

***m*-potent elements in order-preserving transformation semigroups  
and ordered trees**

Joint work by Orhan Sönmez and Yusuf Ünlü

Let  $\mathcal{C}_{n,r}$ ,  $\mathcal{W}_{n,r}$ , and  $\mathcal{U}_{n,r}$  be the sets of *s*-potent elements  $\alpha$  in  $\mathcal{O}_n$ , the semigroup of order-preserving mappings, such that  $Fix(\alpha) = \{1\}$ ,  $|Fix(\alpha)| = 1$ , and  $Fix(\alpha)$  is arbitrary, respectively. We give combinatorial results relating the cardinalities  $|\mathcal{C}_{n,r}|$ ,  $|\mathcal{W}_{n,r}|$ , and  $|\mathcal{U}_{n,r}|$ . We construct a correspondence between  $\mathcal{C}_{n,r}$  and  $T_{n,r}$  the ordered trees with depth at most *r* and *n* nodes. Using this we produce generating functions for  $|\mathcal{W}_{n,r}|$ ,  $|\mathcal{U}_{n,r}|$ .