Transfinite diameter, Chebyshev constants and capacities in $\mathbb{C}^n$

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One of the fundamental results in complex analysis is the classical result of the first third of 20th century (Fekete, Szegö et al) which says that *for any compact set* $K \subset \mathbb{C}$ *the transfinite diameter* $d(K)$, *the Chebyshev constant* $\tau(K)$ *and the capacity* $c(K)$ *coincide*, although they are defined from very different points of view. Indeed, the transfinite diameter derives from a geometrical approach as the *limit of geometrical means of some extremal distances between points of* K :

$$\begin{aligned} d(K) &:= \lim_{s\to\infty} d_s(K), \\ d_s(K) &:= \left(\max\left\{\left|\det\left(z_j^{k-1}\right)_{k,j=1}^{s}\right| : z_j \in K\right\}\right)^{2/s(s-1)}; \end{aligned} \tag{1}$$

the Chebyshev constant is defined in *terms of the least uniform deviation of monic polynomials from zero*:

$$\tau(K) := \lim_{s\to\infty} \inf\left\{\max_{z\in K}\left|z^s + \sum_{j<s} c_j\ z^j\right| : c_j \in \mathbb{C},\ j = 1,\dots,s-1\right\}^{1/s},$$

while the capacity appears from the *potential theory* considerations:

$$c(K) := \exp(-\lambda(K)),\ \lambda(K) := \lim_{z\to\infty}(g_K(z) - \ln|z|)$$

where $g_K(z)$ is the Green function for K with a logarithmic singularity at ∞.

For a compact set K in $\mathbb{C}^n$, the transfinite diameter was introduced by F. Leja in 1957: $d(K) := \limsup_{s\to\infty} d_s(K)$, where $d_s(K)$ is determined analogously to (1). He posed a problem whether there exists the usual limit in his definition. This problem has been solved positively by the speaker in 1975, it was shown also that the *Leja transfinite diameter* coincides with, so-called, *principal Chebyshev constant,* which is expressed as a continual geometric mean of *directional Chebyshev constants.*

In my talk I give a survey of results concerned with the above notions for several complex variables (Bloom, Bos, Levenberg, Calvi, Zeriahi, Rumely, Lau, Varley, Berman, Boucksom, Nystrom et al) and discuss a new approach to the notions of the transfinite diameter and Chebyshev constants on Stein manifolds.