν ἢ ὑπὸ $AH\Theta$ τῆ ὑπὸ $H\Theta\Delta$ · ἴση ἄρα.
ἀλλὰ ἢ ὑπὸ $AH\Theta$ τῆ ὑπὸ EHB ἐστὶν ἴση· καὶ ἢ ὑπὸ EHB ἄρα τῆ ὑπὸ $H\Theta\Delta$ ἐστὶν ἴση· κοινῇ προσκείσθω ἢ ὑπὸ $BH\Theta$ · αἱ ἄρα ὑπὸ EHB , $BH\Theta$ ταῖς ὑπὸ $BH\Theta$, $H\Theta\Delta$ ἴσαι εἰσίν.
ἀλλὰ αἱ ὑπὸ EHB , $BH\Theta$ δύο ὀρθαῖς ἴσαι εἰσίν· καὶ αἱ ὑπὸ $BH\Theta$, $H\Theta\Delta$ ἄρα δύο ὀρθαῖς ἴσαι εἰσίν.

Ἦ ἄρα εἰς τὰς παραλλήλους εὐθείας εὐ-
θεῖα ἐπίπτουσα τὰς τε ἐναλλάξ γωνίας ἴσας ἀλλήλαις ποιεῖ καὶ τὴν ἐκτὸς τῆ ἐντὸς καὶ ἀπεναντίον ἴσην καὶ τὰς ἐντὸς καὶ ἐπὶ τὰ αὐτὰ μέρη δυσὶν ὀρθαῖς ἴσας· ὅπερ ἔδει δεῖξαι.

iç ve karşıt $H\Theta\Delta$ açısına eşit,
ve iç ve aynı taraftaki $BH\Theta$ ile $H\Theta\Delta$ açılarını iki dik açıya eşit.

Çünkü, eğer eşit değilse $AH\Theta$, $H\Theta\Delta$ açısına, biri büyüktür. Büyük olan $AH\Theta$ olsun; eklenmiş olsun her ikisine de $BH\Theta$; dolayısıyla $AH\Theta$ ve $BH\Theta$, $BH\Theta$ ve $H\Theta\Delta$ açılarından büyüktürler. Fakat, $AH\Theta$ ve $BH\Theta$ iki dik açıya eşittirler. Dolayısıyla $BH\Theta$ ve $H\Theta\Delta$ [da] iki dik açıdan küçüktürler. Ve küçük olanlardan, iki dik açıdan, sonsuza uzatılanlar [doğrular], birbirinin üzerine düşerler. Dolayısıyla AB ve $\Gamma\Delta$, uzatılınca sonsuza, birbirinin üzerine düşecekler. Ama onlar birbirinin üzerine düşmezler, paralel oldukları kabul edildiğinden. Dolayısıyla eşit değil değildir $AH\Theta$, $H\Theta\Delta$ açısına. Dolayısıyla eşittir. Ancak, $AH\Theta$, EHB açısına eşittir; dolayısıyla EHB da $H\Theta\Delta$ açısına eşittir; eklenmiş olsun her ikisine de $BH\Theta$; dolayısıyla EHB ve $BH\Theta$, $BH\Theta$ ve $H\Theta\Delta$ açılarda eşittir. Ama EHB ve $BH\Theta$ iki dik açıya eşittirler. Dolayısıyla $BH\Theta$ ve $H\Theta\Delta$ da iki dik açıya eşittirler. Dolayısıyla paralel doğrular üzerine, doğru düşerken ters açılarda eşit yapar birbirine, ve dış açıyı iç ve karşıta eşit, ve iç ve aynı taraftakileri s iki dik açıya eşit; — gösterilmesi gereken tam buydu.



3.30

[STRAIGHTS] to the same STRAIGHT parallel also to one another are parallel.

Let be either of AB and ΓΔ to ΓΔ parallel.

I say that also AB to ΓΔ is parallel.

For let fall on them a STRAIGHT, HK.

Then, since on the parallel STRAIGHTS AB and EZ a STRAIGHT has fallen, [namely] HK, equal therefore is AHK to HΘZ. Moreover, since on the parallel STRAIGHTS EZ and ΓΔ a STRAIGHT has fallen, [namely] HK, equal is HΘZ to HKΔ. And it was shown also that AHK to HΘZ is equal. Also AHK therefore to HKΔ is equal; and they are alternate. Parallel therefore is AB to ΓΔ.

Therefore [STRAIGHTS] to the same STRAIGHT parallel also to one another are parallel; —just what it was necessary to show.

Αἰ τῆ αὐτῆ εὐθείᾳ παράλληλοι καὶ ἀλλήλαις εἰσι παράλληλοι.

Ἐστω ἑκατέρα τῶν AB, ΓΔ τῆ EZ παράλληλος·

λέγω, ὅτι καὶ ἡ AB τῆ ΓΔ ἐστὶ παράλληλος.

Ἐμπιπέτω γὰρ εἰς αὐτὰς εὐθεῖα ἡ HK.

Καὶ ἐπεὶ εἰς παραλλήλους εὐθείας τὰς AB, EZ εὐθεῖα ἐμπέπτωκεν ἡ HK, ἴση ἄρα ἡ ὑπὸ AHK τῆ ὑπὸ HΘZ· πάλιν, ἐπεὶ εἰς παραλλήλους εὐθείας τὰς EZ, ΓΔ εὐθεῖα ἐμπέπτωκεν ἡ HK, ἴση ἐστὶν ἡ ὑπὸ HΘZ τῆ ὑπὸ HKΔ· ἐδείχθη δὲ καὶ ἡ ὑπὸ AHK τῆ ὑπὸ HΘZ ἴση· καὶ ἡ ὑπὸ AHK ἄρα τῆ ὑπὸ HKΔ ἐστὶν ἴση· καὶ εἰσὶν ἐναλλάξ· παράλληλος ἄρα ἐστὶν ἡ AB τῆ ΓΔ.

[Αἰ ἄρα τῆ αὐτῆ εὐθείᾳ παράλληλοι καὶ ἀλλήλαις εἰσι παράλληλοι·] ὅπερ ἔδει δεῖξαι.

Aynı doğruya paralel doğrular birbirlerine de paraleldir.

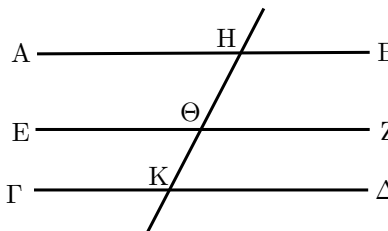
Olsun AB ve ΓΔ doğrularının her biri, ΓΔ doğrusuna paralel.

İddia ediyorum ki AB da ΓΔ doğrusuna paraleldir.

Çünkü üzerlerine bir HK doğrusu düşmüş olsun.

O zaman, paralel AB ve EZ doğrularının üzerine bir doğru düşmüş olduğundan, [yani] HK, eşittir dolayısıyla AHK, HΘZ açısına. Dahası, paralel EZ ve ΓΔ doğrularının üzerine bir doğru düşmüş olduğundan, [yani] HK, eşittir HΘZ, HKΔ açısına. Ve gösterilmişti ki AHK, HΘZ açısına eşittir. VE AHK dolayısıyla HKΔ açısına eşittir; ve bunlar terstirler. Paraleldir dolayısıyla AB, ΓΔ doğrusuna.

Dolayısıyla aynı doğruya paraleller birbirlerine de paraleldir; — gösterilmesi gereken tam buydu.



3.31

Through the given point to the given STRAIGHT parallel

Διὰ τοῦ δοθέντος σημείου τῆ δοθείσης εὐθείᾳ παράλληλον

Verilen bir noktadan verilen bir doğruya paralel

a straight line to draw.

Let be
the given point A,
and the given STRAIGHT BΓ.

It is necessary then
through the point A
to the STRAIGHT BΓ parallel
a straight line to draw.

Suppose there has been chosen
on BΓ
a random point Δ,
and there has been joined AΔ,
and there has been constructed,
on the STRAIGHT ΔA,
and at the point A of it,
to the angle AΔΓ equal,
ΔAE;
and suppose there has been extended,
in STRAIGHTS with EA,
the STRAIGHT AZ.

And because
on the two STRAIGHTS BΓ and EZ
the straight line falling, AΔ,
the alternate angles
EAD and AΔΓ
equal to one another has made,
parallel therefore is EAZ to BΓ.

Therefore, through the given point A,
to the given STRAIGHT BΓ parallel,
a straight line has been drawn, EAZ;
—just what it was necessary to do.

εὐθεῖαν γραμμὴν ἀγαγεῖν.

Ἐστω
τὸ μὲν δοθὲν σημεῖον τὸ Α,
ἢ δὲ δοθεῖσα εὐθεῖα ἢ BΓ.

δεῖ δὴ
διὰ τοῦ Α σημείου
τῇ BΓ εὐθείᾳ παράλληλον
εὐθεῖαν γραμμὴν ἀγαγεῖν.

Εἰλήφθω
ἐπὶ τῆς BΓ
τυχὸν σημεῖον τὸ Δ,
καὶ ἐπεζεύχθω ἢ AΔ.
καὶ συνεστάτω
πρὸς τῇ ΔΑ εὐθείᾳ
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ Α
τῇ ὑπὸ AΔΓ γωνίᾳ ἴση
ἢ ὑπὸ ΔΑΕ.
καὶ ἐκβεβλήσθω
ἐπ' εὐθείας τῇ EA
εὐθεῖα ἢ AZ.

Καὶ ἐπεὶ
εἰς δύο εὐθείας τὰς BΓ, EZ
εὐθεῖα ἐμπέπτουσα ἢ AΔ
τὰς ἐναλλάξ γωνίας
τὰς ὑπὸ EAD, AΔΓ
ἴσας ἀλλήλαις πεποίηκεν,
παράλληλος ἄρα ἐστὶν ἢ EAZ τῇ BΓ.

Διὰ τοῦ δοθέντος ἄρα σημείου τοῦ Α
τῇ δοθείσῃ εὐθείᾳ τῇ BΓ παράλληλος
εὐθεῖα γραμμὴ ἤχεται ἢ EAZ.
ὅπερ ἔδει ποιῆσαι.

bir doğru çizmek.

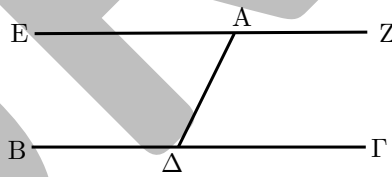
Olsun
verilen nokta A,
ve verilen doğru BΓ.

Şimdi gereklidir
A noktasından
BΓ doğrusuna paralel
bir doğru çizmek.

Varsayalım seçilmiş olduğu
BΓ üzerinde
rastgele bir Δ noktasının,
ve AΔ doğrusunun birleştirilmiş
olduğu,
ve inşa edilmiş olduğu,
ΔA doğrusunda,
ve onun A noktasında,
AΔΓ açısına eşit,
ΔAE açısının;
ve kabul edilsin uzatılmış olsun,
EA ile aynı doğruda,
AZ doğrusu.

Ve çünkü
BΓ ve EZ doğruları üzerine
düşerken AΔ doğrusu,
ters
EAD ve AΔΓ açılarını
eşit yapmıştır birbirine,
paraleldir dolayısıyla EAZ, BΓ
doğrusuna.

Dolayısıyla, verilen A noktasından,
verilen BΓ doğrusuna paralel,
bir doğru EAZ, çizilmiş oldu;
— yapılması gereken tam buydu.



3.32

Of any triangle
one of the sides being extended,
the exterior angle
to the two opposite interior angles
is equal,
and the triangle's three interior angles
to two RIGHTS equal are.

Παντὸς τριγώνου
μᾶς τῶν πλευρῶν προσεκβληθείσης
ἢ ἐκτὸς γωνία
δυσὶ ταῖς ἐντὸς καὶ ἀπεναντίον
ἴση ἐστίν,
καὶ αἱ ἐντὸς τοῦ τριγώνου τρεῖς γωνίαι
δυσὶν ὀρθαῖς ἴσαι εἰσίν.

Herhangi bir üçgenin
kenarlarından biri uzatıldığında,
dış açı
iki karşıt iç açiya
eşittir,
ve üçgenin üç iç açısı
iki dik açiya eşittir.

Let there be
the triangle $AB\Gamma$,
and suppose there has been extended
its one side, $B\Gamma$, to Δ ;

I say that
the exterior angle $A\Gamma$ is equal
to the two interior and opposite angles
 ΓAB and $AB\Gamma$,
and the triangle's three interior angles
 $AB\Gamma$, $B\Gamma A$, and ΓAB
to two RIGHTS equal are.

For, suppose there has been drawn
through the point Γ
to the STRAIGHT AB parallel
 ΓE .

And since parallel is AB to ΓE ,
and on these has fallen $A\Gamma$,
the alternate angles BAG and AGE
equal to one another are.
Moreover, since parallel is
 AB to ΓE ,
and on these has fallen
the STRAIGHT $B\Delta$,
the exterior angle $E\Gamma\Delta$ is equal
to the interior and opposite $AB\Gamma$.
And it was shown that
also AGE to BAG [is] equal.
Therefore the whole angle $A\Gamma\Delta$
is equal
to the two interior and opposite angles
 BAG and $AB\Gamma$.

Let be added in common AGB ;
Therefore $A\Gamma\Delta$ and AGB
to the three $AB\Gamma$, $B\Gamma A$, and ΓAB
equal are.
However, $A\Gamma\Delta$ and AGB
to two RIGHTS equal are;
also AGB , ΓBA , and ΓAB therefore
to two RIGHTS equal are.

Therefore, of any triangle
one of the sides being extended,
the exterior angle
to the two opposite interior angles
is equal,
and the triangle's three interior angles
to two RIGHTS equal are;
—just what it was necessary to show.

Ἐστω
τρίγωνον τὸ $AB\Gamma$,
καὶ προσεξεβλήσθω
αὐτοῦ μία πλευρὰ ἢ $B\Gamma$ ἐπὶ τὸ Δ .

λέγω, ὅτι
ἢ ἔκτος γωνία ἢ ὑπὸ $A\Gamma\Delta$ ἴση ἐστὶ
δυοῖς ταῖς ἐντὸς καὶ ἀπεναντίον
ταῖς ὑπὸ ΓAB , $AB\Gamma$,
καὶ αἱ ἐντὸς τοῦ τριγώνου τρεῖς γωνίαι
αἱ ὑπὸ $AB\Gamma$, $B\Gamma A$, ΓAB
δυοῖν ὀρθαῖς ἴσαι εἰσίν.

Ἦχθω γὰρ
διὰ τοῦ Γ σημείου
τῆ AB εὐθείᾳ παράλληλος
ἢ ΓE .

Καὶ ἐπεὶ παράλληλος ἐστὶν ἢ AB τῆ ΓE ,
καὶ εἰς αὐτὰς ἐμπέπτωκεν ἢ $A\Gamma$,
αἱ ἐναλλὰξ γωνίαι αἱ ὑπὸ BAG , AGE
ἴσαι ἀλλήλαις εἰσίν.
πάλιν, ἐπεὶ παράλληλος ἐστὶν
ἢ AB τῆ ΓE ,
καὶ εἰς αὐτὰς ἐμπέπτωκεν
εὐθεῖα ἢ $B\Delta$,
ἢ ἔκτος γωνία ἢ ὑπὸ $E\Gamma\Delta$ ἴση ἐστὶ
τῆ ἐντὸς καὶ ἀπεναντίον τῆ ὑπὸ $AB\Gamma$.
ἐδείχθη δὲ καὶ ἢ ὑπὸ AGE τῆ ὑπὸ BAG
ἴση.
ὅλη ἄρα ἢ ὑπὸ $A\Gamma\Delta$ γωνία
ἴση ἐστὶ
δυοῖς ταῖς ἐντὸς καὶ ἀπεναντίον
ταῖς ὑπὸ BAG , $AB\Gamma$.

Κοινὴ προσκείσθω ἢ ὑπὸ AGB .
αἱ ἄρα ὑπὸ $A\Gamma\Delta$, AGB
τρισοῖς ταῖς ὑπὸ $AB\Gamma$, $B\Gamma A$, ΓAB
ἴσαι εἰσίν.
ἀλλ' αἱ ὑπὸ $A\Gamma\Delta$, AGB
δυοῖν ὀρθαῖς ἴσαι εἰσίν.
καὶ αἱ ὑπὸ AGB , ΓBA , ΓAB ἄρα
δυοῖν ὀρθαῖς ἴσαι εἰσίν.

Παντὸς ἄρα τριγώνου
μιας τῶν πλευρῶν προσεξεβληθείσης
ἢ ἔκτος γωνία
δυοῖς ταῖς ἐντὸς καὶ ἀπεναντίον
ἴση ἐστίν,
καὶ αἱ ἐντὸς τοῦ τριγώνου τρεῖς γωνίαι
δυοῖν ὀρθαῖς ἴσαι εἰσίν.
ὅπερ ἔδει δεῖξαι.

Verilmiş olsun
 $AB\Gamma$ üçgeni,
ve varsayalım uzatılmış olduğu
bir $B\Gamma$ kenarının Δ noktasına.

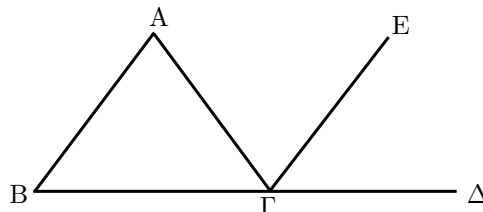
İddia ediyorum ki
 $A\Gamma\Delta$ dış açısı eşittir
iki iç ve karşıt
 ΓAB ve $AB\Gamma$ açısına,
ve üçgenin üç iç açısı
 $AB\Gamma$, $B\Gamma A$ ve ΓAB ,
iki dik açıya eşittir.

Çünkü, varsayalım çizilmiş olduğu
 Γ noktasından
 AB doğrusuna paralel
 ΓE doğrusunun.

Ve paralel olduğundan AB , ΓE
doğrusuna,
ve bunların üzerine düştüğünden $A\Gamma$,
ters BAG ve AGE açıları
eşittirler birbirlerine.
Dahası, paralel olduğundan
 AB , ΓE doğrusuna,
and bunların üzerine düştüğünden
 $B\Delta$ doğrusu,
 $E\Gamma\Delta$ dış açısı eşittir
iç ve karşıt $AB\Gamma$ açısına.
Ve gösterilmişti ki
 AGE da BAG açısına eşittir.
Dolayısıyla açının tamamı $A\Gamma\Delta$
eşittir
iç ve karşıt
 BAG ve $AB\Gamma$ açılara.

Eklenmiş olsun AGB ortak olarak;
Dolayısıyla $A\Gamma\Delta$ ve AGB açıları
 $AB\Gamma$, $B\Gamma A$ ve ΓAB üçlüsüne
eşittir.
Fakat, $A\Gamma\Delta$ ve AGB açıları
iki dik açıya eşittir;
 AGB , ΓBA ve ΓAB da dolayısıyla
iki dik açıya eşittir.

Dolayısıyla, herhangi bir üçgenin
kenarlarından biri uzatıldığında,
dış açı
iki karşıt iç açıya
eşittir,
ve üçgenin üç iç açısı
iki dik açıya eşittir;
— gösterilmesi gereken tam buydu.



3.33

STRAIGHTS joining equals and parallels to the same parts also themselves equal and parallel are.

Let be equals and parallels AB and ΓΔ, and let join these in the same parts STRAIGHTS ΑΓ and ΒΔ.

I say that also ΑΓ and ΒΔ equal and parallel are.

Suppose there has been joined ΒΓ. And since parallel is AB to ΓΔ, and on these has fallen ΒΓ, the alternate angles ΑΒΓ and ΒΓΔ equal to one another are. And since equal is AB to ΓΔ, and common [is] ΒΓ, then the two AB and ΒΓ to the two ΒΓ and ΓΔ equal are; also angle ΑΒΓ to angle ΒΓΔ [is] equal; therefore the base ΑΓ to the base ΒΔ is equal, and the triangle ΑΒΓ to the triangle ΒΓΔ is equal, and the remaining angles to the remaining angles equal will be, either to either, which the equal sides subtend; equal therefore the ΑΓΒ angle to ΓΒΔ. And since on the two STRAIGHTS ΑΓ and ΒΔ the STRAIGHT falling—ΒΓ—alternate angles equal to one another has made, parallel therefore is ΑΓ to ΒΔ. And it was shown to it also equal.

Therefore STRAIGHTS joining equals and parallels to the same parts also themselves equal and parallel are.—just what it was necessary to show.

Αἱ τὰς ἴσας τε καὶ παραλλήλους ἐπὶ τὰ αὐτὰ μέρη ἐπιζευγνύουσαι εὐθεῖαι καὶ αὐταὶ ἴσαι τε καὶ παράλληλοι εἰσιν.

Ἐστῶσαν ἴσαι τε καὶ παράλληλοι αἱ ΑΒ, ΓΔ, καὶ ἐπιζευγνύτωσαν αὐτὰς ἐπὶ τὰ αὐτὰ μέρη εὐθεῖαι αἱ ΑΓ, ΒΔ.

λέγω, ὅτι καὶ αἱ ΑΓ, ΒΔ ἴσαι τε καὶ παράλληλοι εἰσιν.

Ἐπεζεύχθω ἡ ΒΓ. καὶ ἐπεὶ παράλληλός ἐστιν ἡ ΑΒ τῇ ΓΔ, καὶ εἰς αὐτὰς ἐμπίπτωκεν ἡ ΒΓ, αἱ ἐναλλάξ γωνίαι αἱ ὑπὸ ΑΒΓ, ΒΓΔ ἴσαι ἀλλήλαις εἰσίν. καὶ ἐπεὶ ἴση ἐστιν ἡ ΑΒ τῇ ΓΔ κοινὴ δὲ ἡ ΒΓ, δύο δὴ αἱ ΑΒ, ΒΓ δύο ταῖς ΒΓ, ΓΔ ἴσαι εἰσίν· καὶ γωνία ἡ ὑπὸ ΑΒΓ γωνία τῇ ὑπὸ ΒΓΔ ἴση· βάσις ἄρα ἡ ΑΓ βάσει τῇ ΒΔ ἐστὶν ἴση, καὶ τὸ ΑΒΓ τρίγωνον τῷ ΒΓΔ τριγώνῳ ἴσον ἐστίν, καὶ αἱ λοιπαὶ γωνίαι ταῖς λοιπαῖς γωνίαις ἴσαι ἔσονται ἑκατέρα ἑκατέρᾳ, ὅφ' ἂς αἱ ἴσαι πλευραὶ ὑποτείνουσιν· ἴση ἄρα ἡ ὑπὸ ΑΓΒ γωνία τῇ ὑπὸ ΓΒΔ. καὶ ἐπεὶ εἰς δύο εὐθείας τὰς ΑΓ, ΒΔ εὐθεῖα ἐμπίπτουσα ἡ ΒΓ τὰς ἐναλλάξ γωνίας ἴσας ἀλλήλαις πεποίηκεν, παράλληλος ἄρα ἐστὶν ἡ ΑΓ τῇ ΒΔ. ἐδείχθη δὲ αὐτῇ καὶ ἴση.

Αἱ ἄρα τὰς ἴσας τε καὶ παραλλήλους ἐπὶ τὰ αὐτὰ μέρη ἐπιζευγνύουσαι εὐθεῖαι καὶ αὐταὶ ἴσαι τε καὶ παράλληλοι εἰσιν· ὅπερ ἔδει δεῖξαι.

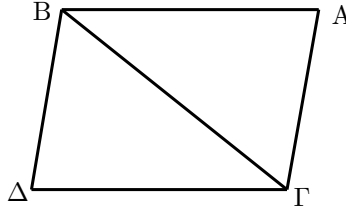
Eἶσι ve paralellerin aynı taraflarını birleştiren doğruların kendileri de eşit ve paraleldirler.

Olsun eşit ve paraleller AB ve ΓΔ, ve bunların birleştiresin aynı taraflarını ΑΓ ve ΒΔ doğruları.

İddia ediyorum ki ΑΓ ve ΒΔ da eşit ve paraleldirler.

Varsayalım birleştirilmiş olduğu ΒΓ doğrusunun. Ve paralel olduğundan AB, ΓΔ doğrusuna, ve bunların üzerine düştüğünden ΒΓ, ters ΑΒΓ ve ΒΓΔ açıları birbirlerine eşittirler. Ve eşit olduğundan AB, ΓΔ doğrusuna, ve ΒΓ ortak, AB ve ΒΓ ikilisi ΒΓ ve ΓΔ ikilisine eşittir; ΑΒΓ açısı da ΒΓΔ açısına eşittir; dolayısıyla ΑΓ tabanı ΒΔ tabanına eşittir, ve ΑΒΓ üçgeni ΒΓΔ üçgenine eşittir, ve kalan açılar kalan açılara eşit olacaklar, her biri birine, eşit kenarları görenler; eşittir dolayısıyla ΑΓΒ, ΓΒΔ açısına. Ve üzerine iki ΑΓ ve ΒΔ doğrularının, düşen doğru—ΒΓ—birbirine eşit ters açılar yapmıştır, paraleldir dolayısıyla ΑΓ, ΒΔ doğrusuna. Ve eşit olduğu da gösterilmişti.

Dolayısıyla eşit ve paralellerin aynı taraflarını birleştiren doğruların kendileri de eşit ve paraleldirler; — gösterilmesi gereken tam buydu.



3·34

Of parallelogram areas,
opposite sides and angles
are equal to one another,
and the diameter cuts them in two.

Let there be
a parallelogram area
ΑΓΔΒ;
a diameter of it, ΒΓ.

I say that
of the ΑΓΔΒ parallelogram
the opposite sides and angles
equal to one another are,
and the ΒΓ diameter it cuts in two.

For, since parallel is
ΑΒ to ΓΔ,
and on these has fallen
a STRAIGHT, ΒΓ,
the alternate angles ΑΒΓ and ΒΓΔ
equal to one another are.
Moreover, since parallel is
ΑΓ to ΒΔ,
and on these has fallen
ΒΓ,
the alternate angles ΑΓΒ and ΓΒΔ
equal to one another are.
Then two triangles there are,
ΑΒΓ and ΒΓΑ,
the two angles ΑΒΓ and ΒΓΑ
to the two ΒΓΔ and ΓΒΔ
equal having,
either to either,
and one side to one side equal,
that near the equal angles,
their common ΒΓ;
also then the remaining sides
to the remaining sides
equal they will have,
either to either,
and the remaining angle
to the remaining angle;
equal, therefore,
the ΑΒ side to ΓΔ,
and ΑΓ to ΒΔ,
and yet equal is the ΒΑΓ angle
to ΓΔΒ.
And since equal is the ΑΒΓ angle
to ΒΓΔ,
and ΓΒΔ to ΑΓΒ,
therefore the whole ΑΒΔ

Τῶν παραλληλογράμμων χωρίων
αἱ ἀπεναντίον πλευραὶ τε καὶ γωνίαι
ἴσαι ἀλλήλαις εἰσίν,
καὶ ἡ διάμετρος αὐτὰ δίχα τέμνει.

Ἐστω
παραλληλόγραμμον χωρίον
τὸ ΑΓΔΒ,
διάμετρος δὲ αὐτοῦ ἡ ΒΓ·

λέγω, ὅτι
τοῦ ΑΓΔΒ παραλληλογράμμου
αἱ ἀπεναντίον πλευραὶ τε καὶ γωνίαι
ἴσαι ἀλλήλαις εἰσίν,
καὶ ἡ ΒΓ διάμετρος αὐτὸ δίχα τέμνει.

Ἐπεὶ γὰρ παράλληλός ἐστιν
ἡ ΑΒ τῇ ΓΔ,
καὶ εἰς αὐτὰς ἐμπέπτωκεν
εὐθεῖα ἡ ΒΓ,
αἱ ἐναλλὰξ γωνίαι αἱ ὑπὸ ΑΒΓ, ΒΓΔ
ἴσαι ἀλλήλαις εἰσίν.
πάλιν ἐπεὶ παράλληλός ἐστιν
ἡ ΑΓ τῇ ΒΔ,
καὶ εἰς αὐτὰς ἐμπέπτωκεν
ἡ ΒΓ,
αἱ ἐναλλὰξ γωνίαι αἱ ὑπὸ ΑΓΒ, ΓΒΔ
ἴσαι ἀλλήλαις εἰσίν.
δύο δὲ τρίγωνά ἐστι
τὰ ΑΒΓ, ΒΓΑ
τὰς δύο γωνίας τὰς ὑπὸ ΑΒΓ, ΒΓΑ
δυσὶ ταῖς ὑπὸ ΒΓΔ, ΓΒΔ
ἴσας ἔχοντα
ἐκατέραν ἐκατέρα
καὶ μίαν πλευρὰν μὲν πλευρᾷ ἴσην
τὴν πρὸς ταῖς ἴσαις γωνίαις
κοινήν αὐτῶν τὴν ΒΓ·
καὶ τὰς λοιπὰς ἄρα πλευρὰς
ταῖς λοιπαῖς
ἴσας ἔξει
ἐκατέραν ἐκατέρα
καὶ τὴν λοιπὴν γωνίαν
τῇ λοιπῇ γωνίᾳ·
ἴση ἄρα
ἡ μὲν ΑΒ πλευρὰ τῇ ΓΔ,
ἡ δὲ ΑΓ τῇ ΒΔ,
καὶ ἔτι ἴση ἐστὶν ἡ ὑπὸ ΒΑΓ γωνία
τῇ ὑπὸ ΓΔΒ.
καὶ ἐπεὶ ἴση ἐστὶν ἡ μὲν ὑπὸ ΑΒΓ γωνία
τῇ ὑπὸ ΒΓΔ,
ἡ δὲ ὑπὸ ΓΒΔ τῇ ὑπὸ ΑΓΒ,
ὅλη ἄρα ἡ ὑπὸ ΑΒΔ

Paralelkenar alanların,
karşit kenar ve açıları
eşittir birbirine,
ve köşegen onları ikiye böler.

Verilmiş olsun
bir paralelkenar alan
ΑΓΔΒ;
ve onun bir köşegeni, ΒΓ.

iddia ediyorum ki
ΑΓΔΒ paralelkenarının
karşit kenar ve açıları
eşittir birbirine,
ve ΒΓ köşegeni onu ikiye böler.

Çünkü, paralel olduğundan
ΑΒ, ΓΔ doğrusuna,
bunların üzerine düştüğünden
bir ΒΓ doğrusu,
ters ΑΒΓ ve ΒΓΔ açıları
eşittir birbirlerine.
Dahası, paralel olduğundan
ΑΓ, ΒΔ doğrusuna,
ve bunların üzerine düştüğünden
ΒΓ,
ters açılar ΑΓΒ ve ΓΒΔ
eşittir birbirlerine..
Şimdi iki üçgen vardır;
ΑΒΓ ve ΒΓΑ,
iki ΑΒΓ ve ΒΓΑ açıları
iki ΒΓΔ ve ΓΒΔ açılara
eşit olan,
her biri birine,
ve bir kenarı, bir kenarına eşit olan,
eşit açılımlar yanında olan,
onların ortak ΒΓ kenarı;
o zaman kalan kenarları da
kalan kenarlarına
eşit olacaklar ,
her biri birine,
ve kalan açı
kalan açiya;
eşit, dolayısıyla,
ΑΒ kenarı ΓΔ kenarına,
ve ΑΓ, ΒΔ kenarına,
ve eşittir ΒΑΓ açısı
ΓΔΒ açısına.
Ve eşit olduğundan ΑΒΓ,
ΒΓΔ açısına,
ve ΓΒΔ, ΑΓΒ açısına,
dolayısıyla açının tamamı ΑΒΔ,

to the whole $AG\Delta$
is equal.
And was shown also
 $BA\Gamma$ to $\Gamma\Delta B$ equal.

Therefore, of parallelogram areas,
opposite sides and angles
equal to one another are.

I say then that
also the diameter them cuts in two.

For, since equal is AB to $\Gamma\Delta$,
and common [is] $B\Gamma$,
the two AB and $B\Gamma$
to the two $\Gamma\Delta$ and $B\Gamma$
equal are,
either to either;
and angle $AB\Gamma$
to angle $B\Gamma\Delta$
equal.
Therefore also the base AG
to the base ΔB
equal.
Therefore also the $AB\Gamma$ triangle
to the $B\Gamma\Delta$ triangle
is equal.

Therefore the $B\Gamma$ diameter cuts in two
the $AB\Gamma\Delta$ parallelogram;
—just what it was necessary to show.

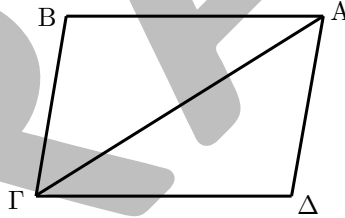
ὅλη τῆ ὑπὸ $AG\Delta$
ἐστὶν ἴση.
ἐδείχθη δὲ καὶ
ἡ ὑπὸ $BA\Gamma$ τῆ ὑπὸ $\Gamma\Delta B$ ἴση.

Τῶν ἄρα παραλληλογράμων χωρίων
αἱ ἀπεναντίον πλευραὶ τε καὶ γωνίαι
ἴσαι ἀλλήλαις εἰσίν.

Λέγω δὴ, ὅτι
καὶ ἡ διάμετρος αὐτὰ διχα τέμνει.

ἐπεὶ γὰρ ἴση ἐστὶν ἡ AB τῆ $\Gamma\Delta$,
κοινὴ δὲ ἡ $B\Gamma$,
δύο δὴ αἱ AB , $B\Gamma$
δυοὶ ταῖς $\Gamma\Delta$, $B\Gamma$
ἴσαι εἰσίν
ἐκατέρα ἐκατέρα·
καὶ γωνία ἡ ὑπὸ $AB\Gamma$
γωνία τῆ ὑπὸ $B\Gamma\Delta$
ἴση.
καὶ βάσις ἄρα ἡ AG
τῆ ΔB
ἴση.
καὶ τὸ $AB\Gamma$ [ἄρα] τρίγωνον
τῷ $B\Gamma\Delta$ τριγῶνῳ
ἴσον ἐστίν.

Ἡ ἄρα $B\Gamma$ διάμετρος διχα τέμνει
τὸ $AB\Gamma\Delta$ παραλληλόγραμμον·
ὅπερ ἔδει δεῖξαι.



açının tamamına, $AG\Delta$
eşittir.
Ve gösterilmişti ayrıca
 $BA\Gamma$ ile $\Gamma\Delta B$ açısının eşitliği.

Dolayısıyla, paralelkenar alanların,
karşıt kenar ve açıları
eşittir birbirlerine.

Şimdi iddia ediyorum ki
köşegen de onları ikiye keser.

Çünkü, eşit olduğundan AB , $\Gamma\Delta$ ke-
narına,
ve $B\Gamma$ ortak,
 AB ve $B\Gamma$ ikilisi
- $\Gamma\Delta$ ve $B\Gamma$ ikilisine
eşittirler,
her biri birine;
ve $AB\Gamma$ açısı
 $B\Gamma\Delta$ açısına
eşittir.
Dolayısıyla AG tabanı da
 ΔB tabanına
eşittir.
Dolayısıyla $AB\Gamma$ üçgeni de
 $B\Gamma\Delta$ üçgenine
eşittir.

Dolayısıyla $B\Gamma$ köşegeni ikiye böler
 $AB\Gamma\Delta$ paralelkenarını;
— gösterilmesi gereken tam buydu.

3.35

Parallelograms
on the same base being
and in the same parallels
equal to one another are.

Let there be
parallelograms
 $AB\Gamma\Delta$ and $EB\Gamma\Delta$
on the same base, ΓB ,
and in the same parallels,
 AZ and $B\Gamma$.

I say that
equal is
 $AB\Gamma\Delta$
to the trapezium $EB\Gamma Z$.

For, since
a parallelogram is $AB\Gamma\Delta$,

Τὰ παραλληλόγραμμα
τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν.

Ἐστω
παραλληλόγραμμα
τὰ $AB\Gamma\Delta$, $EB\Gamma Z$
ἐπὶ τῆς αὐτῆς βάσεως τῆς $B\Gamma$
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς AZ , $B\Gamma$.

λέγω, ὅτι
ἴσον ἐστὶ
τὸ $AB\Gamma\Delta$
τῷ $EB\Gamma Z$ παραλληλόγραμμῳ.

Ἐπεὶ γὰρ
παραλληλόγραμμὸν ἐστὶ τὸ $AB\Gamma\Delta$,

Paralelkenarlar;
aynı tabanda olan
ve aynı paralellerde olanlar,
birbirlerine eşittir.

Verilmiş olsun
paralelkenarlar,
 $AB\Gamma\Delta$ ve $EB\Gamma\Delta$,
aynı ΓB tabanında,
ve aynı
 AZ ve $B\Gamma$ paralellerinde.

İddia ediyorum ki
eşittir
 $AB\Gamma\Delta$
 $EB\Gamma Z$ yamuğuna.

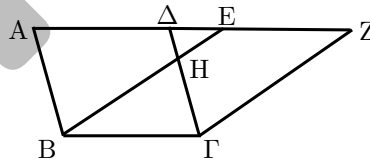
Çünkü
bir paralelkenar olduğundan $AB\Gamma\Delta$,

equal is $A\Delta$ to $B\Gamma$.
 Similarly then also,
 EZ to $B\Gamma$ is equal;
 so that also $A\Delta$ to EZ is equal;
 and common [is] ΔE ;
 therefore AE , as a whole,
 to ΔZ , as a whole,
 is equal.
 Is also AB to $\Delta\Gamma$ equal.
 Then the two EA and AB
 to the two $Z\Delta$ and $\Delta\Gamma$
 equal are
 either to either;
 also angle $Z\Delta\Gamma$
 to EAB
 is equal,
 the exterior to the interior;
 therefore the base EB
 to the base $Z\Gamma$
 is equal,
 and triangle EAB
 to triangle $\Delta Z\Gamma$
 equal will be;
 suppose has been removed, commonly,
 ΔHE ;
 therefore the trapezium $ABH\Delta$ that
 remains
 to the trapezium $EZH\Gamma$ that remains
 is equal;
 let be added in common
 the triangle HBF ;
 therefore the trapezium $AB\Gamma\Delta$ as a
 whole
 to the trapezium $EB\Gamma Z$ as a whole
 is equal.

Therefore parallelograms
 on the same base being
 and in the same parallels
 equal to one another are;
 —just what it was necessary to show.

ἴση ἐστὶν ἡ $A\Delta$ τῆ $B\Gamma$.
 διὰ τὰ αὐτὰ δὴ καὶ
 ἡ EZ τῆ $B\Gamma$ ἐστὶν ἴση·
 ὥστε καὶ ἡ $A\Delta$ τῆ EZ ἐστὶν ἴση·
 καὶ κοινὴ ἡ ΔE ·
 ὅλη ἄρα ἡ AE
 ὅλη τῆ ΔZ
 ἐστὶν ἴση.
 ἔστι δὲ καὶ ἡ AB τῆ $\Delta\Gamma$ ἴση·
 δύο δὴ αἱ EA , AB
 δύο ταῖς $Z\Delta$, $\Delta\Gamma$
 ἴσαι εἰσὶν
 ἑκατέρω ἑκατέρω·
 καὶ γωνία ἡ ὑπὸ $Z\Delta\Gamma$
 γωνία τῆ ὑπὸ EAB
 ἐστὶν ἴση
 ἡ ἐκτὸς τῆ ἐντὸς·
 βάσεις ἄρα ἡ EB
 βάσει τῆ $Z\Gamma$
 ἴση ἐστὶν,
 καὶ τὸ EAB τρίγωνον
 τῷ $\Delta Z\Gamma$ τριγώνῳ
 ἴσον ἔσται·
 κοινὸν ἀφηρήσθω τὸ ΔHE ·
 λοιπὸν ἄρα τὸ $ABH\Delta$ τραπέζιον
 λοιπῷ τῷ $EZH\Gamma$ τραπέζιῳ
 ἐστὶν ἴσον·
 κοινὸν προσκείσθω τὸ HBF τρίγωνον·
 ὅλον ἄρα τὸ $AB\Gamma\Delta$ παραλληλόγραμμον
 ὅλω τῷ $EB\Gamma Z$ παραλληλογράμμῳ
 ἴσον ἐστὶν.

Τὰ ἄρα παραλληλόγραμμα
 τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
 καὶ ἐν ταῖς αὐταῖς παραλλήλοις
 ἴσα ἀλλήλοις ἐστὶν·
 ὅπερ εἶδει δεῖξαι.



eşittir $A\Delta$, $B\Gamma$ kenarına.
 Benzer şekilde o zaman,
 EZ , $B\Gamma$ kenarına eşittir;
 böylece $A\Delta$ da EZ kenarına eşittir;
 ve ortaktır ΔE ;
 dolayısıyla AE , bir bütün olarak,
 ΔZ kenarına
 eşittir.
 AB da $\Delta\Gamma$ kenarına eşittir.
 O zaman EA ve AB ikilisi
 $Z\Delta$ ve $\Delta\Gamma$ ikilisine
 eşittirler
 her biri birine;
 ve $Z\Delta\Gamma$ açısı da
 EAB açısına
 eşittir,
 dış açı, iç açıya;
 dolayısıyla EB tabanı
 $Z\Gamma$ tabanına
 eşittir,
 ve EAB üçgeni
 $\Delta Z\Gamma$ üçgenine
 eşit olacak;
 kaldırılmış olsun, ortak olarak,
 ΔHE ;
 dolayısıyla kalan $ABH\Delta$ yamuğu
 kalan $EZH\Gamma$ yamuğuna
 eşittir;
 eklenmiş olsun her ikisine birden
 HBF üçgeni;
 dolayısıyla $AB\Gamma\Delta$ yamuğunun
 tamamı
 $EB\Gamma Z$ yamuğunun tamamına
 eşittir.

Dolayısıyla paralelkenarlar;
 aynı tabanda olan
 ve aynı paralellerde olanlar,
 birbirlerine eşittir;
 — gösterilmesi gereken tam buydu.

3.36

Parallelograms
 that are on equal bases
 and in the same parallels
 are equal to one another.

Let there be
 parallelograms
 $AB\Gamma\Delta$ and $EZH\Theta$
 on equal bases,
 $B\Gamma$ and ZH ,
 and in the same parallels,
 $A\Theta$ and BH .

Τὰ παραλληλόγραμμα
 τὰ ἐπὶ ἴσων βάσεων ὄντα
 καὶ ἐν ταῖς αὐταῖς παραλλήλοις
 ἴσα ἀλλήλοις ἐστὶν.

Ἐστω
 παραλληλόγραμμα
 τὰ $AB\Gamma\Delta$, $EZH\Theta$
 ἐπὶ ἴσων βάσεων ὄντα
 τῶν $B\Gamma$, ZH
 καὶ ἐν ταῖς αὐταῖς παραλλήλοις
 ταῖς $A\Theta$, BH ·

Paralelkenarlar;
 eşit tabanlarda olanlar
 ve aynı paralellerde olanlar
 eşittirler birbirlerine.

Verilmiş olsun
 paralelkenarlar
 $AB\Gamma\Delta$ ve $EZH\Theta$
 eşit
 $B\Gamma$ ve ZH tabanlarında,
 ve aynı
 $A\Theta$ ve BH paralellerinde.

I say that
equal is
parallelogram $AB\Gamma\Delta$
to $EZH\Theta$.

For, suppose have been joined
 BE and $\Gamma\Theta$.

And since equal are $B\Gamma$ and ZH ,
but ZH to $E\Theta$ is equal,
therefore also $B\Gamma$ to $E\Theta$ is equal.
And [they] are also parallel.
Also EB and $\Theta\Gamma$ join them.
And [STRAIGHTS] that join equals and
parallels in the same parts
are equal and parallel.
[Also therefore EB and $H\Theta$
are equal and parallel.]
Therefore a parallelogram is $EB\Gamma\Theta$.
And it is equal to $AB\Gamma\Theta$.
For it has the same base as it,
 $B\Gamma$,
and in the same parallels
as it it is, $B\Gamma$ and $A\Theta$.
For the same [reason] then,
also $EZH\Theta$ to it, [namely] $EB\Gamma\Theta$,
is equal;
so that parallelogram $AB\Gamma\Delta$
to $EZH\Theta$ is equal.

Therefore parallelograms
that are on equal bases
and in the same parallels
are equal to one another;
—just what it was necessary to show.

λέγω, ὅτι
ἴσον ἐστὶ
τὸ $AB\Gamma\Delta$ παραλληλόγραμμον
τῷ $EZH\Theta$.

Ἐπεξεύχθωσαν γὰρ
αἱ BE , $\Gamma\Theta$.

καὶ ἐπεὶ ἴση ἐστὶν ἡ $B\Gamma$ τῆς ZH ,
ἀλλὰ ἡ ZH τῆς $E\Theta$ ἐστὶν ἴση,
καὶ ἡ $B\Gamma$ ἄρα τῆς $E\Theta$ ἐστὶν ἴση.
εἰσὶ δὲ καὶ παράλληλοι.
καὶ ἐπιζευγνύουσιν αὐτάς αἱ EB , $\Theta\Gamma$.
αἱ δὲ τὰς ἴσας τε καὶ παραλλήλους ἐπι-
τὰ αὐτὰ μέρη ἐπιζευγνύουσαι
ἴσαι τε καὶ παράλληλοι εἰσι
[καὶ αἱ EB , $\Theta\Gamma$ ἄρα
ἴσαι τέ εἰσι καὶ παράλληλοι].
παραλληλόγραμμον ἄρα ἐστὶ τὸ $EB\Gamma\Theta$.
καὶ ἐστὶν ἴσον τῷ $AB\Gamma\Delta$.
βάσιν τε γὰρ αὐτῷ τὴν αὐτὴν ἔχει
τὴν $B\Gamma$,
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἐστὶν αὐτῷ ταῖς $B\Gamma$, $A\Theta$.
διὰ τὰ αὐτὰ δὴ
καὶ τὸ $EZH\Theta$ τῷ αὐτῷ τῷ $EB\Gamma\Theta$
ἐστὶν ἴσον.
ὥστε καὶ τὸ $AB\Gamma\Delta$ παραλληλόγραμμον
τῷ $EZH\Theta$ ἐστὶν ἴσον.

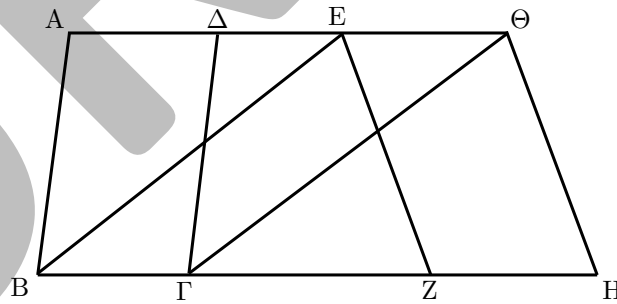
Τὰ ἄρα παραλληλόγραμμα
τὰ ἐπὶ ἴσων βάσεων ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν.
ὅπερ εἶδει δεῖξαι.

İddia ediyorum ki
eşittir
 $AB\Gamma\Delta$,
 $EZH\Theta$ paralelkenarına.

Çünkü, varsayalım birleştirilmiş
olduğu
 BE ile $\Gamma\Theta$ kenarlarımın.

Ve eşit olduğundan $B\Gamma$ ile ZH ,
ama ZH , $E\Theta$ kenarına eşittir,
dolayısıyla $B\Gamma$ da $E\Theta$ kenarına eşittir.
Ve paraleldirler de.
Ayrıca EB ve $\Theta\Gamma$ onları birleştirir.
Ve eşit ve paralelleri aynı tarafta bir-
leştiren doğrular
eşit ve paraleldirler.
[Yine dolayısıyla EB ve $H\Theta$
eşit ve paraleldirler.]
Dolayısıyla $EB\Gamma\Theta$ bir paralelkenardır.
Ve eşittir $AB\Gamma\Theta$ paralelkenarına.
Çünkü onunla aynı,
 $B\Gamma$ tabanı vardır,
ve onunla aynı paralelleri,
 $B\Gamma$ ve $A\Theta$ vardır.
Aynı sebeple o şimdi,
 $EZH\Theta$ da ona, [yani] $EB\Gamma\Theta$ paralelke-
narına,
eşittir;
böylece $AB\Gamma\Delta$,
 $EZH\Theta$ paralelkenarına eşittir.

Dolayısıyla paralelkenarlar;
eşit tabanlarda olanlar
ve aynı paralellerde olanlar
eşittirler birbirlerine;
— gösterilmesi gereken tam buydu.



3.37

Triangles
that are on the same base
and in the same parallels
are equal to one another.

Let there be
triangles $AB\Gamma$ and $\Delta B\Gamma$,
on the same base $B\Gamma$
and in the same parallels
 $A\Delta$ and $B\Gamma$.

Τὰ τρίγωνα
τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν.

Ἐστω
τρίγωνα τὰ $AB\Gamma$, $\Delta B\Gamma$
ἐπὶ τῆς αὐτῆς βάσεως τῆς $B\Gamma$
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς $A\Delta$, $B\Gamma$.

Üçgenler;
aynı tabanda
ve aynı paralellerde olanlar,
eşittir birbirlerine.

Verilmiş olsun
 $AB\Gamma$ ve $\Delta B\Gamma$ üçgenleri,
aynı $B\Gamma$ tabanında
ve aynı
 $A\Delta$ ve $B\Gamma$ paralellerinde.

I say that
equal is
triangle ABΓ
to triangle ΔBΓ.

Suppose has been extended
AΔ on both sides to E and Z,
and through B,
parallel to ΓA
has been drawn BE,
and through Γ
parallel to BΔ
has been drawn ΓZ.

Therefore a parallelogram
is either of EBΓA and ΔBΓZ;
and they are equal;
for they are on the same base,
BΓ,
and in the same parallels,
BΓ and EZ;
and [it] is
of the parallelogram EBΓA
half
—the triangle ABΓ;
for the diameter AB cuts it in two;
and of the parallelogram ΔBΓZ
half
—the triangle ΔBΓ;
for the diameter ΔΓ cuts it in two.
[And halves of equals
are equal to one another.]
Therefore equal is
the triangle ABΓ to the triangle ΔBΓ.

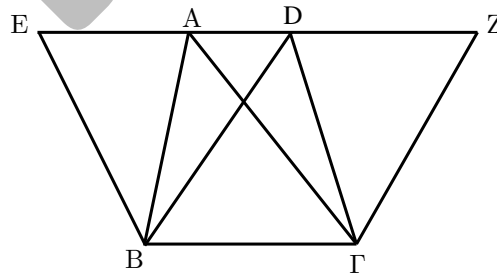
Therefore triangles
that are on the same base
and in the same parallels
are equal to one another;
—just what it was necessary to show.

λέγω, ὅτι
ἴσον ἐστὶ
τὸ ABΓ τρίγωνον
τῷ ΔBΓ τριγώνῳ.

Ἐκβεβλήσθω
ἡ AΔ ἐφ' ἑκάτερα τὰ μέρη ἐπὶ τὰ E, Z,
καὶ διὰ μὲν τοῦ B
τῇ ΓA παράλληλος
ῤχθῶ ἡ BE,
διὰ δὲ τοῦ Γ
τῇ BΔ παράλληλος
ῤχθῶ ἡ ΓZ.

παρλληλόγραμμον ἄρα
ἐστὶν ἑκάτερον τῶν EBΓA, ΔBΓZ·
καὶ εἰσιν ἴσα·
ἐπὶ τε γὰρ τῆς αὐτῆς βάσεως εἰσι
τῆς BΓ
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς BΓ, EZ·
καὶ ἐστὶ
τοῦ μὲν EBΓA παρλληλογράμμου
ἥμισυ
τὸ ABΓ τρίγωνον·
ἡ γὰρ AB διάμετρος αὐτὸ δίχα τέμνει·
τοῦ δὲ ΔBΓZ παρλληλογράμμου
ἥμισυ
τὸ ΔBΓ τρίγωνον·
ἡ γὰρ ΔΓ διάμετρος αὐτὸ δίχα τέμνει.
[τὰ δὲ τῶν ἴσων ἡμίση
ἴσα ἀλλήλοις ἐστίν].
ἴσον ἄρα ἐστὶ
τὸ ABΓ τρίγωνον τῷ ΔBΓ τριγώνῳ.

Τὰ ἄρα τρίγωνα
τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν·
ὅπερ ἔδει δεῖξαι.



İddia ediyorum ki
eşittir
ABΓ üçgeni
ΔBΓ üçgenine.

Varsayalım uzatılmış olduğu
AΔ doğrusunun her iki kenarda E ve
Z noktalarına,
ve B noktasından,
ΓA kenarına paralel
BE çizilmiş olsun,
ve Γ noktasından
BΔ kenarına papalel
ΓZ çizilmiş olsun.

Dolayısıyla birer paralelkenardır
EBΓA ile ΔBΓZ;
ve bunlar eşittir;
aynı
BΓ tabanında,
ve aynı,
BΓ ve EZ paralellerinde oldukları için;
ve
EBΓA paralelkenarının
yarısı
— ABΓ üçgenidir;
AB köşegeni onu ikiye kestiği için;
ΔBΓZ paralelkenarının
yarısı
— ΔBΓ üçgenidir;
ΔΓ köşegeni onu ikiye kestiği için.
[Ve eşitlerin yaruları
eşittirler birbirlerine.]
Dolayısıyla eşittir
ABΓ üçgeni ΔBΓ üçgenine.

Dolayısıyla üçgenler;
aynı tabanda
ve aynı paralellerde olanlar,
eşittir birbirlerine;
— gösterilmesi gereken tam buydu.

3.38

Triangles
that are on equal bases
and in the same parallels
are equal to one another.

Let there be
triangles ABΓ and ΔEZ

Τὰ τρίγωνα
τὰ ἐπὶ ἴσων βάσεων ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν.

Ἐστω
τρίγωνα τὰ ABΓ, ΔEZ

Üçgenler;
eşit tabanlarda
ve aynı paralellerde olanlar,
eşittir birbirlerine.

Verilmiş olsun
ABΓ ve ΔEZ üçgenleri

on equal bases $B\Gamma$ and EZ
and in the same parallels
 BZ and $A\Delta$.

I say that
equal is
triangle $AB\Gamma$
to triangle ΔEZ .

For, suppose has been extended
 $A\Delta$ on both sides to H and Θ ,
and through B ,
parallel to ΓA ,
has been drawn BH ,
and through Z ,
parallel to ΔE ,
has been drawn $Z\Theta$.

Therefore a parallelogram
is either of $HB\Gamma A$ and $\Delta EZ\Theta$;
and $HB\Gamma A$ [is] equal to $\Delta EZ\Theta$;
for they are on equal bases,
 $B\Gamma$ and EZ ,
and in the same parallels,
 BZ and $H\Theta$;
and [it] is
of the parallelogram $HB\Gamma A$
half
—the triangle $AB\Gamma$.
For the diameter AB cuts it in two;
and of the parallelogram $\Delta EZ\Theta$
half
—the triangle $Z\Theta\Delta$;
for the diameter ΔZ cuts it in two.
[And halves of equals
are equal to one another.]
Therefore equal is
the triangle $AB\Gamma$ to the triangle ΔEZ .

Therefore triangles
that are on equal bases
and in the same parallels
are equal to one another;
—just what it was necessary to show.

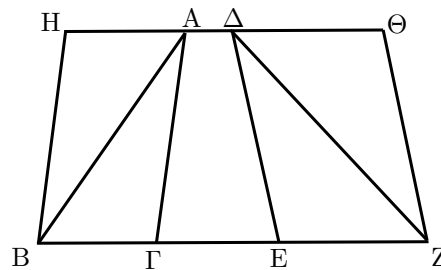
ἐπι ἴσων βάσεων τῶν $B\Gamma$, EZ
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς BZ , $A\Delta$.

λέγω, ὅτι
ἴσον ἐστὶ
τὸ $AB\Gamma$ τρίγωνον
τῷ ΔEZ τριγώνῳ.

Ἐκβεβλήσθω γὰρ
ἡ $A\Delta$ ἐφ' ἑκάτερα τὰ μέρη ἐπὶ τὰ H , Θ ,
καὶ διὰ μὲν τοῦ B
τῇ ΓA παράλληλος
ἦχθω ἡ BH ,
διὰ δὲ τοῦ Z
τῇ ΔE παράλληλος
ἦχθω ἡ $Z\Theta$.

παρλληλόγραμμον ἄρα
ἐστὶν ἑκάτερον τῶν $HB\Gamma A$, $\Delta EZ\Theta$.
καὶ ἴσον τὸ $HB\Gamma A$ τῷ $\Delta EZ\Theta$.
ἐπὶ τε γὰρ ἴσων βάσεων εἰσι
τῶν $B\Gamma$, EZ
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς BZ , $H\Theta$.
καὶ ἐστὶ
τοῦ μὲν $HB\Gamma A$ παρλληλογράμμου
ἡμισυ
τὸ $AB\Gamma$ τρίγωνον.
ἡ γὰρ AB διάμετρος αὐτὸ δίχα τέμνει
τοῦ δὲ $\Delta EZ\Theta$ παρλληλογράμμου
ἡμισυ
τὸ $Z\Theta\Delta$ τρίγωνον.
ἡ γὰρ ΔZ διάμετρος αὐτὸ δίχα τέμνει
[τὰ δὲ τῶν ἴσων ἡμίση
ἴσα ἀλλήλοις ἐστίν].
ἴσον ἄρα ἐστὶ
τὸ $AB\Gamma$ τρίγωνον τῷ ΔEZ τριγώνῳ.

Τὰ ἄρα τρίγωνα
τὰ ἐπὶ ἴσων βάσεων ὄντα
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ἴσα ἀλλήλοις ἐστίν.
ὅπερ εἶδει δεῖξαι.



eşit $B\Gamma$ ve EZ tabanlarında
ve aynı
 BZ ve $A\Delta$ paralellerinde.

İddia ediyorum ki
eşittir
 $AB\Gamma$ üçgeni
 ΔEZ üçgenine.

Çünkü varsayalım uzatılmış olduğu
 $A\Delta$ kenarının her iki kenarda H ve Θ
noktalarına,
ve B noktasından,
 ΓA kenarına paralel,
 BH çizilmiş olsun,
ve Z noktasından,
 ΔE kenarına paralel,
 $Z\Theta$ çizilmiş olsun.

Dolayısıyla birer paralelkenardır
 $HB\Gamma A$ ile $\Delta EZ\Theta$;
ve $HB\Gamma A$ eşittir $\Delta EZ\Theta$ paralelke-
narına;
eşit,
 $B\Gamma$ ve EZ tabanlarında,
ve aynı,
 BZ ve $H\Theta$ paralellerinde oldukları
için;
ve
 $HB\Gamma A$ paralelkenarının
yarısı
— $AB\Gamma$ üçgenidir.
 AB köşegeni onu ikiye kestiği için;
ve $\Delta EZ\Theta$ paralelkenarının
yarısı
— $Z\Theta\Delta$ üçgenidir;
 ΔZ köşegeni onu ikiye kestiği için.
[Ve eşitlerin yarıları
eşittirler birbirlerine.]
Dolayısıyla eşittir
 $AB\Gamma$ üçgeni ΔEZ üçgenine.

Dolayısıyla üçgenler;
eşit tabanlarda
ve aynı paralellerde olanlar,
eşittir birbirlerine;
— gösterilmesi gereken tam buydu.

3.39

Equal triangles
that are on the same base

Τὰ ἴσα τρίγωνα
τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα

Eşit üçgenler;
aynı tabanda

and in the same parts
are also in the same parallels.

Let there be
equal triangles $AB\Gamma$ and $\Delta B\Gamma$,
being on the same base
and on the same side of $B\Gamma$.

I say that
they are also in the same parallels.

For suppose has been joined $A\Delta$.

I say that
parallel is $A\Delta$ to $B\Gamma$.

For if not,
suppose there has been drawn
through the point A
parallel to the STRAIGHT $B\Gamma$
 AE ,
and there has been joined $E\Gamma$.
Equal therefore is
the triangle $AB\Gamma$
to the triangle $EB\Gamma$;
for on the same base
as it is, $B\Gamma$,
and in the same parallels.
But $AB\Gamma$ is equal to $\Delta B\Gamma$.
Also therefore $\Delta B\Gamma$ to $EB\Gamma$ is equal,
the greater to the less;
which is impossible.
Therefore is not parallel AE to $B\Gamma$.
Similarly then we shall show that
neither is any other but $A\Delta$;
therefore $A\Delta$ is parallel to $B\Gamma$.

Therefore equal triangles
that are on the same base
and in the same parts
are also in the same parallels;
—just what it was necessary to show.

καὶ ἐπὶ τὰ αὐτὰ μέρη
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν.

Ἐστω
ἴσα τρίγωνα τὰ $AB\Gamma$, $\Delta B\Gamma$
ἐπὶ τῆς αὐτῆς βάσεως ὄντα
καὶ ἐπὶ τὰ αὐτὰ μέρη τῆς $B\Gamma$.

λέγω, ὅτι
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν.

Ἐπεζεύχθω γὰρ ἡ $A\Delta$.

λέγω, ὅτι
παράλληλός ἐστιν ἡ $A\Delta$ τῇ $B\Gamma$.

Εἰ γὰρ μή,
ῥηθῶ
διὰ τοῦ A σημείου
τῇ $B\Gamma$ εὐθείᾳ παράλληλος
ἡ AE ,
καὶ ἐπεζεύχθω ἡ $E\Gamma$.
ἴσον ἄρα ἐστὶ
τὸ $AB\Gamma$ τρίγωνον
τῷ $EB\Gamma$ τριγώνῳ·
ἐπὶ τε γὰρ τῆς αὐτῆς βάσεως
ἐστὶν αὐτῷ τῆς $B\Gamma$
καὶ ἐν ταῖς αὐταῖς παραλλήλοις.
ἀλλὰ τὸ $AB\Gamma$ τῷ $\Delta B\Gamma$ ἐστὶν ἴσον·
καὶ τὸ $\Delta B\Gamma$ ἄρα τῷ $EB\Gamma$ ἴσον ἐστὶ
τὸ μείζον τῷ ἐλάσσονι·
ὅπερ ἐστὶν ἀδύνατον·
οὐκ ἄρα παράλληλός ἐστιν ἡ AE τῇ $B\Gamma$.
ὁμοίως δὲ δεῖξομεν, ὅτι
οὐδ' ἄλλη τις πλὴν τῆς $A\Delta$
ἡ $A\Delta$ ἄρα τῇ $B\Gamma$ ἐστὶ παράλληλος.

Τὰ ἄρα ἴσα τρίγωνα
τὰ ἐπὶ τῆς αὐτῆς βάσεως ὄντα
καὶ ἐπὶ τὰ αὐτὰ μέρη
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν·
ὅπερ ἔδει δεῖξαι.

ve onun aynı tarafında olan,
aynı paralellerdedirler de.

Verilmiş olsun
 $AB\Gamma$ ve $\Delta B\Gamma$ eşit üçgenleri,
aynı $B\Gamma$ tabanında
ve onun aynı tarafında olan .

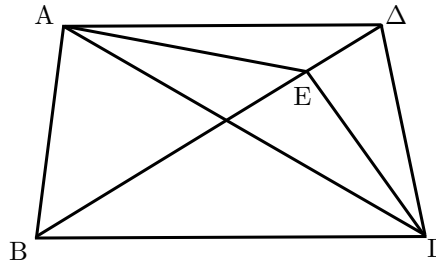
İddia ediyorum ki
aynı paralellerdedirler de.

Çünkü $A\Delta$ doğrusunun birleştirilmiş
olduğu varsayılımsın.

İddia ediyorum ki
paraleldir $A\Delta$, $B\Gamma$ tabanına.

Çünkü eğer değil ise,
çizilmiş olduğu varsayılımsın
 A noktasından
 $B\Gamma$ doğrusuna paralel
 AE doğrusunun,
ve birleştirildiği $E\Gamma$ doğrusunun.
Eşittir dolayısıyla
 $AB\Gamma$ üçgeni
 $EB\Gamma$ üçgenine;
onunla aynı
 $B\Gamma$ tabanında,
ve aynı paralellerde olduğu için.
Ama $AB\Gamma$ eşittir $\Delta B\Gamma$ üçgenine.
Ve dolayısıyla $\Delta B\Gamma$, $EB\Gamma$ üçgenine
eşittir,
büyük küçüğe;
ki bu imkansızdır.
Dolayısıyla paralel değildir AE , $B\Gamma$
doğrusuna.
Benzer şekilde o zaman göstereceğiz ki
 $A\Delta$ dışındakiler de paralel değildi ;
dolayısıyla $A\Delta$, $B\Gamma$ doğrusuna paar-
aleldir.

Dolayısıyla eşit üçgenler;
aynı tabanda
ve onun aynı tarafında olan,
aynı paralellerdedirler de;
— gösterilmesi gereken tam buydu.



3.40

Equal triangles
that are on equal bases
and in the same parts

Τὰ ἴσα τρίγωνα
τὰ ἐπὶ ἴσων βάσεων ὄντα
καὶ ἐπὶ τὰ αὐτὰ μέρη

Eşit üçgenler,
eşit tabanlarda
ve aynı tarafta olan,

are also in the same parallels.

Let there be
equal triangles $AB\Gamma$ and $\Gamma\Delta E$,
on equal bases $B\Gamma$ and ΓE ,
and in the same parts.

I say that
they are also in the same parallels.

For suppose $A\Delta$ has been joined.

I say that
parallel is $A\Delta$ to BE .

For if not,
suppose there has been drawn
through the point A ,
parallel to BE ,
 AZ ,
and there has been joined ZE .
Equal therefore is
the triangle $AB\Gamma$
to the triangle $Z\Gamma E$;
for they are on equal bases,
 $B\Gamma$ and ΓE ,
and in the same parallels,
 BE and AZ .

But the triangle $AB\Gamma$
is equal to the [triangle] $\Delta\Gamma E$;
also therefore the [triangle] $\Delta\Gamma E$
is equal to the triangle $Z\Gamma E$,
the greater to the less;
which is impossible.
Therefore is not parallel AZ to BE .
Similarly then we shall show that
neither is any other but $A\Delta$;
therefore $A\Delta$ to BE is parallel.

Therefore equal triangles
that are on equal bases
and in the same parts
are also in the same parallels;
—just what it was necessary to show.

καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν.

Ἐστω
ἴσα τρίγωνα τὰ $AB\Gamma$, $\Gamma\Delta E$
ἐπὶ ἴσων βάσεων τῶν $B\Gamma$, ΓE
καὶ ἐπὶ τὰ αὐτὰ μέρη.

λέγω, ὅτι
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν.

Ἐπεζεύχθω γὰρ ἡ $A\Delta$.

λέγω, ὅτι
παράλληλός ἐστιν ἡ $A\Delta$ τῇ BE .

Εἰ γὰρ μή,
ἦχθω
διὰ τοῦ A
τῇ BE παράλληλος
ἡ AZ ,
καὶ ἐπεζεύχθω ἡ ZE .
ἴσον ἄρα ἐστὶ
τὸ $AB\Gamma$ τρίγωνον
τῷ $Z\Gamma E$ τριγώνῳ·
ἐπὶ τε γὰρ ἴσων βάσεων εἰσι
τῶν $B\Gamma$, ΓE
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς BE , AZ .
ἀλλὰ τὸ $AB\Gamma$ τρίγωνον
ἴσον ἐστὶ τῷ $\Delta\Gamma E$ [τρίγωνῳ].
καὶ τὸ $\Delta\Gamma E$ ἄρα [τρίγωνον]
ἴσον ἐστὶ τῷ $Z\Gamma E$ τριγώνῳ·
τὸ μείζον τῷ ἐλάσσονι·
ὅπερ ἐστὶν ἀδύνατον·
οὐκ ἄρα παράλληλος ἡ AZ τῇ BE .
ὁμοίως δὲ δεῖξομεν, ὅτι
οὐδ' ἄλλη τις πλὴν τῆς $A\Delta$
ἡ $A\Delta$ ἄρα τῇ BE ἐστὶ παράλληλος.

Τὰ ἄρα ἴσα τρίγωνα
τὰ ἐπὶ ἴσων βάσεων ὄντα
καὶ ἐπὶ τὰ αὐτὰ μέρη
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἐστίν·
ὅπερ εἶδει δεῖξαι.

ayn paralelerdedirler de.

Verilmiş olsun
eşit $AB\Gamma$ ve $\Gamma\Delta E$ üçgenleri,
eşit $B\Gamma$ ve ΓE tabanlarında,
ve aynı tarafta olan.

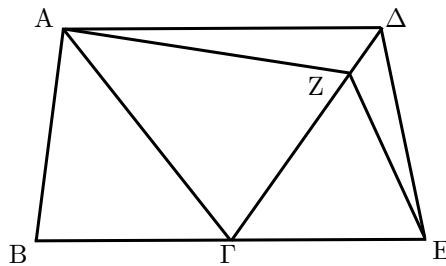
İddia ediyorum ki
aynı paralellerdedirler de.

Çünkü varsayalım $A\Delta$ doğrusunun
birleştirildiği.

İddia ediyorum ki
paraleldir $A\Delta$, BE doğrusuna.

Çünkü eğer değil ise,
varsayalım birleştirildiği
 A noktasından,
 BE doğrusuna paralel,
 AZ doğrusunun,
ve birleştirildiği ZE doğrusunun.
Dolayısıyla eşittir
 $AB\Gamma$ üçgeni
 $Z\Gamma E$ üçgenine;
eşit,
 $B\Gamma$ ve ΓE tabanlarında,
ve aynı,
 BE ve AZ paralellerinde oldukları için.
Fakat $AB\Gamma$ üçgeni
eşittir $\Delta\Gamma E$ üçgenine;
ve dolayısıyla $\Delta\Gamma E$ üçgenini
eşittir $Z\Gamma E$ üçgenine,
büyük küçüğe;
ki bu imkansızdır.
Dolayısıyla paralel değildir AZ , BE
doğrusuna.
Benzer şekilde o zaman göstereceğiz ki
 $A\Delta$ dışındakiler de paralel değildi ;
dolayısıyla $A\Delta$, BE doğrusuna par-
aleldir.

Dolayısıyla eşit üçgenler,
eşit tabanlarda
ve aynı tarafta olan,
aynı paralelerdedirler de;
— gösterilmesi gereken tam buydu.



3.41

If a parallelogram
have the same base as a triangle,

Ἐὰν παραλληλόγραμμον
τριγώνῳ βάσιν τε ἔχη τὴν αὐτὴν

Eğer bir paralelkenar
bir üçgenle aynı tabana sahipse,

and be in the same parallels,
double is
the parallelogram of the triangle.

For, the parallelogram $AB\Gamma\Delta$
as the triangle $EB\Gamma$,
—suppose it has the same base, $B\Gamma$,
and is in the same parallels,
 $B\Gamma$ and AE .

I say that
double is
the parallelogram $AB\Gamma\Delta$
of the triangle $BE\Gamma$.

For, suppose $A\Gamma$ has been joined.

Equal is the triangle $AB\Gamma$
to the triangle $EB\Gamma$;
for it is on the same base as it,
 $B\Gamma$,
and in the same parallels,
 $B\Gamma$ and AE .

But the parallelogram $AB\Gamma\Delta$
is double of the triangle $AB\Gamma$;
for the diameter $A\Gamma$ cuts it in two;
so that the parallelogram $AB\Gamma\Delta$
also of the triangle $EB\Gamma$ is double.

Therefore, if a parallelogram
have the same base as a triangle,
and be in the same parallels,
double is
the parallelogram of the triangle;
—just what it was necessary to show.

καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἦ,
διπλάσιόν ἐστί
τὸ παραλληλόγραμμον τοῦ τριγώνου.

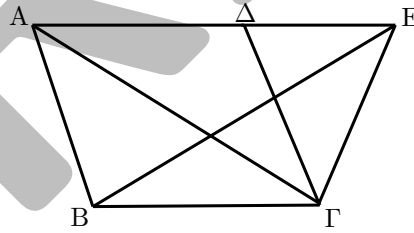
Παραλληλόγραμμον γὰρ τὸ $AB\Gamma\Delta$
τριγώνω $\tau\tilde{\omega}$ $EB\Gamma$
βάσιν τε ἔχέτω τὴν αὐτὴν τὴν $B\Gamma$
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἔστω
ταῖς $B\Gamma$, AE .

λέγω, ὅτι
διπλάσιόν ἐστί
τὸ $AB\Gamma\Delta$ παραλληλόγραμμον
τοῦ $BE\Gamma$ τριγώνου.

Ἐπεξεύχθω γὰρ ἡ $A\Gamma$.

ἴσον δὲ ἐστὶ τὸ $AB\Gamma$ τρίγωνον
 $\tau\tilde{\omega}$ $EB\Gamma$ τριγώνω·
ἐπὶ τε γὰρ τῆς αὐτῆς βάσεως ἐστὶν αὐτῶ
τῆς $B\Gamma$
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς $B\Gamma$, AE .
ἀλλὰ τὸ $AB\Gamma\Delta$ παραλληλόγραμμον
διπλάσιόν ἐστί τοῦ $AB\Gamma$ τριγώνου·
ἢ γὰρ $A\Gamma$ διάμετρος αὐτὸ δίχα τέμνει·
ὥστε τὸ $AB\Gamma\Delta$ παραλληλόγραμμον
καὶ τοῦ $EB\Gamma$ τριγώνου ἐστὶ διπλάσιον.

Ἐὰν ἄρα παραλληλόγραμμον
τριγώνω βάσιν τε ἔχη τὴν αὐτὴν
καὶ ἐν ταῖς αὐταῖς παραλλήλοις ἦ,
διπλάσιόν ἐστί
τὸ παραλληλόγραμμον τοῦ τριγώνου·
ὅπερ ἔδει δεῖξαι.



ve aynı paralellerdeyse,
iki katıdır
paralelkenar, üçgenin.

Çünkü $AB\Gamma\Delta$ paralelkenarının
 $EB\Gamma$ üçgeniyle,
—sayını $B\Gamma$ tabanı olduğu varsayılsın,
ve aynı
 $B\Gamma$ ve AE paralellerinde oldukları.

İddia ediyorum ki
iki katıdır
 $AB\Gamma\Delta$ paralelkenarı
 $BE\Gamma$ üçgeninin.

Çünkü, varsayılsın $A\Gamma$ doğrusunun
birleştirildiği.

Eşittir $AB\Gamma$ üçgeni
 $EB\Gamma$ üçgenine;
onunla aynı,
 $B\Gamma$ tabanına sahip,
ve aynı
 $B\Gamma$ ve AE paralellerinde olduğu için.
Fakat $AB\Gamma\Delta$ paralelkenarı
iki katıdır $AB\Gamma$ üçgeninin;
 $A\Gamma$ köşegeni onu ikiye kestiğinden;
böylece $AB\Gamma\Delta$ paralelkenarı da
gr $EB\Gamma$ üçgeninin iki katıdır.

Dolayısıyla, eğer bir paralelkenar
bir üçgenle aynı tabana sahipse,
ve aynı paralellerdeyse,
iki katıdır
paralelkenar, üçgenin;
— gösterilmesi gereken tam buydu.

3.42

To the given triangle equal,
a parallelogram to construct
in the given rectilinear angle.

Let be
the given triangle $AB\Gamma$,
and the given rectilinear angle, Δ .

It is necessary then
to the triangle $AB\Gamma$ equal
a parallelogram to construct
in the rectilinear angle Δ .

Suppose $B\Gamma$ has been cut in two at E ,

Τῶ δοθέντι τριγώνω ἴσον
παραλληλόγραμμον συστήσασθαι
ἐν τῇ δοθείσῃ γωνίᾳ εὐθύγραμμω.

Ἐστω
τὸ μὲν δοθὲν τρίγωνον τὸ $AB\Gamma$,
ἢ δὲ δοθεῖσα γωνία εὐθύγραμμος ἡ Δ .

δεῖ δὲ
τῶ $AB\Gamma$ τριγώνω ἴσον
παραλληλόγραμμον συστήσασθαι
ἐν τῇ Δ γωνίᾳ εὐθύγραμμω.

Τετμήσθω ἡ $B\Gamma$ δίχα κατὰ τὸ E ,

Verilen bir üçgene eşit,
bir paralelkenarı
verilen bir düzkenar açıda inşa etmek.

Verilen
üçgen $AB\Gamma$,
ve verilen düzkenar açısı Δ olsun.

Şimdi gerklidir
 $AB\Gamma$ üçgenine eşit
bir paralelkenarın
 Δ düzkenar açısına inşa edilmesi.

Varsayılsın $B\Gamma$ kenarının E nok-

and there has been joined AE,
and there has been constructed
on the STRAIGHT EG,
and at the point E on it,
to angle Δ equal,
GEZ,
also, through A, parallel to EG,
suppose AH has been drawn,
and through G, parallel to EZ,
suppose GH has been drawn;
therefore a parallelogram is ZEGH.

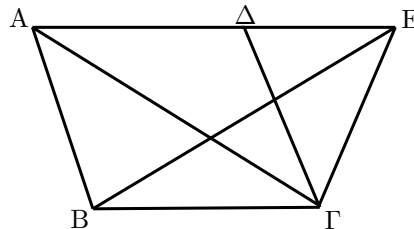
And since equal is BE to EG,
equal is also
triangle ABE to triangle AEG;
for they are on equal bases,
BE and EG,
and in the same parallels,
BG and AH;
double therefore is
triangle ABG of triangle AEG.
also is
parallelogram ZEGH
double of triangle AEG;
for it has the same base as it,
and
is in the same parallels as it;
therefore is equal
the parallelogram ZEGH
to the triangle ABG.
And it has angle GEZ
equal to the given Δ .

Therefore, to the given triangle ABG
equal,
a parallelogram has been constructed,
ZEGH,
in the angle GEZ,
which is equal to Δ ;
—just what it was necessary to do.

καὶ ἐπεζεύχθω ἡ AE,
καὶ συνεστάτω
πρὸς τῇ EG εὐθείᾳ
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ E
τῇ Δ γωνίᾳ ἴση
ἢ ὑπὸ GEZ,
καὶ διὰ μὲν τοῦ A τῇ EG παράλληλος
ῤχθω ἡ AH,
διὰ δὲ τοῦ G τῇ EZ παράλληλος
ῤχθω ἡ GH·
παράλληλόγραμμον ἄρα ἐστὶ τὸ ZEGH.

καὶ ἐπεὶ ἴση ἐστὶν ἡ BE τῇ EG,
ἴσον ἐστὶ καὶ
τὸ ABE τρίγωνον τῷ AEG τριγώνῳ·
ἐπὶ τε γὰρ ἴσων βάσεων εἰσι
τῶν BE, EG
καὶ ἐν ταῖς αὐταῖς παραλλήλοις
ταῖς BG, AH·
διπλάσιον ἄρα ἐστὶ
τὸ ABG τρίγωνον τοῦ AEG τριγώνου.
ἔστι δὲ καὶ
τὸ ZEGH παράλληλόγραμμον
διπλάσιον τοῦ AEG τριγώνου·
βάσιν τε γὰρ αὐτῷ τὴν αὐτὴν ἔχει
καὶ
ἐν ταῖς αὐταῖς ἐστὶν αὐτῷ παραλλήλοις·
ἴσον ἄρα ἐστὶ
τὸ ZEGH παράλληλόγραμμον
τῷ ABG τριγώνῳ,
καὶ ἔχει τὴν ὑπὸ GEZ γωνίαν
ἴσην τῇ δοθείσῃ τῇ Δ .

Τῷ ἄρα δοθέντι τριγώνῳ τῷ ABG
ἴσον
παράλληλόγραμμον συνέσταται
τὸ ZEGH
ἐν γωνίᾳ τῇ ὑπὸ GEZ,
ἣτις ἐστὶν ἴση τῇ Δ ·
ὅπερ ἔδει ποιῆσαι.



tasında ikiye kesildiği
ve AE doğrusunun birleştirildiği,
ve inşa edildiği
EG doğrusunda,
ve üzerindeki E noktasında,
 Δ açısına eşit,
GEZ açısının,
ayrıca, A noktasından, EG doğrusuna
paralel,
AH doğrusunun çizilmiş olduğu
varsayalım,
ve G noktasından, EZ doğrusuna par-
alel,
GH doğrusunun çizilmiş olduğu
varsayalım;
dolayısıyla ZEGH bir paralelkenardır.

Ve eşit olduğundan BE, EG
doğrusuna,
eşittir
ABE üçgeni de AEG üçgenine;
tabanları
BE ve EG eşit,
ve aynı
BG ve AH paralelerinde oldukları için;
iki katıdır dolayısıyla
ABG üçgeni AEG üçgeninin,
ayrıca
ZEGH paralelkenarı
iki katıdır AEG üçgeninin;
onunla aynı tabanı olduğu,
ve
onunla aynı paralellerde olduğu için;
dolayısıyla eşittir
ZEGH paralelkenarı
ABG üçgenine.
Ve onun GEZ açısı
eşittir verilen Δ açısına.

Dolayısıyla, verilen ABG üçgenine
eşit,
bir paralelkenar,
ZEGH, inşa edilmiş oldu
GEZ açısında,
 Δ açısına eşit olan;
— yapılması gereken tam buydu.

3.43

Of any parallelogram,
of the parallelograms about the diam-
eter,
the complements

Παντὸς παραλληλογράμμου
τῶν περὶ τὴν διάμετρον παραλληλο-
γράμμων
τὰ παραπληρώματα

Herhangi bir paralelkenarın,
köşegeni etrafındaki paralelkenarların,
tümleyenleri
eşittir birbirlerine.

are equal to one another.

Let there be
a parallelogram $AB\Gamma\Delta$,
and its diameter, AG ,
and about AG
let be parallelograms,
 $E\Theta$ and ZH ,¹
and the so-called² complements,
 BK and $K\Delta$.

I say that
equal is the complement BK
to the complement $K\Delta$.

For, since a parallelogram is
 $AB\Gamma\Delta$,
and its diameter, AG ,
equal is
triangle $AB\Gamma$ to triangle $A\Gamma\Delta$.
Moreover, since a parallelogram is
 $E\Theta$,
and its diameter, AK ,
equal is
triangle AEK to triangle $A\Theta K$.
Then for the same [reasons] also
triangle $KZ\Gamma$ to $K\eta\Gamma$ is equal.
Since then triangle AEK
is equal to triangle $A\Theta K$,
and $KZ\Gamma$ to $K\eta\Gamma$,
triangle AEK with $K\eta\Gamma$
is equal
to triangle $A\Theta K$ with $KZ\Gamma$;
also is triangle $AB\Gamma$, as a whole,
equal to $A\Delta\Gamma$, as a whole;
therefore the complement BK remain-
ing
to the complement $K\Delta$ remaining
is equal.

Therefore, of any parallelogram area,
of the about-the-diameter
parallelograms,
the complements
are equal to one another;
—just what it was necessary to show.

ἴσα ἀλλήλοις ἐστίν.

Ἐστω
παράλληλογράμμον τὸ $AB\Gamma\Delta$,
διάμετρος δὲ αὐτοῦ ἡ AG ,
περὶ δὲ τὴν AG
παράλληλογράμματα μὲν ἔστω
τὰ $E\Theta$, ZH ,
τὰ δὲ λεγόμενα παραπληρώματα
τὰ BK , $K\Delta$.

λέγω, ὅτι
ἴσον ἐστὶ τὸ BK παραπλήρωμα
τῷ $K\Delta$ παραπληρώματι.

Ἐπεὶ γὰρ παράλληλογράμμον ἐστὶ
τὸ $AB\Gamma\Delta$,
διάμετρος δὲ αὐτοῦ ἡ AG ,
ἴσον ἐστὶ
τὸ $AB\Gamma$ τρίγωνον τῷ $A\Gamma\Delta$ τριγώνῳ.
πάλιν, ἐπεὶ παράλληλογράμμον ἐστὶ
τὸ $E\Theta$,
διάμετρος δὲ αὐτοῦ ἐστὶν ἡ AK ,
ἴσον ἐστὶ
τὸ AEK τρίγωνον τῷ $A\Theta K$ τριγώνῳ.
διὰ τὰ αὐτὰ δὴ καὶ
τὸ $KZ\Gamma$ τρίγωνον τῷ $K\eta\Gamma$ ἐστὶν ἴσον.
ἐπεὶ οὖν τὸ μὲν AEK τρίγωνον
τῷ $A\Theta K$ τριγώνῳ ἐστὶν ἴσον,
τὸ δὲ $KZ\Gamma$ τῷ $K\eta\Gamma$,
τὸ AEK τρίγωνον μετὰ τοῦ $K\eta\Gamma$
ἴσον ἐστὶ
τῷ $A\Theta K$ τριγώνῳ μετὰ τοῦ $KZ\Gamma$.
ἔστι δὲ καὶ ὅλον τὸ $AB\Gamma$ τρίγωνον
ὅλῳ τῷ $A\Delta\Gamma$ ἴσον.
λοιπὸν ἄρα τὸ BK παραπλήρωμα
λοιπῷ τῷ $K\Delta$ παραπληρώματι
ἐστὶν ἴσον.

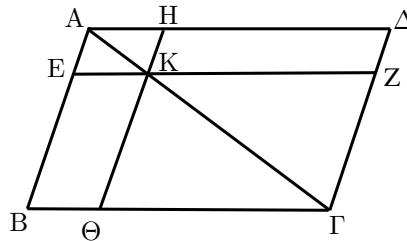
Παντὸς ἄρα παραλληλογράμμου χωρίου
τῶν περὶ τὴν διάμετρον
παράλληλογράμμων
τὰ παραπληρώματα
ἴσα ἀλλήλοις ἐστίν·
ὅπερ ἔδει δεῖξαι.

Verilmiş olsun
bir $AB\Gamma\Delta$ paralelkenarı,
ve onun AG köşegeni,
ve AG etrafında
paralelkenarlar,
 $E\Theta$ ve ZH ,
ve bunların tümleyenleri,
 BK ile $K\Delta$.

İddia ediyorum
eşittir BK tümleyeni
 $K\Delta$ tümleyenine.

Çünkü, bir paralelkenar olduğundan
 $AB\Gamma\Delta$,
ve $grAG$, onun köşegeni,
eşittir
 $AB\Gamma$ üçgeni $A\Gamma\Delta$ üçgenine.
Dahası, bir paralelkenar olduğundan
 $E\Theta$,
 AK , onun köşegeni,
eşittir
 AEK üçgeni $A\Theta K$ üçgenine.
Şimdiki aynı nedenle
 $KZ\Gamma$ eşittir $K\eta\Gamma$ üçgenine.
O zaman AEK
eşit olduğundan $A\Theta K$ üçgenine,
ve $KZ\Gamma$, $K\eta\Gamma$ üçgenine,
 AEK ile $K\eta\Gamma$ üçgenleri
eşittir
 $A\Theta K$ ile $KZ\Gamma$ üçgenlerine;
ayrıca $AB\Gamma$ üçgeninin tümü
eşittir $A\Delta\Gamma$ üçgeninin tümüne;
dolayısıyla geriye kalan BK tümleyeni,
geriye kalan $K\Delta$ tümleyenine
eşittir.

Dolayısıyla, herhangi bir paralelke-
narın,
köşegeni etrafındaki paralelkenarların,
tümleyenleri
eşittir birbirlerine;
— gösterilmesi gereken tam buydu.



3·44

¹Here Euclid can use two letters without qualification for a parallelogram, because they are not unqualified in the Greek: they take the neuter article, while a line takes the feminine article.

²This is Heath's translation. The Greek does not require any-

thing corresponding to 'so-'. The LSJ lexicon [10] gives the present proposition as the original geometrical use of παραπλήρωμα—other meanings are 'expletive' and a certain flowering herb.

Along the given STRAIGHT,
equal to the given triangle,
to apply a parallelogram
in the given rectilinear angle.

Let be
the given STRAIGHT AB,
and the given triangle, Γ ,
and the given rectilinear angle, Δ .

It is necessary then
along the given STRAIGHT AB
equal to the given triangle Γ
to apply a parallelogram
in an equal to the angle Δ .

Suppose has been constructed
equal to triangle Γ ,
a parallelogram BEZH
in angle EBH,
which is equal to Δ ;
and let it be laid down
so that on a STRAIGHT is BE
with AB,
and suppose has been drawn through
ZH to Θ ,
and through A,
parallel to either of BH and EZ,
suppose there has been drawn
A Θ ,
and suppose there has been joined
 Θ B.

And since on the parallels A Θ and EZ
fell the STRAIGHT Θ Z,
the angles A Θ Z and Θ ZE
are equal to two RIGHTS.
Therefore B Θ H and HZE
are less than two RIGHTS.
And [STRAIGHTS] from [angles] that
are less
than two RIGHTS,
extended to the infinite,
fall together.
Therefore Θ B and ZE, extended,
fall together.

Suppose they have been extended,
and they have fallen together at K,
and through the point K,
parallel to either of EA and Z Θ ,
suppose has been drawn KL,
and suppose have been extended Θ A
and HB
to the points Λ and M.

A parallelogram therefore is Θ AKZ,
a diameter of it is Θ K,
and about Θ K [are]
the parallelograms AH and ME,
and the so-called complements,
AB and BZ;

Παρά τὴν δοθεῖσαν εὐθεῖαν
τῷ δοθέντι τριγώνῳ ἴσον
παράλληλόγραμμον παραβαλεῖν
ἐν τῇ δοθείσῃ γωνίᾳ εὐθύγραμμῳ.

Ἐστω
ἡ μὲν δοθεῖσα εὐθεῖα ἡ AB,
τὸ δὲ δοθὲν τρίγωνον τὸ Γ ,
ἡ δὲ δοθεῖσα γωνία εὐθύγραμμος ἡ Δ .

δεῖ δὲ
παρά τὴν δοθεῖσαν εὐθεῖαν τὴν AB
τῷ δοθέντι τριγώνῳ τῷ Γ ἴσον
παράλληλόγραμμον παραβαλεῖν
ἐν ἴσῃ τῇ Δ γωνίᾳ.

Συνεστάτω
τῷ Γ τριγώνῳ ἴσον
παράλληλόγραμμον τὸ BEZH
ἐν γωνίᾳ τῇ ὑπὸ EBH,
ἣ ἐστὶν ἴση τῇ Δ .
καὶ κείσθω
ὥστε ἐπ' εὐθείας εἶναι τὴν BE
τῇ AB,
καὶ διήχθω
ἡ ZH ἐπὶ τὸ Θ ,
καὶ διὰ τοῦ A
ὁποτέρᾳ τῶν BH, EZ
παράλληλος ἦχθω ἡ A Θ ,
καὶ ἐπεζεύχθω ἡ Θ B.

καὶ ἐπεὶ εἰς παράλληλους τὰς A Θ , EZ
εὐθεῖα ἐνέπεσεν ἡ Θ Z,
αἱ ἄρα ὑπὸ A Θ Z, Θ ZE γωνίαι
δυσὶν ὀρθαῖς εἰσὶν ἴσαι.
αἱ ἄρα ὑπὸ B Θ H, HZE
δύο ὀρθῶν ἐλάσσονές εἰσιν.
αἱ δὲ ἀπὸ ἐλασσόνων ἢ δύο ὀρθῶν εἰς
ἄπειρον ἐχβαλλόμεναι
συμπίπτουσιν.
αἱ Θ B, ZE ἄρα ἐχβαλλόμεναι
συμπεσοῦνται.

ἐχβεβλήσθωσαν
καὶ συμπίπτωσαν κατὰ τὸ K,
καὶ διὰ τοῦ K σημείου
ὁποτέρᾳ τῶν EA, Z Θ παράλληλος
ἦχθω ἡ KL,
καὶ ἐχβεβλήσθωσαν αἱ Θ A, HB
ἐπὶ τὰ Λ , M σημεία.

παράλληλόγραμμον ἄρα ἐστὶ τὸ Θ AKZ,
διάμετρος δὲ αὐτοῦ ἡ Θ K,
περὶ δὲ τὴν Θ K
παράλληλόγραμμα μὲν τὰ AH, ME,
τὰ δὲ λεγόμενα παραπληρώματα
τὰ AB, BZ.

Verilen bir doğru boyunca
verilen bir üçgene eşit,
bir paralel kenarı yerleştirmek
verilen bir düz kenar açıda.

Verilen doğru AB,
ve verilen üçgen Γ ,
ve verilen düzkenar açı Δ olsun.

Şimdi gereklidir
verilen AB doğrusu boyunca
 Γ üçgenine eşit
bir paralelkenarı
 Δ açısında yerleştirmek.

Varsayalım inşa edildiği
 Γ üçgenine eşit,
bir BEZH paralelkenarının
EBH açısında,
eşit olan Δ açısına;
ve öyle yerleştirilmiş olsun ki
bir doğruda kalsın BE,
AB ile,
ve çizilmiş olsun
ZH doğrusundan Θ noktasına,
ve A noktasından,
paralel olan BH ve EZ doğrularından
birine,
çizilmiş olsun
A Θ ,
ve birleştirilmiş olsun
 Θ B.

Ve A Θ ile EZ paralellerinin üzerine
düştüğünden Θ Z doğrusu,
A Θ Z ve Θ ZE açıları
eşittir iki dik açıya.
Dolayısıyla B Θ H ve HZE
küçüktür iki dik açıdan.
Ve küçük olanlardan
iki dik açıdan,
uzatıldıklarında sonsuza,
birbirlerine düşerler doğrular.
Dolayısıyla Θ B ve ZE, uzatılırsa,
birbirlerine düşerler.

Varsayalım uzatıldıkları,
ve K noktasında kesiştikleri,
ve K noktasından,
paralel olan EA veya Z Θ doğrusuna,
çizilmiş olsun KL,
ve uzatılmış olsunlar Θ A ve HB doğru-
ları
 Λ ve M noktalarından.

Bir paralelkenardır dolayısıyla Θ AKZ,
ve onun köşegeni Θ K,
ve Θ K etrafındadır
AH ve ME paralelkenarları,
ve bunların tümleyenleris,
AB ile BZ;

equal therefore is ΛB to BZ .
But BZ to triangle Γ is equal.
Also therefore ΛB to Γ is equal.
And since equal is
angle HBE to ABM ,
but HBE to Δ is equal,
also therefore ABM to Δ
is equal.

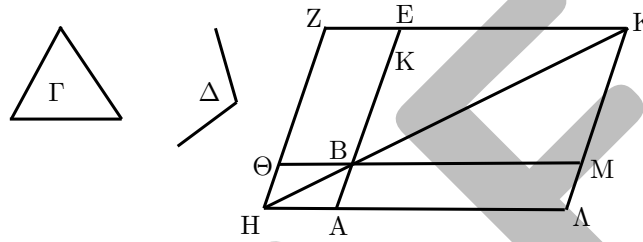
Therefore, along the given STRAIGHT,
 AB ,
equal to the given triangle, Γ ,
a parallelogram has been applied,
 ΛB ,
in the angle ABM ,
which is equal to Δ ;
—just what it was necessary to do.

ἴσον ἄρα ἐστὶ τὸ ΛB τῷ BZ .
ἀλλὰ τὸ BZ τῷ Γ τριγώνῳ ἐστὶν ἴσον·
καὶ τὸ ΛB ἄρα τῷ Γ ἐστὶν ἴσον.
καὶ ἐπεὶ ἴση ἐστὶν
ἡ ὑπὸ HBE γωνία τῇ ὑπὸ ABM ,
ἀλλὰ ἡ ὑπὸ HBE τῇ Δ ἐστὶν ἴση,
καὶ ἡ ὑπὸ ABM ἄρα τῇ Δ γωνία
ἐστὶν ἴση.

Παρά τὴν δοθεῖσαν ἄρα εὐθεΐαν
τὴν AB
τῷ δοθέντι τριγώνῳ τῷ Γ ἴσον
παρὰ τὴν AB παραβέβληται
τὸ ΛB
ἐν γωνίᾳ τῇ ὑπὸ ABM ,
ἣ ἐστὶν ἴση τῇ Δ .
ὅπερ ἔδει ποιῆσαι.

eşittirler dolayısıyla ΛB ile BZ
tümleyenlerine.
Ama BZ , Γ üçgenine eşittir.
Dolayısıyla ΛB da Γ üçgenine eşittir.
Ve eşit olduğundan
 HBE , ABM açısına,
fakat HBE , Δ açısına eşit,
dolayısıyla ABM de Δ açısına
eşittir.

Dolayısıyla, verilen bir,
 AB doğrusu boyunca,
verilen bir Γ üçgenine eşit,
bir,
 ΛB paralelkenarı yerleştirilmiş oldu,
 ABM açısında,
eşit olan Δ açısına;
— yapılması gereken tam buydu.



3.45

To the given rectilinear [figure] equal
a parallelogram to construct
in the given rectilinear angle.

Let be
the given rectilinear [figure] $AB\Gamma\Delta$,
and the given rectilinear angle, E .

It is necessary then
to the rectilinear $AB\Gamma\Delta$ equal
a parallelogram to construct
in the given angle E .

Suppose has been joined ΔB ,
and suppose has been constructed,
equal to the triangle $AB\Delta$,
a parallelogram, $Z\Theta$,
in the angle ΘKZ ,
which is equal to E ;
and suppose there has been applied
along the STRAIGHT $H\Theta$,
equal to triangle $\Delta B\Gamma$,
a parallelogram, HM ,
in the angle $H\Theta M$,
which is equal to E .

And since angle E
to either of ΘKZ and $H\Theta M$
is equal,
therefore also ΘKZ to $H\Theta M$
is equal.
Let $K\Theta H$ be added in common;

Τῷ δοθέντι εὐθύγραμμῳ ἴσον
παρὰ τὴν AB παραβέβληται
τὸ ΛB
ἐν γωνίᾳ τῇ ὑπὸ ABM ,
ἣ ἐστὶν ἴση τῇ Δ .

Ἐστὼ
τὸ μὲν δοθέν εὐθύγραμμον τὸ $AB\Gamma\Delta$,
ἡ δὲ δοθεῖσα γωνία εὐθύγραμμος ἡ E .

δεῖ δὴ
τῷ $AB\Gamma\Delta$ εὐθύγραμμῳ ἴσον
παρὰ τὴν AB παραβέβληται
τὸ ΛB
ἐν γωνίᾳ τῇ ὑπὸ ABM ,
ἣ ἐστὶν ἴση τῇ Δ .

Ἐπεζεύχθω ἡ ΔB ,
καὶ συνεστάτω
τῷ $AB\Delta$ τριγώνῳ ἴσον
παρὰ τὴν AB παραβέβληται
τὸ $Z\Theta$
ἐν τῇ ὑπὸ ΘKZ γωνίᾳ,
ἣ ἐστὶν ἴση τῇ E .
καὶ παραβέβλησθω
παρὰ τὴν $H\Theta$ εὐθεΐαν
τῷ $\Delta B\Gamma$ τριγώνῳ ἴσον
παρὰ τὴν $H\Theta$ εὐθεΐαν
τὸ HM
ἐν τῇ ὑπὸ $H\Theta M$ γωνίᾳ,
ἣ ἐστὶν ἴση τῇ E .

καὶ ἐπεὶ ἡ E γωνία
ἐκάτερά τῶν ὑπὸ ΘKZ , $H\Theta M$
ἐστὶν ἴση,
καὶ ἡ ὑπὸ ΘKZ ἄρα τῇ ὑπὸ $H\Theta M$
ἐστὶν ἴση.
κοινὴ προσκείσθω ἡ ὑπὸ $K\Theta H$.

Verilen bir düzkenar [figüre] eşitli
bir paralelkenar inşa etmek,
verilen düzkenar açıda.

Verilmiş olsun
 $AB\Gamma\Delta$ düzkenar [figürü],
ve düzkenar E açısı.

Gereklidir şimdi
 $AB\Gamma\Delta$ düzkenarına eşit
bir paralelkenar inşa etmek,
verilen E açısında.

Birleştirilmiş olduğu ΔB doğrusunun,
ve inşa edilmiş olsun,
 $AB\Delta$ üçgenine eşit,
bir $Z\Theta$ paralelkenarı,
 ΘKZ açısında,
eşit olan E açısına;
ve yerleştirilmiş olsun
 $H\Theta$ doğrusu boyunca,
 $\Delta B\Gamma$ üçgenine eşit,
bir HM paralelkenarı,
 $H\Theta M$ açısında,
eşit olan E açısına.

Ve E açısı
 ΘKZ ve $H\Theta M$ açılarının her birine
eşit olduğundan,
 ΘKZ da $H\Theta M$ açısına
eşittir.
Eklenmiş olsun $K\Theta H$ ortak olarak;

therefore $ZK\Theta$ and $K\Theta H$
to $K\Theta H$ and $H\Theta M$
are equal.

But $ZK\Theta$ and $K\Theta H$
are equal to two RIGHTS;
therefore also $K\Theta H$ and $H\Theta M$
are equal to two RIGHTS.

Then to some STRAIGHT, $H\Theta$,
and at the same point, Θ ,
two STRAIGHTS, $K\Theta$ and ΘM ,
not lying in the same parts,
the adjacent angles
make equal to two RIGHTS.

In a STRAIGHT then are $K\Theta$ and ΘM ;
and since on the parallels KM and ZH
fell the STRAIGHT ΘH ,
the alternate angles $M\Theta H$ and $\Theta H Z$
are equal to one another.

Let $\Theta H A$ be added in common;
therefore $M\Theta H$ and $\Theta H A$
to $\Theta H Z$ and $\Theta H A$
are equal.

But $M\Theta H$ and $\Theta H A$
are equal to two RIGHTS;
therefore also $\Theta H Z$ and $\Theta H A$
are equal to two RIGHTS;
therefore on a STRAIGHT are ZH and
 $H A$.

And since ZK to ΘH
is equal and parallel,
but also ΘH to $M A$,
therefore also KZ to $M A$
is equal and parallel;
and join them

KM and $Z A$, which are STRAIGHTS;
therefore also KM and $Z A$
are equal and parallel;
a parallelogram therefore is $KZ A M$.
And since equal is
triangle $A B \Delta$
to the parallelogram $Z \Theta$,
and $\Delta B \Gamma$ to $H M$,
therefore, as a whole,
the rectilinear $A B \Gamma \Delta$
to parallelogram $KZ A M$ as a whole
is equal.

Therefore, to the given rectilinear [fig-
ure], $A B \Gamma \Delta$, equal,
a parallelogram has been constructed,
 $KZ A M$,
in the angle $Z K M$,
which is equal to the given E ;
—just what it was necessary to do.

αἱ ἄρα ὑπὸ $ZK\Theta$, $K\Theta H$
ταῖς ὑπὸ $K\Theta H$, $H\Theta M$
ἴσαι εἰσίν.

ἀλλ' αἱ ὑπὸ $ZK\Theta$, $K\Theta H$
δυσὶν ὀρθαῖς ἴσαι εἰσίν·
καὶ αἱ ὑπὸ $K\Theta H$, $H\Theta M$ ἄρα
δύο ὀρθαῖς ἴσαι εἰσίν.
πρὸς δὴ τινὶ εὐθεῖᾳ τῇ $H\Theta$
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ Θ
δύο εὐθεῖαι αἱ $K\Theta$, ΘM
μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
δύο ὀρθαῖς ἴσας ποιοῦσιν·

ἐπ' εὐθείας ἄρα ἐστὶν ἡ $K\Theta$ τῇ ΘM ·
καὶ ἐπεὶ εἰς παραλλήλους τὰς KM , ZH
εὐθεῖα ἐνέπεσεν ἡ ΘH ,
αἱ ἐναλλάξ γωνίαι αἱ ὑπὸ $M\Theta H$, $\Theta H Z$
ἴσαι

ἀλλήλαις εἰσίν.
κοινὴ προσκεῖσθω ἡ ὑπὸ $\Theta H A$ ·
αἱ ἄρα ὑπὸ $M\Theta H$, $\Theta H A$ ταῖς ὑπὸ $\Theta H Z$,
 $\Theta H A$

ἴσαι εἰσίν.
ἀλλ' αἱ ὑπὸ $M\Theta H$, $\Theta H A$
δύο ὀρθαῖς ἴσαι εἰσίν·
καὶ αἱ ὑπὸ $\Theta H Z$, $\Theta H A$ ἄρα
δύο ὀρθαῖς ἴσαι εἰσίν·
ἐπ' εὐθείας ἄρα ἐστὶν ἡ ZH τῇ $H A$.

καὶ ἐπεὶ ἡ ZK τῇ ΘH
ἴση τε καὶ παράλληλός ἐστιν,
ἀλλὰ καὶ ἡ ΘH τῇ $M A$,
καὶ ἡ KZ ἄρα τῇ $M A$
ἴση τε καὶ παράλληλός ἐστιν·
καὶ ἐπιζευγνύουσιν αὐτάς εὐθεῖαι αἱ
 KM , $Z A$ ·
καὶ αἱ KM , $Z A$ ἄρα
ἴσαι τε καὶ παράλληλοί εἰσιν·
παραλληλόγραμμον ἄρα ἐστὶ τὸ
 $KZ A M$.

καὶ ἐπεὶ ἴσον ἐστὶ
τὸ μὲν $A B \Delta$ τρίγωνον τῷ $Z \Theta$ παραλ-
ληλογράμμῳ,
τὸ δὲ $\Delta B \Gamma$ τῷ $H M$,
ὅλον ἄρα τὸ $A B \Gamma \Delta$ εὐθύγραμμον
ὅλω τῷ $KZ A M$ παραλληλογράμμῳ
ἐστὶν ἴσον.

Τῷ ἄρα δοθέντι εὐθυγράμμῳ τῷ $A B \Gamma \Delta$
ἴσον
παραλληλόγραμμον συνέσταται
τὸ $KZ A M$
ἐν γωνίᾳ τῇ ὑπὸ $Z K M$,
ἣ ἐστὶν ἴση τῇ δοθείσῃ τῇ E ·
ὅπερ ἔδει ποιῆσαι.

dolayısıyla $ZK\Theta$ ve $K\Theta H$,
 $K\Theta H$ ve $H\Theta M$ açılına
eşittirler.

Fakat $ZK\Theta$ ve $K\Theta H$
eşittirler iki dik açıya;
dolayısıyla $K\Theta H$ ve $H\Theta M$ açılarında
eşittirler iki dik açıya.

Şimdi bir $H\Theta$ doğrusuna,
ve aynı Θ noktasında,
iki $K\Theta$ ve ΘM doğruları,
aynı tarafta kalmayan,
komşu açıları
iki dik açıya eşit yapar.

O zaman bir doğrudadır $K\Theta$ ve ΘM ;
ve KM ve ZH paralelleri üzerine
düştüğünden ΘH doğrusu,
ters $M\Theta H$ ve $\Theta H Z$ açıları
eşittir birbirine.

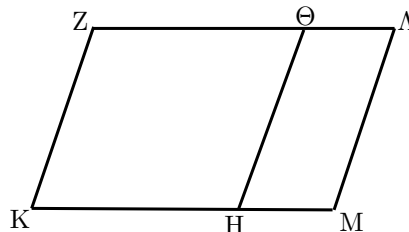
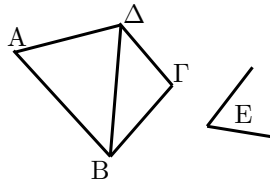
eklenmiş olsun $\Theta H A$ ortak olarak;
dolayısıyla $M\Theta H$ ve $\Theta H A$,
 $\Theta H Z$ ve $\Theta H A$ açılına
eşittirler.

Fakat $M\Theta H$ ve $\Theta H A$
eşittirler iki dik açıya;
dolayısıyla $\Theta H Z$ ve $\Theta H A$ da
eşittirler iki dik açıya;
dolayısıyla bir doğru üzerindedir ZH
ve $H A$.

Ve olduğundan ZK , ΘH doğrusuna
eşit ve paralel,
ve de ΘH , $M A$ doğrusuna,
dolayısıyla KZ da $M A$ doğrusuna
eşit ve paraleldir;
ve birleştirir onları KM ile $Z A$, ki bun-
larda doğrulardır;

dolayısıyla KM ve $Z A$ da
eşit ve paraleldirler;
dolayısıyla $KZ A M$ bir paralelkenardır.
Ve eşit olduğundan
 $A B \Delta$ üçgeni
 $Z \Theta$ paralelkenarına,
ve $\Delta B \Gamma$, $H M$ paralelkenarına,
dolayısıyla, bir bütün olarak,
 $A B \Gamma \Delta$ düzkenarı
bir bütün olarak $KZ A M$ paralelke-
narına
eşittir.

Dolayısıyla, verilen düzkenar $A B \Gamma \Delta$
figürüne eşit,
bir $KZ A M$ paralelkenarı inşa edilmiş
oldu,
 $Z K M$ açısında,
eşit olan verilmiş E açısına;
— yapılması gereken tam buydu.



3.46

On the given STRAIGHT
to set up a square.

Let be
the given STRAIGHT AB.

It is required then
on the STRAIGHT AB
to set up a square.

Suppose there has been drawn
to the STRAIGHT AB,
at the point A of it,
at a RIGHT,
ΑΓ,
and suppose there has been laid down,
equal to AB,
ΑΔ;
and through the point Δ,
parallel to AB,
suppose there has been drawn ΔΕ;
and through the point Β,
parallel to ΑΔ,
suppose there has been drawn ΒΕ.

A parallelogram therefore is ΑΔΕΒ;
equal therefore is ΑΒ to ΔΕ,
and ΑΔ to ΒΕ.
But ΑΒ to ΑΔ is equal.
Therefore the four
ΒΑ, ΑΔ, ΔΕ, and ΕΒ
are equal to one another;
equilateral therefore
is the parallelogram ΑΔΕΒ.

I say then that
it is also right-angled.

For, since on the parallels ΑΒ and ΔΕ
fell the STRAIGHT ΑΔ,
therefore the angles ΒΑΔ and ΑΔΕ
are equal to two RIGHTS.
And ΒΑΔ is right;
right therefore is ΑΔΕ.
And of parallelogram areas
the opposite sides and angles
are equal to one another.
Right therefore is either
of the opposite angles ΑΒΕ and ΒΕΔ;
right-angled therefore is ΑΔΕΒ.
And it was shown also equilateral.

A square therefore it is;
and it is on the STRAIGHT ΑΒ
set up;
—just what it was necessary to do.

Ἀπὸ τῆς δοθείσης εὐθείας
τετράγωνον ἀναγράψαι.

Ἐστω
ἡ δοθεῖσα εὐθεῖα ἡ ΑΒ·

δεῖ δὴ
ἀπὸ τῆς ΑΒ εὐθείας
τετράγωνον ἀναγράψαι.

Ἦχθω
τῆ ΑΒ εὐθεία
ἀπὸ τοῦ πρὸς αὐτῇ σημείου τοῦ Α
πρὸς ὀρθὰς
ἡ ΑΓ,
καὶ κείσθω
τῆ ΑΒ ἴση
ἡ ΑΔ·
καὶ διὰ μὲν τοῦ Δ σημείου
τῆ ΑΒ παράλληλος
ἦχθω ἡ ΔΕ,
διὰ δὲ τοῦ Β σημείου
τῆ ΑΔ παράλληλος
ἦχθω ἡ ΒΕ.

παρὰλληλόγραμμον ἄρα ἐστὶ τὸ ΑΔΕΒ·
ἴση ἄρα ἐστὶν ἡ μὲν ΑΒ τῆ ΔΕ,
ἡ δὲ ΑΔ τῆ ΒΕ.
ἀλλὰ ἡ ΑΒ τῆ ΑΔ ἐστὶν ἴση·
αἱ τέσσαρες ἄρα
αἱ ΒΑ, ΑΔ, ΔΕ, ΕΒ
ἴσαι ἀλλήλαις εἰσὶν·
ἰσόπλευρον ἄρα
ἐστὶ τὸ ΑΔΕΒ παρὰλληλόγραμμον.

λέγω δὴ, ὅτι
καὶ ὀρθογώνιον.

ἐπεὶ γὰρ εἰς παρὰλλήλους τὰς ΑΒ, ΔΕ
εὐθεῖα ἐνέπεσεν ἡ ΑΔ,
αἱ ἄρα ὑπὸ ΒΑΔ, ΑΔΕ γωνία
δύο ὀρθαῖς ἴσαι εἰσὶν.
ὀρθὴ δὲ ἡ ὑπὸ ΒΑΔ·
ὀρθὴ ἄρα καὶ ἡ ὑπὸ ΑΔΕ.
τῶν δὲ παρὰλληλογράμμων χωρίων
αἱ ἀπεναντίον πλευραὶ τε καὶ γωνία
ἴσαι ἀλλήλαις εἰσὶν·
ὀρθὴ ἄρα καὶ ἑκατέρω
τῶν ἀπεναντίον τῶν ὑπὸ ΑΒΕ, ΒΕΔ
γωνιῶν·
ὀρθογώνιον ἄρα ἐστὶ τὸ ΑΔΕΒ.
ἐδείχθη δὲ καὶ ἰσόπλευρον.

Τετράγωνον ἄρα ἐστὶν
καὶ ἐστὶν ἀπὸ τῆς ΑΒ εὐθείας
ἀναγεγραμμένον·
ὅπερ ἔδει ποιῆσαι.

Verilen bir doğruya
bir kare kurmak.

Verilmiş olsun
ΑΒ doğrusu.

Şimdi gereklidir
ΑΒ doğrusunda
bir kare kurmak.

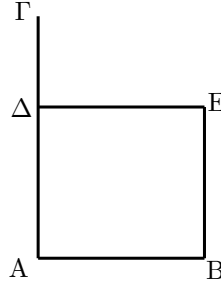
Çizilmiş olsun
ΑΒ doğrusunda,
onun Α noktasında,
dik açıya,
ΑΓ,
ve yerleştirilmiş olsun,
ΑΒ doğrusuna eşit,
ΑΔ;
ve Δ noktasından,
ΑΒ doğrusuna paralel,
çizilmiş olsun ΔΕ;
ve Β noktasından,
ΑΔ doğrusuna paralel,
ΒΕ çizilmiş olsun.

Bir paralelkenardır dolayısıyla
ΑΔΕΒ;
eşittir dolayısıyla ΑΒ, ΔΕ doğrusuna,
ve ΑΔ, ΒΕ doğrusuna.
Ama ΑΒ, ΑΔ doğrusuna eşittir.
Dolayısıyla şu dördü
ΒΑ, ΑΔ, ΔΕ ve ΕΒ
birbirlerine eşittirler;
eşkenardır dolayısıyla
ΑΔΕΒ paralelkenarı.

Şimdi iddia ediyorum ki
aynı zamanda dik açıdır.

Çünkü, ΑΒ ve ΔΕ paralellerinin üzer-
ine
düştüğünden ΑΔ doğrusu,
eşittir dolayısıyla ΒΑΔ ve ΑΔΕ
iki dik açıya.
Ve ΒΑΔ diktir;
diktir dolayısıyla ΑΔΕ.
Ve paralelkenar alanların
karşıt kenar ve açıları
eşittir birbirlerine.
Diktir dolayısıyla her bir
karşıt açı ΑΒΕ ve ΒΕΔ;
dik açıdır dolayısıyla ΑΔΕΒ.
Ve gösterilmişti ki eşkenardır da.

Bir karedir dolayısıyla ο;
ve ο ΑΒ doğrusu üzerine
kurulmuştur;
— yapılması gereken tam buydu.



3.47

In right-angled triangles,
the square on the side that subtends
the right angle
is equal
to the squares on the sides that con-
tain the right angle.

Let be
a right-angled triangle, ABΓ,
having the angle BAG right.

I say that
the square on BΓ
is equal
to the squares on BA and AG.

For, suppose there has been set up
on BΓ
a square, BΔΕΓ,
and on BA and AG,
HB and ΘΓ,
and through A,
parallel to either of BΔ and ΓΕ,
suppose AA has been drawn;
and suppose have been joined
AΔ and ZΓ.

And since right is
either of the angles BAG and BAH,
on some STRAIGHT, BA,
to the point A on it,
two STRAIGHTS, AG and AH,
not lying in the same parts,
the adjacent angles
make equal to two RIGHTS;
on a STRAIGHT therefore is ΓA with
AH.

Then for the same [reason]
also BA with AΘ is on a STRAIGHT.
And since equal is
angle ΔBΓ to angle ZBA;
for either is RIGHT;
let ABΓ be added in common;
therefore ΔBA as a whole
to ZBΓ as a whole
is equal.
And since equal is

Ἐν τοῖς ὀρθογωνίοις τριγώνοις
τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτε-
νοῦσης πλευρᾶς τετραγώνον
ἴσον ἐστὶ
τοῖς ἀπὸ τῶν τὴν ὀρθὴν γωνίαν περιε-
χουσῶν πλευρῶν τετραγώνοις.

Ἐστω
τρίγωνον ὀρθογώνιον τὸ ABΓ
ὀρθὴν ἔχον τὴν ὑπὸ BAG γωνίαν.

λέγω, ὅτι
τὸ ἀπὸ τῆς BΓ τετραγώνον
ἴσον ἐστὶ
τοῖς ἀπὸ τῶν BA, AG τετραγώνοις.

Ἀναγεγράφθω γὰρ
ἀπὸ μὲν τῆς BΓ
τετραγώνον τὸ BΔΕΓ,
ἀπὸ δὲ τῶν BA, AG
τὰ HB, ΘΓ,
καὶ διὰ τοῦ A
ὀποτέρᾳ τῶν BΔ, ΓΕ παράλληλος
ἦχθω ἡ AA.¹
καὶ ἐπεζεύχθωσαν
αἱ AΔ, ZΓ.

καὶ ἐπεὶ ὀρθὴ ἐστὶν
ἑκάτερα τῶν ὑπὸ BAG, BAH γωνιῶν,
πρὸς δὴ τινὶ εὐθείᾳ τῆς BA
καὶ τῷ πρὸς αὐτῇ σημείῳ τῷ A
δύο εὐθεῖαι αἱ AG, AH
μὴ ἐπὶ τὰ αὐτὰ μέρη κείμεναι
τὰς ἐφεξῆς γωνίας
δυσὶν ὀρθαῖς ἴσας ποιῶσιν.
ἐπ' εὐθείας ἄρα ἐστὶν ἡ ΓA τῆς AH.
διὰ τὰ αὐτὰ δὲ
καὶ ἡ BA τῆς AΘ ἐστὶν ἐπ' εὐθείας.
καὶ ἐπεὶ ἴση ἐστὶν
ἡ ὑπὸ ΔBΓ γωνία τῆς ὑπὸ ZBA.
ὀρθὴ γὰρ ἑκάτερα·
κοινὴ προσκείμεθα ἡ ὑπὸ ABΓ.
ὅλη ἄρα ἡ ὑπὸ ΔBA
ὅλη τῆς ὑπὸ ZBΓ
ἐστὶν ἴση.
καὶ ἐπεὶ ἴση ἐστὶν
ἡ μὲν ΔB τῆς BΓ,

Dik açılı üçgenlerde,
dik açının gördüğü kenar üzerindeki
kare
eşittir
dik açığı içeren kenarların üzerindeki
karelere.

Verilmiş olsun
dik açılı bir ABΓ üçgeni
BAG açısı dik olan.

İddia ediyorum ki
BΓ üzerindeki kare
eşittir
BA ve AG üzerlerindeki karelere.

Çünkü, kurulmuş olsun
BΓ üzerinde
bir BΔΕΓ karesi,
ve BA ile AG üzerlerinde,
HB ve ΘΓ,
ve A noktasından,
BΔ ve ΓΕ doğrularına paralel olan,
AA çizilmiş olsun;
ve birleştirilmiş olsun
AΔ ve ZΓ.

Ve dik olduğundan
BAG ve BAH açıların her biri,
bir BA doğrusunda,
üzerindeki A noktasına,
AG ve AH doğruları,
aynı tarafta kalmayan,
bitişik açılar
oluştururlar eşit iki dik açığa;
bir doğrudadır dolayısıyla ΓA ile AH.
Sonra aynı nedenle
BA ile AΘ da bir doğrudadır.
Ve eşit olduğundan
ΔBΓ, ZBA açısına;
her ikisinde dikdir;
eklenmiş olsun ABΓ her ikisine de;
dolayısıyla ΔBA açısının tamamı
ZBΓ açısının tamamına
eşittir.
Ve eşit olduğundan
ΔB, BΓ doğrusuna,

¹Heiberg's text [1, p. 110] has Δ for Λ at this place and else-
where (though not in the diagram). Probably this is a compositor's

mistake, owing to the similarity in appearance of the two letters,
especially in the font used.

ΔB to $B\Gamma$,
and ZB to BA ,
the two ΔB and BA
to the two ZB and $B\Gamma$ ²
are equal,
either to either;
and angle ΔBA
to angle $ZB\Gamma$
is equal;
therefore the base AA
to the base $Z\Gamma$
[is] equal,
and the triangle ABA
to the triangle $ZB\Gamma$
is equal;
and of the triangle $AB\Delta$
the parallelogram BA is double;
for they have the same base, BA ,
and are in the same parallels,
 $B\Delta$ and AA ;
and of the triangle $ZB\Gamma$
the square HB is double;
for again they have the same base,
 ZB ,
and are in the same parallels,
 ZB and $H\Gamma$.
[And of equals,
the doubles are equal to one another.]
Equal therefore is
also the parallelogram BA
to the square HB .
Similarly then,
there being joined AE and BK ,
it will be shown that
also the parallelogram ΓA
[is] equal to the square $\Theta\Gamma$.
Therefore the square $\Delta BE\Gamma$ as a
whole
to the two squares HB and $\Theta\Gamma$
is equal.
Also is
the square $B\Delta E\Gamma$ set up on $B\Gamma$,
and HB and $\Theta\Gamma$ on BA and $A\Gamma$.
Therefore the square on the side $B\Gamma$
is equal
to the squares on the sides BA and
 $A\Gamma$.

Therefore in right-angled triangles
the square on the side subtending the
right angle
is equal
to the squares on the sides subtending
the right [angle];
—just what it was necessary to show.

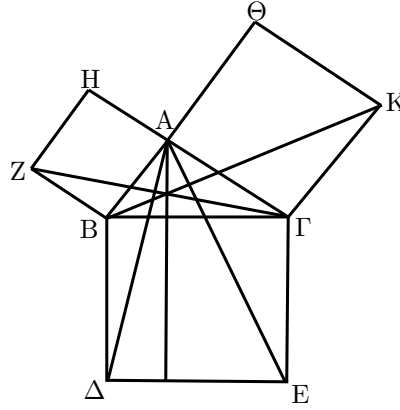
ἡ δὲ ZB τῆ BA ,
δύο δὴ αἱ ΔB , BA
δύο ταῖς ZB , $B\Gamma$
ἴσαι εἰσὶν
ἑκατέρα ἑκατέρῃ·
καὶ γωνία ἡ ὑπὸ ΔBA
γωνία τῆ ὑπὸ $ZB\Gamma$
ἴση·
βάσις ἄρα ἡ AA
βάσει τῆ $Z\Gamma$
[ἔστιν] ἴση,
καὶ τὸ ABA τρίγωνον
τῷ $ZB\Gamma$ τριγώνῳ
ἔστιν ἴσον·
καὶ [ἔστι] τοῦ μὲν $AB\Delta$ τριγώνου
διπλάσιον τὸ BA παραλληλόγραμμον·
βάσιν τε γὰρ τὴν αὐτὴν ἔχουσι τὴν $B\Delta$
καὶ ἐν ταῖς αὐταῖς εἰσι παραλλήλοις
ταῖς $B\Delta$, AA ·
τοῦ δὲ $ZB\Gamma$ τριγώνου
διπλάσιον τὸ HB τετράγωνον·
βάσιν τε γὰρ πάλιν τὴν αὐτὴν ἔχουσι
τὴν ZB
καὶ ἐν ταῖς αὐταῖς εἰσι παραλλήλοις
ταῖς ZB , $H\Gamma$.
[τὰ δὲ τῶν ἴσων
διπλάσια ἴσα ἀλλήλοις ἔστιν·]
ἴσον ἄρα ἔστι
καὶ τὸ BA παραλληλόγραμμον
τῷ HB τετραγώνῳ.
ὁμοίως δὴ
ἐπιζευγνυμένων τῶν AE , BK
δειχθήσεται
καὶ τὸ ΓA παραλληλόγραμμον
ἴσον τῷ $\Theta\Gamma$ τετραγώνῳ·
ὅλον ἄρα τὸ $B\Delta E\Gamma$ τετράγωνον
δυοῖς τοῖς HB , $\Theta\Gamma$ τετραγώνοις
ἴσον ἔστιν.
καὶ ἔστι
τὸ μὲν $B\Delta E\Gamma$ τετράγωνον ἀπὸ τῆς $B\Gamma$
ἀναγραφέν,
τὰ δὲ HB , $\Theta\Gamma$ ἀπὸ τῶν BA , $A\Gamma$.
τὸ ἄρα ἀπὸ τῆς $B\Gamma$ πλευρᾶς τετράγω-
νον
ἴσον ἔστι
τοῖς ἀπὸ τῶν BA , $A\Gamma$ πλευρῶν τε-
τραγώνοις.

Ἐν ἄρα τοῖς ὀρθογωνίοις τριγώνοις
τὸ ἀπὸ τῆς τὴν ὀρθὴν γωνίαν ὑποτεί-
νούσης πλευρᾶς τετράγωνον
ἴσον ἔστι
τοῖς ἀπὸ τῶν τὴν ὀρθὴν [γωνίαν] περιε-
χουσῶν πλευρῶν τετραγώνοις·
ὅπερ ἔδει δεῖξαι.

ve ZB , BA doğrusuna
 ΔB ve BA ikilisi
 ZB ve $B\Gamma$ ikilisine³
eşittirler,
her biri birine;
ve ΔBA açısı
 $ZB\Gamma$ açısına
eşittir;
dolayısıyla AA tabanı
 $Z\Gamma$ tabanına
eşittir,
ve ABA üçgeni
 $ZB\Gamma$ üçgenine
eşittir;
ve $AB\Delta$ üçgeninin
 BA paralelkenarı iki katıdır;
aynı BA tabanları olduğu,
ve aynı
 $B\Delta$ ve AA paralellerinde oldukları için;
ve $ZB\Gamma$ üçgeninin
 HB karesi iki katıdır;
yine aynı
 ZB tabanları olduğu
ve aynı
 ZB ve $H\Gamma$ paralellerinde oldukları için.
[Ve eşitlerin,
iki katları birbirlerine eşittirler.]
Eşittir dolayısıyla
 BA paralelkenarı da
 HB karesine.
Şimdi benzer şekilde,
birleştirildiğinde AE ve BK ,
gösterilecek ki
 ΓA paralelkenarı da
eşittir $\Theta\Gamma$ karesine.
Dolayısıyla $\Delta BE\Gamma$ bir bütün olarak
 HB ve $\Theta\Gamma$ iki karesine
eşittir.
Ayrıca
 $B\Delta E\Gamma$ karesi $B\Gamma$ üzerine kurulmuştur,
ve HB ve $\Theta\Gamma$, BA ve $A\Gamma$ üzerine.
Dolayısıyla $B\Gamma$ kenarındaki kare
eşittir
 BA ve $A\Gamma$ kenarlarındaki karelere.

Dolayısıyla dik açılı üçgenlerde,
dik açının gördüğü kenar üzerindeki
kare
eşittir
dik açığı içeren kenarların üzerindeki
ilere;
— gösterilmesi gereken tam buydu.

²Fitzpatrick considers this ordering of the two straight lines to be 'obviously a mistake'. But if it is a mistake, how could it have been made?



3.48

If of a triangle
the square on one of the sides
be equal
to the squares on the remaining sides
of the triangle,
the angle contained
by the two remaining sides of the tri-
angle
is right.

For, of the triangle ABΓ
the square on the one side BΓ
—suppose it is equal
to the squares on the sides BA and
AΓ.

I say that
right is the angle BAΓ.

For, suppose has been drawn
from the point A
to the STRAIGHT AΓ
at RIGHTS
AΔ,
and let be laid down
equal to BA
AΔ,
and suppose ΔΓ has been joined.

Since equal is ΔA to AB,
equal is
also the square on ΔA
to the square on AB.
Let be added in common
the square on AΓ;
therefore the squares on ΔA and AΓ
are equal
to the squares on BA and AΓ.
But those on ΔA and AΓ
are equal
to that on ΔΓ;
for right is the angle ΔAΓ;
and those on BA and AΓ
are equal

Ἐὰν τριγώνου
τὸ ἀπὸ μιᾶς τῶν πλευρῶν τετράγωνον
ἴσον ᾗ
τοῖς ἀπὸ τῶν λοιπῶν τοῦ τριγώνου δύο
πλευρῶν τετραγώνοις,
ἢ περιεχομένη γωνία
ὑπὸ τῶν λοιπῶν τοῦ τριγώνου δύο
πλευρῶν
ὀρθή ἐστίν.

Τριγώνου γὰρ τοῦ ABΓ
τὸ ἀπὸ μιᾶς τῆς BΓ πλευρᾶς τετράγω-
νον
ἴσον ἔστω
τοῖς ἀπὸ τῶν BA, AΓ πλευρῶν τε-
τραγώνοις·

λέγω, ὅτι
ὀρθή ἐστίν ἢ ὑπὸ BAΓ γωνία.

Ἦχθω γὰρ
ἀπὸ τοῦ A σημείου
τῆς AΓ εὐθείᾳ
πρὸς ὀρθὰς
ἢ AΔ,
καὶ κείσθω
τῆς BA ἴση
ἢ AΔ,
καὶ ἐπεζεύχθω ἢ ΔΓ.

ἐπεὶ ἴση ἐστίν ἢ ΔA τῆς AB,
ἴσον ἐστὶ
καὶ τὸ ἀπὸ τῆς ΔA τετράγωνον
τῶ ἀπὸ τῆς AB τετραγώνῳ.
κοινὸν προσκείσθω
τὸ ἀπὸ τῆς AΓ τετράγωνον·
τὰ ἄρα ἀπὸ τῶν ΔA, AΓ τετράγωνα
ἴσα ἐστὶ
τοῖς ἀπὸ τῶν BA, AΓ τετραγώνοις.
ἀλλὰ τοῖς μὲν ἀπὸ τῶν ΔA, AΓ
ἴσον ἐστὶ
τὸ ἀπὸ τῆς ΔΓ·
ὀρθή γὰρ ἐστίν ἢ ὑπὸ ΔAΓ γωνία·
τοῖς δὲ ἀπὸ τῶν BA, AΓ
ἴσον ἐστὶ

Eğer bir üçgende
bir kenarın üzerindeki kare
eşitse
üçgenin geriye kalan kenarlarındaki
karelere,
üçgenin geriye kalan kenarlarınca içere-
ilen
açı
diktir.

Çünkü, ABΓ üçgeninin
bir BΓ kenarındaki karesi
—varsayalım eşit
BA ve AΓ kenarlarındaki karelere.

İddia ediyorum ki
BAΓ açısı diktir.

Çünkü, çizilmiş olsun
A noktasından
AΓ doğrusuna
dik açılarda
AΔ,
ve yerleştirilmiş olsun
BA doğrusuna eşit
AΔ,
ve ΔΓ birleştirilmiş olsun.

Eşit olduğundan ΔA, AB kenarına,
eşittir
ΔA üzerindeki kare de
AB üzerindeki kareye.
Eklenmiş olsun ortak
AΓ üzerindeki kare;
dolayısıyla ΔA ve AΓ üzerlerindeki
kareler
eşittir
BA ve AΓ üzerlerindeki karelere.
Ama ΔA ve AΓ'ler üzerlerindeki
eşittir
ΔΓ üzerlerindeki;
ΔAΓ açısı dik olduğundan;
ve BA ile AΓ üzerlerindeki

to that on $B\Gamma$;
 for it is supposed;
 therefore the square on $\Delta\Gamma$
 is equal
 to the square on $B\Gamma$;
 so that the side $\Delta\Gamma$
 to the side $B\Gamma$
 is equal;
 and since equal is ΔA to AB ,
 and common [is] $A\Gamma$,
 the two ΔA and $A\Gamma$
 to the two BA and $A\Gamma$
 are equal;
 and the base ΔA
 to the base $B\Gamma$
 [is] equal;
 therefore the angle $\Delta A\Gamma$
 to the angle $B A\Gamma$
 [is] equal.
 And right [is] $\Delta A\Gamma$;
 right therefore [is] $B A\Gamma$.

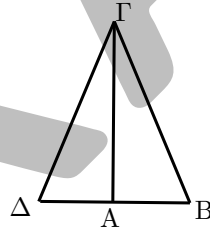
If, therefore, of a triangle,
 the square on one of the sides
 be equal
 to the squares on the remaining two
 sides,
 the angle contained
 by the remaining two sides of the tri-
 angle
 is right;
 —just what it was necessary to show.

τὸ ἀπὸ τῆς $B\Gamma$.
 ὑπόκειται γάρ.
 τὸ ἄρα ἀπὸ τῆς $\Delta\Gamma$ τετράγωνον
 ἴσον ἐστὶ
 τῷ ἀπὸ τῆς $B\Gamma$ τετραγώνῳ.
 ὥστε καὶ πλευρὰ
 ἢ $\Delta\Gamma$ τῆ $B\Gamma$
 ἐστὶν ἴση.
 καὶ ἐπεὶ ἴση ἐστὶν ἢ ΔA τῆ AB ,
 κοινὴ δὲ ἢ $A\Gamma$,
 δύο δὴ αἰ ΔA , $A\Gamma$
 δύο ταῖς BA , $A\Gamma$
 ἴσαι εἰσίν.
 καὶ βάσις ἢ $\Delta\Gamma$
 βάσει τῆ $B\Gamma$
 ἴση.
 γωνία ἄρα ἢ ὑπὸ $\Delta A\Gamma$
 γωνία τῆ $\text{ὑπὸ } B A\Gamma$
 [ἐστὶν] ἴση.
 ὀρθὴ δὲ ἢ ὑπὸ $\Delta A\Gamma$.
 ὀρθὴ ἄρα καὶ ἢ ὑπὸ $B A\Gamma$.

Ἐὰν ἄρα τριγώνου
 τὸ ἀπὸ μιᾶς τῶν πλευρῶν τετράγωνον
 ἴσον ᾖ
 τοῖς ἀπὸ τῶν λοιπῶν τοῦ τριγώνου δύο
 πλευρῶν τετραγώνοις,
 ἢ περιεχομένη γωνία
 ὑπὸ τῶν λοιπῶν τοῦ τριγώνου δύο
 πλευρῶν
 ὀρθὴ ἐστὶν.
 ὅπερ ἔδει δεῖξαι.

are equal
 $B\Gamma$ üzerlerindeki;
 çünkü varsayıldı;
 dolayısıyla $\Delta\Gamma$ üzerlerindeki
 eşittir
 $B\Gamma$ üzerlerindeki kareye;
 böylece $\Delta\Gamma$ kenarı
 $B\Gamma$ kenarına
 eşittir;
 ve ΔA , AB kenarına eşit olduğundan,
 ve $A\Gamma$ ortak,
 ΔA ve $A\Gamma$ ikilisi
 BA ve $A\Gamma$ ikilisine
 eşittirler;
 ve ΔA tabanı
 $B\Gamma$ tabanına
 eşittir;
 dolayısıyla $\Delta A\Gamma$ açısı
 $B A\Gamma$ açısına
 eşittir.
 Ve $\Delta A\Gamma$ diktir;
 diktir dolayısıyla $B A\Gamma$.

Eğer dolayısıyla bir üçgende
 bir kenarın üzerindeki kare
 eşitse
 üçgenin geriye kalan kenarlarındaki
 karelere,
 üçgenin geriye kalan kenarlarınca iç-
 ilen
 açı
 diktir;
 — gösterilmesi gereken tam buydu.



Bibliography

- [1] Euclid, *Euclidis Elementa*, Euclidis Opera Omnia, Teubner, 1883, Edited, and with Latin translation, by I. L. Heiberg.
- [2] ———, *The thirteen books of Euclid's Elements translated from the text of Heiberg. Vol. I: Introduction and Books I, II. Vol. II: Books III–IX. Vol. III: Books X–XIII and Appendix*, Dover Publications Inc., New York, 1956, Translated with introduction and commentary by Thomas L. Heath, 2nd ed. MR 17,814b
- [3] ———, *Euclid's Elements*, Green Lion Press, Santa Fe, NM, 2002, All thirteen books complete in one volume, the Thomas L. Heath translation, edited by Dana Densmore. MR MR1932864 (2003j:01044)
- [4] H. W. Fowler, *A dictionary of modern English usage*, second ed., Oxford University Press, 1982, revised and edited by Ernest Gowers.
- [5] ———, *A dictionary of modern English usage*, Wordsworth Editions, Ware, Hertfordshire, UK, 1994, reprint of the original 1926 edition.
- [6] Homer C House and Susan Emolyn Harman, *Descriptive english grammar*, second ed., Prentice-Hall, Englewood Cliffs, N.J., USA, 1950, Revised by Susan Emolyn Harman. Twelfth printing, 1962.
- [7] Rodney Huddleston and Geoffrey K. Pullum, *The Cambridge grammar of the English language*, Cambridge University Press, Cambridge, UK, 2002, reprinted 2003.
- [8] Wilbur R. Knorr, *The wrong text of Euclid: on Heiberg's text and its alternatives*, *Centaurus* **38** (1996), no. 2-3, 208–276. MR 1384401 (97c:01007)
- [9] ———, *On Heiberg's Euclid*, *Sci. Context* **14** (2001), no. 1-2, 133–143, Intercultural transmission of scientific knowledge in the middle ages: Graeco-Arabic-Latin (Berlin, 1996). MR 1892196 (2003b:01009)
- [10] Henry George Liddell and Robert Scott, *A Greek-English lexicon*, Clarendon Press, Oxford, 1996, revised and augmented throughout by Sir Henry Stuart Jones, with the assistance of Roderick McKenzie and with the cooperation of many scholars. With a revised supplement.
- [11] James Morwood and John Taylor (eds.), *The pocket Oxford classical Greek dictionary*, Oxford University Press, Oxford, 2002.
- [12] Reviel Netz, *The shaping of deduction in Greek mathematics*, *Ideas in Context*, vol. 51, Cambridge University Press, Cambridge, 1999, A study in cognitive history. MR MR1683176 (2000f:01003)
- [13] Allardyce Nicoll (ed.), *Chapman's Homer: The Iliad*, paperback ed., Princeton University Press, Princeton, New Jersey, 1998, With a new preface by Garry Wills. Original publication, 1956.
- [14] Proclus, *A commentary on the first book of Euclid's Elements*, Princeton Paperbacks, Princeton University Press, Princeton, NJ, 1992, Translated from the Greek and with an introduction and notes by Glenn R. Morrow, Reprint of the 1970 edition, With a foreword by Ian Mueller. MR MR1200456 (93k:01008)
- [15] Atilla Özkırmı, *Türk dili, dil ve anlatım [the turkish language, language, and expression]*, İstanbul Bilgi Üniversitesi Yayınları, 2001, Yaşayan Türkçe Üzerine Bir Deneme [An Essay on Living Turkish].
- [16] Herbert Weir Smyth, *Greek grammar*, Harvard University Press, Cambridge, Massachusetts, 1980, Revised by Gordon M. Messing, 1956. Eleventh Printing. Original edition, 1920.
- [17] Ivor Thomas (ed.), *Selections illustrating the history of Greek mathematics. Vol. II. From Aristarchus to Pappus*, Harvard University Press, Cambridge, Mass, 1951, With an English translation by the editor. MR 13,419b
- [18] Henry David Thoreau, *Walden [1854] and other writings*, Bantam Books, New York, 1962, Edited and with an introduction by Joseph Wood Krutch.